PROCEEDINGS of the
23rd International Congress on Acoustics

9 to 13 September 2019 in Aachen, Germany

Acoustic transfer admittance of cylindrical cavities in infrasonic frequency range

Paul VINCENT(1),(2), Dominique RODRIGUES(2), Franck LARSONNIER(1), Cécile GUIANVARC'H(2), Stéphane DURAND(4)

(1)Commissariat à l'Énergie Atomique (CEA), France
(2)Laboratoire Commun de Métrologie LNE-CNAM (LCM), France
(3)Laboratoire d'Acoustique de l'Université du Maine (LAUM), France

Abstract

Demand for calibration at infrasonic frequencies has emerged in response to earth monitoring problems. The primary standard for sound pressure is defined through the reciprocity calibration method specified in the International Electrotechnical Commission (IEC) Standard 61094-2:2009. This method is based on the use of closed couplers and is routinely applied by the National Metrology Institutes for a large frequency range; however, infrasonic frequencies below 2 Hz have not been explored until recently. The acoustic transfer admittance of the coupler, including the heat conduction effects of the fluid, must be modelled precisely to obtain accurate microphone sensitivity. IEC 61094-2:2009 provides two standardised solutions for the correction of heat conduction. However, researchers have noted significant deviations between these corrections at low frequencies in plane wave couplers, indicating that one or both techniques incorrectly calculate the influence of heat conduction. In this paper, two alternative solutions are proposed. An experiment is also reported, which highlights the limitations of the standardised formulations for acoustic transfer admittance, while also demonstrating the validity of the proposed alternative formulations at frequencies down to 0.04 Hz.

Keywords: Infrasound, Calibration, Microphones, Reciprocity, Admittance

1 INTRODUCTION

The pressure reciprocity calibration method as specified in the International Electrotechnical Commission (IEC) Standard 61094-2:2009 [3] is currently used worldwide for absolute pressure calibration of laboratory standard microphones, and provides the basis for primary measurement standards for sound pressure. This method, which is based on the use of closed couplers, is routinely applied by the National Metrology Institutes at frequencies of up to 25 kHz and, recently, down to 2 Hz [4]. While the reciprocity method has been used for a long time to determine microphone pressure sensitivity over an extended audible frequency range, the demand for calibration at infrasonic frequencies below 2 Hz has not been identified until recently. This is revealed by the absence of Calibration and Measurement Capabilities (CMCs) in the Bureau International des Poids et Mesures (BIPM) database [7] for frequencies below 2 Hz, except for static pressures, for which CMCs have been obtained using specific techniques such as the pressure balance [13] (see Figure 1).

To perform calibration at infrasonic frequencies, the validity and performance of the pressure reciprocity method must be examined for this frequency range. In the most usual configuration, the pressure reciprocity method requires three reciprocal microphones coupled by pairs using a cavity, generally with a cylindrical shape. The coupler ends are closed by the microphone diaphragms, with one being used as a transmitter and the other one as a receiver. The product of the microphone sensitivities is determined from electrical measurements and from analytical calculation of the acoustic transfer admittance of the system. This operation is repeated with three microphone couples.

---

(1)paul.vincent@cea.fr

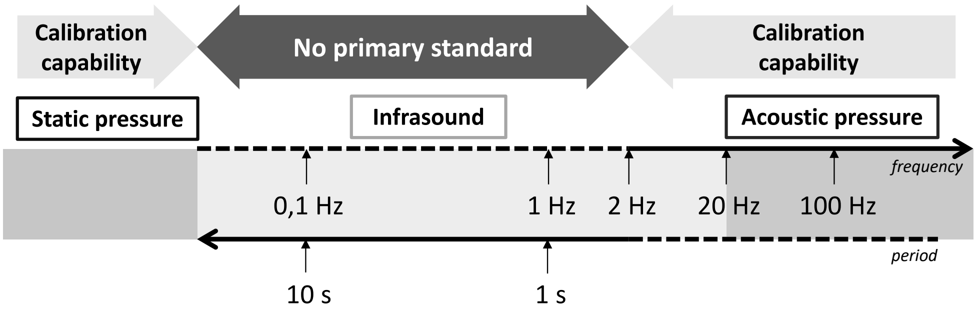

Figure 1. CMC status from static pressure range to acoustic pressure range.

Calculation of the acoustic transfer admittance is a key aspect of microphone pressure reciprocity calibration. The acoustic transfer admittance, defined as the ratio of the short-circuit volume velocity produced by the transmitter microphone to the sound pressure acting on the diaphragm of the receiver microphone has been extensively explored and discussed considering both influence of heat conduction and viscous losses [5, 6, 8–12, 14, 16–18]. In particular, the effects of heat conduction is an important issue of the calculation of the acoustic transfer admittance especially in small closed volumes and at low frequencies where the expansion and compression processes of the gas are somewhere in between an isothermal process and adiabatic process, or said to be polytropic process. The IEC Standard 61094-2:2009 [3] provides two formulations for calculation of the acoustic transfer admittance. These formulations have been revised in the context of the above problem since the first edition, IEC 327 [2], which was published in 1971. However, significant behavioural differences between the standardised models at very low frequencies have recently been highlighted; these discrepancies yield inconsistent calibration results [11, 14].

With the objective of achieving an acoustic primary standard in the infrasonic frequency range, the second section of this paper proposes alternatives and reports an experimental study in the frequency range of 0.04–100 Hz. A detailed presentation of the measurement setup and methodology is provided, and the experiment results are reported and discussed.

2 ACOUSTIC TRANSFER ADMITTANCE: MODEL PRESENTATION

The IEC Standard 61094-2:2009 [3] specifies the requirements for pressure reciprocity calibration of laboratory standard (LS) microphones and includes models for calculating the acoustic transfer admittance of cylindrical couplers. In Appendix A of the standard, two formulations for correcting heat conduction under polytropic conditions are presented:

1. the *'broadband solution'*, which considers both thermal and viscous effects in plane wave couplers and is applicable to higher frequencies;
2. the *'low-frequency solution'*, which considers only thermal effects for cylindrical coupler assuming uniform pressure, and which is based on a solution presented by Gerber [9].

Recently, Jackett [11] highlighted significant deviations between these models at low frequencies for plane wave couplers, indicating that one or both models incorrectly calculate the influence of heat conduction. To realise a primary standard for the infrasonic frequency range, validation of an appropriate acoustic modelling technique appears to be an essential preliminary step. The limitations of the normalized models are reported in [19].

Two new theoretical formulation are also presented in [19]: the *general alternative low-frequency solution* and the *short-term alternative low-frequency solution*.

The acoustic transfer admittance of cylindrical couplers Y_a for these models is given by

$$Y_a = \frac{j\omega V}{\gamma P_0}\left[\gamma - (\gamma - 1)E_P\right] + Y_r + Y_t\,, \tag{1}$$

with V the coupler volume, ω the angular frequency, γ the ratio of the specific heat capacities, P_0 the static pressure under measurement conditions, Y_r and Y_t the acoustical admittances of receiver and transmitter microphones, respectively. For the *general alternative low-frequency solution* [19], E_P is given by

$$\begin{gathered} E_P = \sum_{m=0}^{+\infty}\sum_{n=1}^{+\infty}\left[\frac{8/\pi^2}{(m+1/2)^2\lambda_n^2}F_{m,n}\right],\ \text{with}\ F_{m,n} = \left(1 + \frac{\lambda_n^2R^2 + (m+1/2)^2\pi^2}{(1+2R)^2}X_P^2\right)^{-1}, \\ X_P = \frac{A}{V}\frac{1-j}{\sqrt{2}}\sqrt{\frac{\alpha_t}{\omega}}\,, \end{gathered} \tag{2}$$

where λ_n corresponds to the zeros of integer-order $J_n(x)$ Bessel function, A is the total internal area of the cylinder surface (length ℓ, radius a), $R = \ell/(2a)$ and α_t the thermal diffusivity of the enclosed gas. A short-term Laplace asymptotic development of the previous general solution can also be obtained, which gives the *short-term alternative low-frequency solution* [19]:

$$E_P = 1 - X_P + \frac{\pi R^2 + 8R}{\pi(2R+1)^2}X_P^2 + \frac{3}{4}\sqrt{\pi}\frac{R^3 - 6R^2}{3\sqrt{\pi}(2R+1)^3}X_P^3\,. \tag{3}$$

3 VALIDITY TEST FOR ACOUSTIC TRANSFER ADMITTANCE FORMULATIONS

3.1 Methodology

The experimental protocol implemented to test the accuracy of the above formulations for the acoustic transfer admittance was derived from [15] and was based on the pressure reciprocity method. Figure 2 presents an overview of the measurement system. Two electrical transfer impedances (defined as the ratio of the open-circuit voltage u_{r0} of the receiving microphone to the current i_t through the transmitter microphone) were measured for a pair of microphones using two cavities of different lengths, hereafter referred to as the short and long cavities.

The products of the sensitivities M_r and M_t of the receiver and transmitter microphones, respectively, are given by the well–known equations

$$M_tM_r|_s = Z_{e,s}Y_{a,s}\,,\ \text{and}\ M_tM_r|_\ell = Z_{e,\ell}Y_{a,\ell}\,, \tag{4}$$

for the short and long cavities (subscripts s and ℓ), respectively. Here, $Y_{a,(s,\ell)}$ are the previously defined and discussed acoustic transfer admittances of the cavities. By considering the microphones as stable during the experiment, the products of the sensitivities $M_tM_r|_s$ and $M_tM_r|_\ell$ should be invariant as functions of the cavity, insofar as the models of the acoustic transfer admittances are perfectly valid. The objective of the experiment was to test this validity. Therefore, the error estimator δ_m was defined as the ratio

$$\delta_m = \frac{M_tM_r|_s}{M_tM_r|_\ell} = \frac{Z_{e,s}Y_{a,s}}{Z_{e,\ell}Y_{a,\ell}}\,. \tag{5}$$

This ratio should tend towards unity (or 0 dB) for a perfect model of the acoustic transfer admittances $Y_{a,(s,\ell)}$. Otherwise, the estimated M_tM_r depends on the cavity dimensions, so the model is invalid.

During a reciprocity calibration, the electrical transfer impedance is measured using the insert voltage technique [3] to determine the u_{r0} of the receiver microphone. The current i_t through the transmitter microphone is deduced from the voltage developed across a series-connected capacitor $u = i_t/(j\omega C)$, knowing the value of C (Figure 2). Thus, the electrical transfer impedance is measured based on two voltage ratios, as follows:

$$Z_e = \frac{-1}{j\omega C}\frac{u_r}{u_t}\frac{u_t'}{u_r'}\,, \tag{6}$$

where u_r/u_t and u'_t/u'_r are the ratios of voltages measured at the outputs of the microphone power supply of the receiver (u_r and u'_r) and transmitter (u_t and u'_t) microphones, respectively, during the main measurement phase and voltage insertion phase.

A defined measurement process with two cavities is implemented by fixing the following variables: (a.) the microphone, preamplifier, and conditioner combinations for the receiver and transmitter; (b.) the respective settings of the conditioners, assuming that both measurement channels are stable.

Here, the error estimator δ_m is given by

$$\delta_m = \frac{u_{r,s}/u_{t,s}}{u_{r,\ell}/u_{t,\ell}} \frac{Y_{a,s}}{Y_{a,\ell}}\,, \tag{7}$$

where the subscripts s and ℓ represent the short and long cavities, respectively, in the voltage ratios. Note that this simplification of the measurement process is of interest in the context of infrasound measurement, as these measurements are time-consuming.

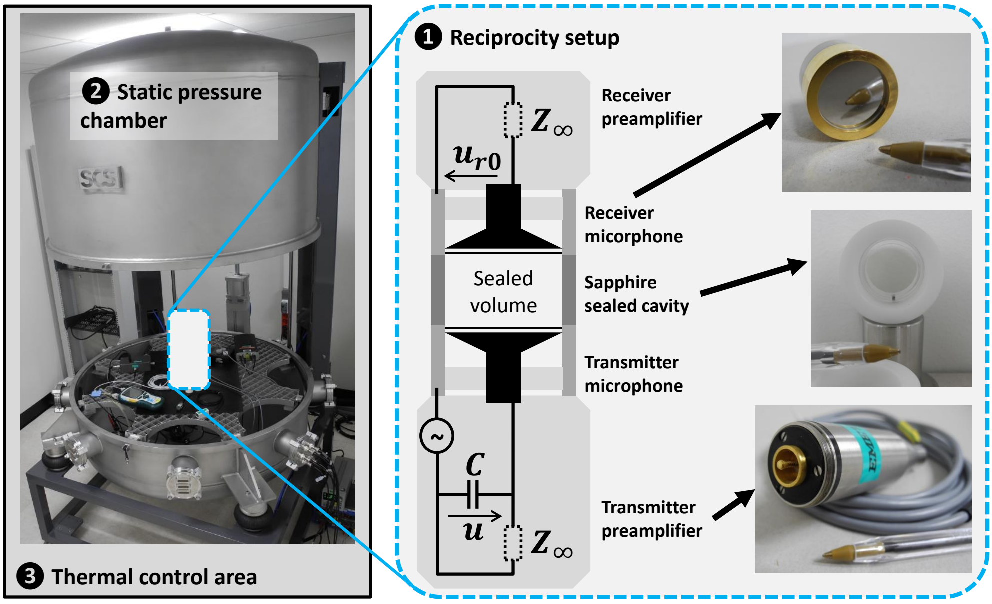

Figure 2. Measurement setup: ❶ reciprocity setup with microphones, preamplifiers, and sealed sapphire cavity; ❷ static pressure chamber; ❸ thermal control area.

3.2 Measurement Setup

As it was essential to perform the experiment in the infrasonic frequency range, *i.e.*, from 0.04 Hz to 100 Hz, some changes were required to the measurement setup employed in this study. The measurements were performed inside a regulated static pressure chamber (Figure 2) installed in a laboratory with a dedicated thermally controlled area. This controlled environment was required to avoid microphone instability due to static pressure and temperature changes. Note that this is particularly important for measurements at infrasonic frequencies for which very long integration times are required.

The reciprocity system was composed of two microphones and their preamplifiers, which were sequentially coupled by two sapphire cavities. The transmitter and receiver microphones were two B&K Type 4160 1 inch microphones, which are usually used for pressure reciprocity calibration (LS1p microphones). Two cavities were especially designed and manufactured for the purposes of this study: a short (6 mm long) and long (10 mm long) cavity. Their diameters fit the microphone membranes (18.6 mm) (see Figure 2).

The cavity lengths were chosen to be sufficiently different to allow measurement of the deviation between the thermal corrections $\gamma-(\gamma-1)E_p$ incorporated in (1) for both cavities. For these cavity lengths, the deviation of the thermal corrections reached 0.3 dB in the isothermal-adiabatic transition frequency range; this could be measured with the given reciprocity system accuracy.

The receiver microphone was connected to a B&K 2669-L-004 preamplifier. The transmitter microphone was connected to a specific preamplifier designed and manufactured for the purposes of this study (Figure 2). The latter had a cut-off frequency of approximately 0.005 Hz given by a 500 GΩ polarisation resistor, with addition of a 100 pF capacitance in parallel with the microphone. The output preamplifiers were connected to a 4 channel B&K Type 2829 microphone power supply. This conditioner was modified by bypassing the high-pass filter. The signals were digitised by a VTI Instruments CMX09 chassis and an EMX4350 digitiser card. The digitising system had a negligible noise level compared to that of the signal to be measured. The amplitude and phase of the signals were computed using a standardised method given in [1].

To avoid acoustic short-circuiting and to obtain a sufficiently high signal-to-noise ratio at lower frequencies, special attention was paid to the sealing of the reciprocity system. That is, the back cavity vents of both microphones were sealed and the cavities were designed with gaskets to ensure optimal sealing conditions.

Another reason for sealing the microphones is to simplify the modelling of the microphone acoustic admittances, which is required for all acoustic transfer admittance formulations. Consequently, complex modelling [14] of the microphone vent effects at low frequencies is not required, which places the experiment focus on validation of the thermal effects on the acoustic transfer admittance only. Given the frequency range of interest (lower than 100 Hz), the microphone admittance (Y_t and Y_r) is given in its simplest form [3] by

$$Y_{r,t}=\frac{j\omega V_{eq,(r,t)}}{\gamma_{ref}P_{ref}}, \tag{8}$$

where $V_{eq,(r,t)}$ is the equivalent volume of the microphones, and γ_{ref} and P_{ref} are the specific heat ratio and static pressure at reference environmental conditions, respectively. For a fully rigorous discussion, is should be noted that the back cavity of the microphone is also subject to thermal effects and its equivalent volume should be dependent on the frequency when the acoustic behaviour is no longer adiabatic. However, in the experiment conducted in this study, the equivalent volumes of the LS1p microphones were much lower than the volume of the smallest cavity ($V/V_{eq,(r,t)} \approx 20$). Therefore, these complex effects were assumed to be negligible.

3.3 Measurement processing

As the cavities and microphones are necessarily sealed, the local environmental variations inside the reciprocity system (*i.e.*, those of the back cavities of the microphones and coupler) have an important effect on its stability. This is true even if the environment inside the static pressure chamber is controlled.

To overcome this problem, a specific measurement process was implemented, explained in [19].

4 Results and discussion

Figure 3 shows the error estimator δ_m as defined in (7) as a function of frequency, for the acoustic transfer admittances derived from the formulations discussed. The uncertainties are mainly due to the repeatability process. Analysis of the error estimator δ_m results yields the following findings:

1. If δ_m does not tend towards zero (unit: dB), the estimated M_tM_r depends on the cavity dimensions. Thus, the formulation of the acoustic transfer admittance is invalid.
2. If δ_m tends towards zero (unit: dB), the estimated M_tM_r does not depend on the cavity dimensions. Thus, the formulation of the acoustic transfer admittance is valid.
3. Other unknown and unaccounted for effects somehow compensate for each other by coincidence, for the chosen coupler sizes. However, such possibility appears to be unlikely.

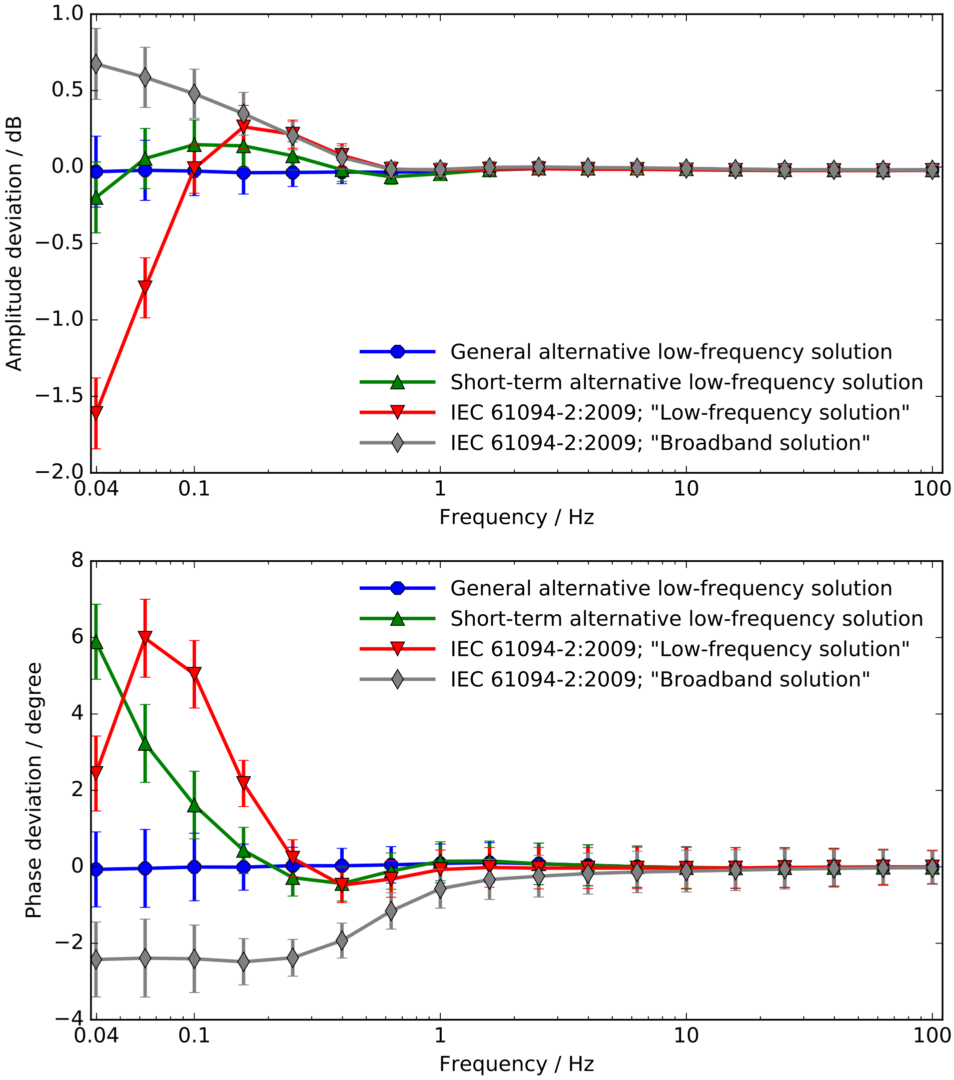

Figure 3. Amplitude (unit: dB, upper graph) and phase (unit: degrees, lower graph) of error estimator δ_m as function of frequency for four different acoustic transfer admittance formulations.

It is worth noting that, for case (2.), the formulation can be considered as valid provided the cavity lengths are sufficiently different that the effects under investigation (here, the heat conduction effects) can be measured. This hypothesis was verified in the present study (see Section 3.2).

It is clearly apparent from Figure 3 that, among the studied formulations, the *general alternative low-frequency solution* (1) is the unique valid model in the targeted frequency range (for amplitude and phase). It is also reminded here that the results for the phase were obtained without applying the environmental correction process described in the previous section. These results are comparable to those obtained for the amplitude where the correction process has been applied. The polytropic condition that occurs in the frequency range of 0.1–10 Hz (for these cavity dimensions) was well corrected by this formulation. The result provided by the *general alternative low-frequency solution* was calculated using $(m,n) = (100,100)$ for E_P in (2); convergence study shows results within 0.01 dB for $(m,n) = (17,17)$ at 100 Hz and $(m,n) = (2,2)$ at 0.04 Hz.

As expected, the *short-term alternative low-frequency solution* (3) provided a better result than the standardised 'low-frequency solution' and 'broadband solution'. These results highlight the limitations of the current standardised formulations of acoustic transfer admittance for the purpose of microphone infrasound calibration.

To obtain information on the possible error of microphone calibration using the pressure reciprocity technique, a method is to present the acoustic transfer admittances relative to the *general alternative low-frequency solution*, taken as a reference. As an example, for the standardised 'low-frequency solution', the possible errors in the acoustic transfer admittance, and thus, in M_tM_r reached 0.1 dB and 0.5 degrees at 1 Hz, and up to 3 dB and 30 degrees at 0.04 Hz. The error in the sensitivity estimation was potentially half these values. Therefore, traceability to the International System of Units (SI) for current calibrations is possibly incorrect.

5 CONCLUSION

The main motivation of this study was to perform groundwork for future primary calibration of microphones in the infrasonic frequency range. Therefore, it was essential to verify the validity of the acoustic transfer admittance formulations for cylindrical cavities at infrasonic frequencies, which are currently standardised and used for primary reciprocity calibration of microphones.

An experiment performed to test the validity of the formulations discussed in this paper clearly indicated that the *general alternative low-frequency solution* (1) is the only valid model among the studied formulations in the targeted frequency range (for amplitude and phase). From the experiment results, it was also concluded that the *short-term alternative low-frequency solution* (3) yields lower errors than the standardised solutions. Finally, the experiment highlighted the limitations of the current standardised formulations of acoustic transfer admittance for infrasound calibration of microphones.

In conclusion, the models quoted in IEC Standard 61094-2:2009 are not suitable at low frequencies. The following recommendations can be made for future revision of the IEC standard:

1. The current standardised 'low-frequency solution' should be modified by the *short-term alternative low-frequency solution* as defined in (1) and (3), as the validity of the former solution is limited at low frequencies by the asymptotic development of the general formulation of E_P, presented in (2).
2. At lower frequencies, where the previous solution is no longer valid, the *general alternative low-frequency solution* should be implemented, as defined in (1) and (2).

The findings of this work have implications for calibration of infrasound sensors, which is particularly important for earth monitoring applications. It would be advisable that in the near future, calibrations of sensors at infrasonic frequencies through reciprocity method as well as others methods based on closed couplers, such as the laser pistonphone, take into consideration these recommendations.

REFERENCES

[1] IEEE. 1057:2017. Standard for digitizing waveform recorders. Standard, IEEE, 2017.

[2] IEC. 327:1971. Precision method for pressure calibration of one-inch standard condenser microphones by the reciprocity technique. Standard, IEC, 1971.

[3] IEC. 61094-2:2009. Measurement microphones-Part2: Primary method for the pressure calibration of laboratory standard microphones by the reciprocity method. Standard, IEC, 2009.

[4] J. Avison and R. Barham. Final report on key comparison CCAUV.A-K5: pressure calibration of laboratory standard microphones in the frequency range 2 Hz to 10 kHz. *Metrologia*, 51(1A):09007, 2014.

[5] K. Ballagh. Acoustical admittance of cylindrical cavities. *Journal of Sound and Vibration*, 112(3):567–569, 1987.

[6] F. Biagi and R. Cook. Acoustic impedance of a right circular cylindrical enclosure. *Journal of the Acoustical Society of America*, 26(4), 1954.

[7] BIPM. *Acoustics, Ultrasound and Vibration - Calibration and measurement capabilities*, 2018 (accessed June 8, 2018).

[8] F. Daniels. Acoustical impedance of enclosures. *The Journal of the Acoustical Society of America*, 19(4):569–571, 1947.

[9] H. Gerber. Acoustic properties of fluid-filled chambers at infrasonic frequencies in the absence of convection. *Journal of the Acoustical Society of America*, 36(8):1427–1434, 1964.

[10] C. Guianvarc'h, J.-N. Durocher, M. Bruneau, and A.-M. Bruneau. Acoustic tranfer admittance of cylindrical cavities. *Journal of Sound and Vibration*, 292, 2006.

[11] R. Jackett. The effect of heat conduction on the realization of the primary standard for sound pressure. *Metrologia*, 51(5):423, 2014.

[12] D. Jarvis. Acoustical admittance of cylindrical cavities. *Journal of Sound and Vibration*, 117(2):390–392, 1987.

[13] G. Molinar *et al.* CCM key comparison in the pressure range 0.05 MPa to 1 MPa (gas medium, gauge mode). Phase A1: Dimensional measurements and calculation of effective area. *Metrologia*, 36(6):657, 1999.

[14] E. Olsen and E. Frederiksen. Microphone acoustic impedance in reciprocity calibration of laboratory standard microphone. *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, 247(8):679–686, 2013.

[15] K. Rasmussen. Radial motion-wave in cylindrical plane-wave couplers. *Acta acustica*, 1:145–151, 1993.

[16] K. Rasmussen. Datafiles simulating a pressure reciprocity calibration of microphones. *EUROMET Project 294 - Report*, PL-13a, 2004.

[17] P. Riéty and M. Lecollinet. Le dispositif d'étalonnage primaire des microphones de laboratoire de l'institut national de métrologie. *Metrologia*, 10(1):17, 1974.

[18] A. Svete *et al.* Investigation of polytropic corrections for the piston-in-cylinder primary standard used in dynamic calibrations of pressure sensors. *Sensors and Actuators A: Physical*, 274:262 – 271, 2018.

[19] P. Vincent, D. Rodrigues, F. Larsonnier, C. Guianvarc'h, and S. Durand. Acoustic transfer admittance of cylindrical cavities in infrasonic frequency range. *Metrologia*, 56(1):015003, 2019.