Global sensitivity analysis of elastomer joints between substructures using random balance design method

Dooho LEE¹; Young-Woo WON²

¹ Dongeui University, Republic of Korea
² Dongeui University, Republic of Korea

ABSTRACT
Plug-In Digital Framework is a system response analysis tool when the system components are composed of black-box modules. Generally, the dynamic characteristics of joints between the system components affect largely on system responses, and have displacement- and frequency-dependent stiffness and loss factor. Thus, the sensitivity of each joint parameters should be estimated in global sense. In this study, we introduce a global sensitivity analysis procedure under the Plug-In Digital Framework. To efficiently calculate the system responses, the FRF-based substructuring method is introduced. Global first-order sensitivity is obtained from the analysis of variance for system responses calculated at randomly sampled points. The random sample points are generated by the random balance design (RBD) which uses equal space sampling in cumulative distribution function for a parameter and random permutation. The Fourier transform for the sorted responses for a parameter provides the global first-order sensitivities of each joint stiffness and loss factor. The proposed global sensitivity analysis method are applied to an interior noise problem of a passenger car and the efficiency of the global sensitivity analysis method is discussed.

Keywords: Global Sensitivity, Substructuring, Joint

INCE Classification of Subjects Number(s): 75.4

1. INTRODUCTION
Numerical models are widely utilizing in predicting responses such as vibration amplitudes and noise levels in vibro-acoustic systems. Generally, a structural system consists of many substructures and changes of dynamic characteristics in a substructure can affect the system responses. Plug-In Digital Frameworks is a system response analysis tool when the system components are composed of black-box modules (1).

Elastomer joints are very crucial components in preventing from the transmission of vibrational power to other substructures. Sensitivity analysis of the dynamic properties of elastomer joints on system responses can provide information on the relative importance to the system responses (2, 3). However, the dynamic characteristics of elastomers usually show nonlinear behaviors: i.e. the dynamic properties are dependent on displacement and frequency. In those cases, a global sensitivity analysis is required to consider the uncertainties of the nonlinear behaviors (4). Among the global sensitivity methods, variance-based methods provide the global sensitivity information from randomly sampled responses using, for example, the Monte Carlo simulation. To apply the sampling-based sensitivity methods for a large vibro-acoustic numerical model, a reduced degree-of-freedom (DOF) model is necessary due to high computational costs. The frequency response function (FRF)-based substructuring method is a good solution to build a reduced model in vibro-acoustic problems which consist of many substructures and joints.

In this paper, we introduce a global sensitivity analysis method for an elastomer joint in vibro-acoustic systems of which material properties are nonlinear and have uncertainty. Section 2 explains the FRF-based substructuring formulation for a reduced system response model. Additionally, distributions represent nonlinear material properties of elastomer joints which are also frequency-

¹ dooho@deu.ac.kr
² whittm@nate.com
dependent. Section 3 describes the variance-based global sensitivity analysis method. The proposed method are applied to an engine mount problem in Section 4. Section 5 concludes the proposed method.

2. REDUCED VIBRO-ACOUSTIC MODEL

2.1 FRF-Based Substructuring Method

Substructure synthesis methods can reduce the DOFs of a vibro-acoustic problem. Amongst many substructuring techniques, the FRF-based substructuring method is used in this study. The FRF-based substructuring method represents the dynamic characteristics of a substructure in terms of the FRFs of the substructure from whether a numerical model or experimental measurements. We explains briefly the FRF-based substructuring method. For details, refer to Ref. (2).

Consider a vibro-acoustic system which consists of \( n \) substructures and elastomer joints as shown in Fig. 1(a). On the substructures, external harmonic forces \( f^k \) exert. Then, the harmonic displacement on \( i \)-th joint of arbitrary \( k \)-th substructure is written with the notations shown in Fig. 1(b).

\[
x_i^k = \sum_{j=k, j\neq k}^{n} H_{ij}^k \cdot R_{ij}^k + H_{ii}^k \cdot f^k, \quad i = 1, \ldots, n, \ i \neq k
\]

where \( H^k \) and \( R^k \) represent the FRFs between two points and internal force acting on an elastomer, respectively. The elastomer which connects two substructures is represented by massless spring with structural damping. Then, the internal forces and the displacements on the joint boundary must satisfy the action-reaction and force-displacement relations. Substituting these relations into Eq. (1) provides the linear algebraic equations in which one can obtain the internal forces acting on the elastomers.

\[
H \cdot R = Q
\]

\[
R = \{ R_1^k, R_2^k, \ldots, R_{n-2}^k, R_{n-1}^k \}^T
\]

Here, \( H \) is a square matrix which consists of FRFs assembled according to the connecting relations of the substructures and has \( n(n-1)/2 \times n(n-1)/2 \) dimensions. \( Q \) is a known force vector. Solving Eq. (2) at each frequency, all external forces acting on a substructure are known. Thus, one can calculate any vibro-acoustic responses in a substructure with known FRFs using the superposition principle as follows.

\[
x_i^k = \sum_{i=k, i\neq k}^{n} H_{ii}^k \cdot R_{ii}^k + H_{ii}^k \cdot f^k
\]

Preparing all substructural FRFs needed for the FRF-based substructuring method, the most time consuming calculations occur in Eq. (2) to calculate the unknown internal forces acting on the elastomers. However, the dimensions of the linear equation is relatively tiny compared to the whole system model, which makes the FRF-based substructuring method competitive as a reduce model for...
the sampling-based global sensitivity analysis.

2.2 Representation of Elastomer Joints

In a vibro-acoustic system with elastomer joints, a linear model for harmonic responses in frequency domain gives good approximation of the physical system in almost cases. However, the equivalent stiffness and loss factor of elastomers are dependent upon the displacement amplitudes as well as the frequency. For the global sensitivity analysis, the variability of input parameters has to be quantified. The material properties of elastomer joints which are represented as a spring and loss factor have variations according to the amplitudes of harmonic response as shown in Fig. 2. In addition, the material properties have uncertainty even under the same amplitude due to the intrinsic randomness of the elastomers. Thus, both the variation of the amplitudes and the variation of the elastomers contribute on the variability of the material properties as shown in Fig. 2. In this study, a cumulative distribution function (CDF) describes the variability of input parameter for the global sensitivity analysis.

Figure 2 – Representation of variability for the stiffness of an elastomer

3. GLOBAL SENSITIVITY ANALYSIS

3.1 Variance-Based Global Sensitivity Analysis

Response of interest (ROI), $Y$, in a vibro-acoustic system varies with changes of input parameters $X_k (k=1, \ldots, q)$. The sampling-based methods generates random samples in the input parameter space and calculate the ROIs at each combination of the input parameters. The global first-order sensitivity of $i$-th input parameter on an ROI can be defined as the variance of the conditional expectation $E(Y|X_i)$ normalized by the overall variance as follows (4):

$$S_i = \frac{V(E(Y|X_i))}{V(Y)}$$

where $V$ refers to the variance. In the variance-based sensitivity analysis, uniformly sampled points over the input parameter space are very important to provide accurate global sensitivity information.

3.2 Random Balance Design Method

Extending the Fourier amplitude sensitivity test, Tarantola et al. proposed the random balance design (RBD) method (5). In the RBD method, the input parameters are sampled in the CDF domain with a frequency $\omega$. To sample evenly over whole input parameter space, a variable $s$ is defined on $[- \pi, +\pi]$ and $N$ points are generated with equal intervals on the $s$-axis: $s(k), k=1, \ldots, N$. Then, a random permutation for each input parameter determines the sampling sequence for the input parameter: i.e. $k$-th point of $i$-th input parameter is determined as follows:
\[ X_i(k) = G_i(0.5 + \frac{1}{\pi} \sin^{-1} \left[ \sin(\omega x_i(k)) \right]), \quad i = 1, \ldots, q, \quad k = 1, \ldots, N \]  

(6)

where \( P_i \) and \( G_i \) refer to the random permutation and the CDF for \( i \)-th input parameter. A numerical model provides the ROIs at the sampled \( N \) points as:

\[ Y(k) = Y(X_i(k), X_{i+1}(k), \ldots, X_q(k)), \quad k = 1, \ldots, N \]  

(7)

To calculate the global sensitivity of \( i \)-th input parameter, the ROI sequence is reordered such that the corresponding sequence of \( X_i \) (\( k=1, \ldots, N \)) is to be increasing one. The Fourier transform of the reordered ROI sequence \( Y_R(k) \) provides the frequency spectrum of the ROI with respect to the harmonic variation of the input parameter. The squared amplitude of the Fourier spectrum is written as:

\[ F_i(\omega) = \left| \frac{1}{\pi} \sum_{k=1}^{N} Y_R(k) \exp(-j\omega k) \right|^2 \]  

(8)

Then, the conditional variance of the ROI is the summation of the squared amplitude up to higher harmonics as follows.

\[ V_i = V(E(Y|X_i)) = \sum_{\omega=1}^{M} F_i(\omega) \]  

(9)

Repetitions of Eqs. (8)~(9) provide the global sensitivity for other input parameters. It should be noted that the global sensitivity calculation for the other input parameters does not require the recalculation of the ROIs besides reordering of the ROIs and the Fourier transforms. In this study, \( M \) is set to 6.

4. APPLICATION TO AN ENGINE MOUNT PROBLEM

The proposed global sensitivity analysis method was applied to calculate the influences of engine mounts and the bushes on the interior noise in a passenger car. The passenger car has 4 engine mounts and 6 bushes to support 4 substructures as shown in Fig. 3. In Fig. 3, substructure A is the engine, B the center member, C the cross member, and D the trimmed body structure.

To simulate the interior noise level in the cabin cavity, the FBS model developed in Refs. (2, 6) was used in this study. The ROI was the graph area of the interior noise of which level is beyond 80 dB over from 33 Hz to 134 Hz range.

The input parameters selected were the stiffnesses of the engine mounts and the bushes for all directions, which leads to 30 input parameters in total. Table 1 shows the uncertainties of the input parameters used in the global sensitivity analysis. The RBD method provided \( N(=10,000) \) sampled points from the distributions of Table 1. The FBS model of the passenger car calculated the ROIs at the sampled points. It is noticeable that the computational time for an ROI is tiny in spite of vibro-acoustic full vehicle model, for example, 0.227 second in a PC workstation (HP Z600) with two Intel Xeon QC CPUs under MATLAB environment. Equations (5) and (9) provided the global sensitivity information for the engine mounts and the bushes. Figure 4 shows the global sensitivity analysis results for the passenger car problem. Figure 4 clearly illustrates that two stiffness components in the \( z \)-direction have dominant influences on the ROI, which also shows that the proposed global sensitivity analysis is very effective in parameter prioritization. Figure 5 shows a scatter plot for the most dominant input parameter, \( X_9 \). Figure 6 illustrates the correlation between the most dominant two input parameter.
Table 1 – Uncertainty of the input parameters

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<th>Joints No</th>
<th>Lower Bound (%)</th>
<th>Upper Bound (%)</th>
<th>Distribution type</th>
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Figure 4 – Global sensitivity analysis results for the passenger car problem

Figure 5 – Scatter plot between the input parameter X9 and the ROI
5. CONCLUSIONS

In this study, the global sensitivity analysis method for elastomer joints was proposed and applied to the vibro-acoustic model of a passenger car. To consider the uncertainties of elastomers’ properties due to both the nonlinearity and the production variation, the variance-based method was introduced with distributions of elastomers’ properties. Additionally, the FRF-based substructuring method for the response calculation in a vibro-acoustic system and the random balance method for the calculation of the variance minimized the computation costs for the global sensitivity analysis. The introduced numerical example for a passenger car showed that the proposed method are very effective in parameter prioritization and efficient in computational cost.

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