



A two-stage noise source identification technique using a far-field random array

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ABSTRACT

A far-field random array is implemented for localization and separation of noise sources. Microphone positions are optimized, with the aid of the simulated annealing (SA) method as a supervised Monte Carlo approach, random samples of sensor position are drawn from Gaussian distribution to minimize the side-lobe maximum of the far-field beam-pattern. A two-stage algorithm is devised on the basis of the spherical-wave model on the image plane. In the localization stage, the active source regions are located by using the delay-and-sum (DAS) method, followed by parametric array localization with improved resolution. In the separation stage, source amplitude extraction can be achieved by formulating an inverse problem based on the steering matrix relating the sound pressures received by the microphones and the source amplitudes. The number of sources is selected to be less than the number of microphones to render an overdetermined problem which can be solved by using the Tikhonov regularization (TIKR) and Maximum Likelihood Estimation (MLE). Alternatively, the separation problem can be augmented into an underdetermined problem which can be solved using the compressive sensing (CS) method. Numerical and experimental results are presented to validate the proposed technique.

Keywords: simulated annealing method, side-lobe maximum, delay-and-sum method, Tikhonov regularization, Maximum Likelihood Estimation.

1. INTRODUCTION

Source identification including acoustic localization and separation has long been an important issue in array signal processing. Algorithms are widely applied in various kind of fields, such as psychoacoustics, teleconferencing and automatic speech recognition, etc.¹⁻³ However, before localization and separation process, the deployment of microphone positions is essential for high quality far-field image. Microphone array with random and sparse positions is known to have the ability to eliminate the grating lobes due to spatial aliasing.⁴ Nevertheless, the presence of side-lobes in array pattern will still result in false source localization. A better way to design microphone array geometry is to find maximum side-lobe level (MSL) as lower as possible in the different array pattern.⁵ Optimization algorithms including Monte Carlo (MC) approach and simulated annealing (SA) method, etc. are investigated to improve the design efficiency and the far-field image quality in previous research.⁶ In this paper, in order to find the optimal microphone positions efficiently, we take the advantage of the SA method to minimize the side-lobe maximum value of the far-field beam-pattern. During optimization process, each microphone position is determined by Gaussian distribution in their limited region for the purpose of manufacturing convenience. Therefore, microphone samples won't be too close or too sparse to manufacture.

Source localization techniques has been well established, for example, beamforming methods, parametric methods, etc.⁷⁻⁸ A better known conventional beamforming method is delay and sum method which is simple to implement. Unfortunately, limitation on resolution issue still suffers when sources are too close to each other. In order to alleviate the limitations, Capon proposed the minimum variance distortionless method to minimize the noise power and signals from other directions and constrain the main signal power in the look direction. Also, MUSIC method proposed by Schmidt can improve the resolution of imaging map.⁹ However, when the source signals are coherent, the suboptimal techniques mentioned above are ineffective. Therefore, more powerful method, Maximum Likelihood (ML) technique, can deal with not only fully correlated signals but also the case in which

the number of snapshots is very small.¹⁰ Sherman further applied simulated annealing method to ML technique to estimate the angle of arrival of identically polarized sources¹¹ and Ilan Ziskind and Mati Wax applied it to diversely polarized sources.¹² This concept is used to our randomized microphone array. Moreover, DAS is implement to determine the initial value of SA method and provide constrained solution region for this kind of nonlinear searching process to improve the convergence time.

Methods of source separation can be divided into two categories. One is blind source separation (BSS) method and the other is model-based method.¹³⁻¹⁴As an effective technique, BSS method and independent component analysis (ICA) still require more research to ensure its robustness.¹⁴ Model-based methods separate the source signals with the aid of propagation model. In this paper, the propagation model is assumed to be spherical wave model which is appropriate for air medium. The sources are located in previous localization stage. Thus, the remaining unknowns are waveform magnitude in the model that can be solved by deconvolution-based approaches. Tikhonov regularization (TIKR) and Maximum likelihood estimation (MLE)¹⁵ are used to extract the located source signals. The number of sources are set to be less than the number of sensors which can be formed as an overdetermined problem. The objective test is undertaken by Perceptual Evaluation of Audio Quality (PEAQ)¹⁶⁻¹⁷ to evaluate the performance of simulation result.

2. Optimal deployment of microphone positions

2.1 Cost function in terms of far-field beam-pattern

Uniform array will lead to undesired grating lobes because of the periodicity of microphone positions. Thus, the sensor deployment must satisfy the half wavelength rule. For random array, the phenomenon can be eliminated and the sensor positions can be arranged with no limitation. However, the presence of the side-lobes in beam-pattern will cause source wave from non-focusing direction to appear in the pattern.

For far-field planar array with look direction focusing on the front, the beam-pattern can be expressed in wave number domain

$$b = \frac{1}{M} \sum_{m=1}^M e^{j\mathbf{k} \cdot \mathbf{r}_m}, \quad (1)$$

where \mathbf{r}_m is the position vector of m th microphone, $j = \sqrt{-1}$, and $\mathbf{k} = -k\boldsymbol{\kappa}$ is wave number vector, $\boldsymbol{\kappa}$ is the unit vector of direction of plane wave, $k = \omega/c$ is the wave number, where ω is the angular velocity, c is the speed of sound in air, M is the number of microphone. During the optimization process, the beam-pattern is drawn in $k_x - k_y$ plane and the final goal is to search the optimal microphone deployment with minimum side-lobe maximum. In order to achieve the purpose, simulated annealing method is used to find the minimum value of the cost function.

2.2 Optimization procedure

Similar to the annealing process of a physical system, the simulated annealing (SA) method can effectively search the global minimum of a function with several local minima.¹⁸ The optimization steps of the random microphone array are shown in Figure 1 In the beginning, the microphone position are uniformly deployed in the $m \times n$ region shown in Figure 2 and the initial position vector is denoted as \mathbf{x}_i . Each size of region is divided with dimensions $d \times d$. Then, the far-field beam-pattern \mathbf{b}_i is calculated to determine the maximum side-lobe level Q_i . Set the initial temperature T_i , final temperature T_f , and annealing coefficient α and assign $\mathbf{x}_{op} = \mathbf{x}_i$, $\mathbf{b}_{op} = \mathbf{b}_i$, $Q_{op} = Q_i$. Next, each of the microphone position is reassigned in $d \times d$ region by means of Normal Gaussian distribution and the position vector is denoted as \mathbf{x} . Again, new far-field beam-pattern \mathbf{b} and the MSL Q are calculated. Next, compare the cost function between present and optimal value. If $Q < Q_{op}$, accept the present value as optimal solutions. If $Q > Q_{op}$, calculate the probability function as follows to decide whether the present solution should be taken or not.

$$PR(\Delta Q, T) = e^{\frac{-\Delta Q}{T_k}}, \tag{2}$$

where ΔQ is the difference between present and optimal value of cost function and T_k is the present temperature.

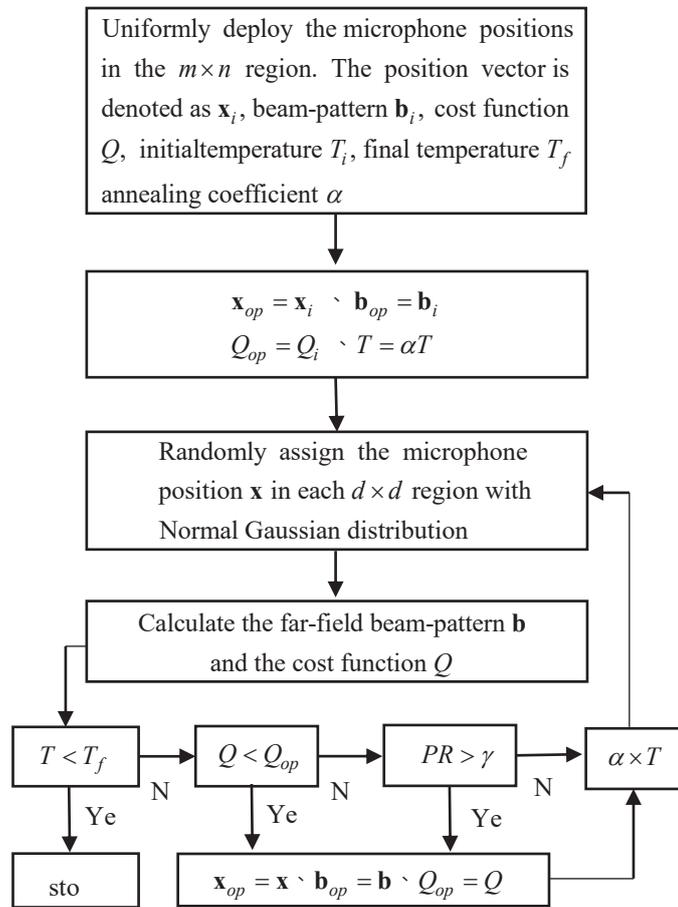


Figure 1 – The flowchart of microphone array position optimization in SA method

The probability will be further compared with a random γ which is between 0 and 1. If $PR(\Delta Q, T) > \gamma$, the present value of cost function Q will be accepted as optimal solution. Otherwise, the present value will be rejected and the annealing process continues with new Gaussian distribution positions. That is, in the early state of optimization process, SA has better opportunity to accept relatively worse solutions than other optimization algorithms. Therefore, with the advantage of SA method, the global minimum of the cost function can be effectively found. As the annealing process goes, the temperature decreases to final temperature T_f . Once the present temperature is lower than the final temperature, the whole process will be terminated.¹⁸⁻²⁰

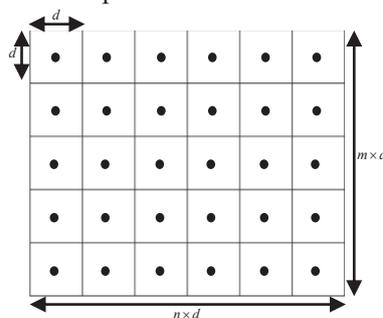


Figure 2 – Illustration of uniform microphone deployment in $m \times n$ region

The optimal microphone array positions are shown in Figure 3. Because each microphone position is constrained in specific region, the positions will not be too sparse. The related beam-pattern is shown in Figure 4. Figure 4(b) shows that the side-lobes decrease after optimization compared with Figure 4(a). Therefore, the design of the microphone array is suitable for source identification process.

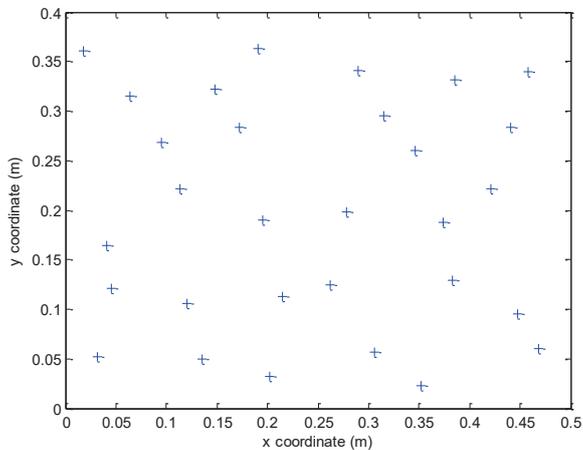


Figure 3 –Optimal microphone array positions

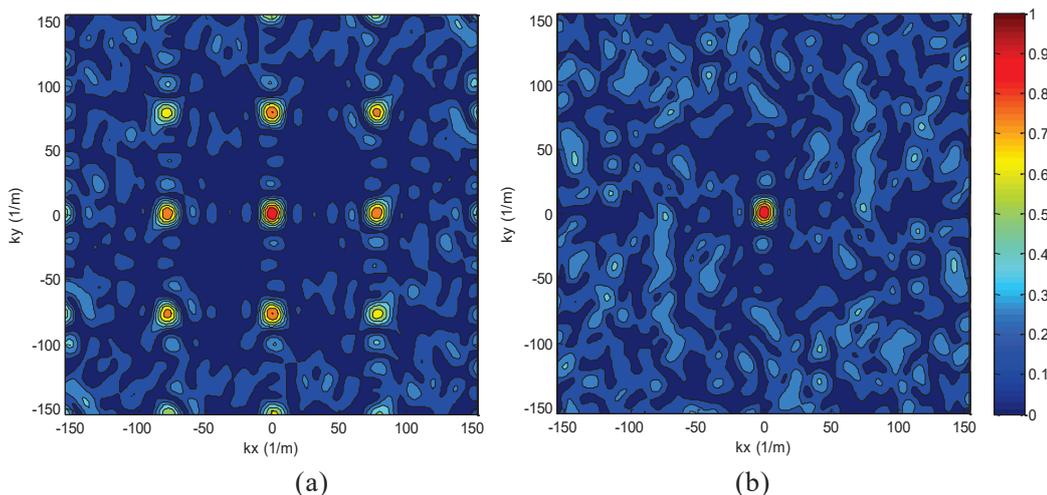


Figure 4 – (a) Beam-pattern before optimization. (b) beam-pattern after optimization

3. Stage 1: acoustic localization algorithm

Consider the sound sources are far away from the array. The acoustic sources can be seen as point sources and have narrowband characteristics. Therefore, the pressure vector $\mathbf{x}(\omega)$ received by M random microphones can be expressed in frequency domain as

$$\mathbf{x}(\omega) = \mathbf{a}(\mathbf{k})s(\omega) + \mathbf{n}(\omega), \tag{3}$$

where $\mathbf{a}(\mathbf{k}) = [e^{j\mathbf{k}\cdot\mathbf{r}_1} \quad e^{j\mathbf{k}\cdot\mathbf{r}_2} \quad \dots \quad e^{j\mathbf{k}\cdot\mathbf{r}_M}]^T$ is called steering vector, $\mathbf{r}_m, m=1, 2, \dots, M$, is microphone position vector, $s(\omega)$ is source waveform amplitude and $\mathbf{n}(\omega)$ is noise vector. If the noise can be neglect, the beam-pattern for far-field array can be expressed as

$$y(\omega) = \mathbf{w}^H \mathbf{x}(\omega) = \mathbf{w}^H \mathbf{a}(\mathbf{k})s(\omega). \tag{4}$$

For delay and sum beamforming, the weighting coefficients \mathbf{w} can be derived as

$$\mathbf{w} = \frac{\mathbf{a}}{\mathbf{a}^H \mathbf{a}}. \tag{5}$$

However, the resolution of DAS generally performs not well, especially when sources are too close

to each other. With the help of parameter estimation, the localization positions will be more accurate than DAS.

3.1 Deterministic maximum likelihood estimation

Assume that source signal $\mathbf{s}(t_i)$ and noise signal $\mathbf{n}(t_i)$ zero-mean Gaussian processes. The noise is assumed to be spatially white and is independent to the source signals. Thus, the covariance matrix is derived as

$$\mathbf{R}_{xx} = E\{\mathbf{x}(t_i)\mathbf{x}^H(t_i)\} = \mathbf{A}\mathbf{S}\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (6)$$

where $\mathbf{A} = [\mathbf{a}(\mathbf{k}_1) \cdots \mathbf{a}(\mathbf{k}_N)]$ is steering matrix that contains spatial information, \mathbf{S} is the covariance matrix of signals, σ^2 is an unknown coefficient and \mathbf{I} is an identity matrix. In DML method, the noise received by the microphones can be thought of as a great amount of independent sources while the source signals are not the case. That is, the source signals $\mathbf{s}(t_i)$ are deterministic and unknown. The noise that is assumed to be spatially white can be modeled as probability density function of circularly Gaussian distribution. Thus, the maximum likelihood function is derived as²¹

$$f(\mathbf{x} | \Theta) = \prod_{n=1}^{N_s} \frac{1}{\pi^M |\mathbf{R}_{mm}|^{N_s}} e^{-[\mathbf{x}(n) - \mathbf{A}\mathbf{s}(n)]^H \mathbf{R}_{mm}^{-1} [\mathbf{x}(n) - \mathbf{A}\mathbf{s}(n)]} = \frac{1}{\pi^{MN_s} |\mathbf{R}_{mm}|^{N_s}} e^{-\sum_{n=1}^{N_s} [\mathbf{x}(n) - \mathbf{A}\mathbf{s}(n)]^H \mathbf{R}_{mm}^{-1} [\mathbf{x}(n) - \mathbf{A}\mathbf{s}(n)]}, \quad (7)$$

where $|\mathbf{R}_{mm}| = |\sigma^2\mathbf{I}|$ is the determinant of noise covariance matrix. The likelihood function is rewrite as the form of negative log function as

$$L_{DML} = -\ln f(\mathbf{x} | \Theta) = M \ln \sigma_n^2 + \frac{1}{\sigma_n^2 N_s} \sum_{n=1}^{N_s} \|\mathbf{x}(n) - \mathbf{A}\mathbf{s}(n)\|_2^2. \quad (8)$$

Then, the likelihood function is differentiate with respect to σ^2 and we get

$$\hat{\sigma}^2 = \frac{1}{M} \text{tr}\{\mathbf{P}_A^\perp \hat{\mathbf{R}}_{xx}\}. \quad (9)$$

Least square method is taken to minimize the function with respect to $\mathbf{s}(n)$:

$$\hat{\mathbf{s}}(n) = \mathbf{A}^+ \mathbf{x}(n). \quad (10)$$

$\mathbf{A}^+ = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H$ is the Moore-Penrose pseudo-inverse of \mathbf{A} , $\hat{\mathbf{R}}_{xx}$ is the covariance matrix of sample data. $\mathbf{P}_A^\perp = \mathbf{I} - \mathbf{A}\mathbf{A}^+$ is the orthogonal projector onto \mathbf{A}^H . Substitute σ^2 and $\mathbf{s}(n)$ in Eq. (7) with Eq. (9) and (10). Therefore, the parameter estimation of DML becomes a minimization problem:

$$\hat{\Theta} = \arg \min_{\Theta} \text{tr}(\mathbf{P}_A^\perp \hat{\mathbf{R}}_{xx}). \quad (11)$$

3.2 Stochastic maximum likelihood estimation

The signal waveforms are further assumed as a Gaussian random processes in stochastic ML technique. Substituting the array signal data to Gaussian model gets²¹

$$f(\mathbf{x} | \Theta) = \prod_{n=1}^{N_s} \frac{1}{\pi^M |\mathbf{R}_{xx}|^{N_s}} e^{-\mathbf{x}(n)^H \mathbf{R}_{xx}^{-1} \mathbf{x}(n)} = \frac{1}{\pi^{MN_s} |\mathbf{R}_{xx}|^{N_s}} e^{-\sum_{n=1}^{N_s} \mathbf{x}(n)^H \mathbf{R}_{xx}^{-1} \mathbf{x}(n)}. \quad (12)$$

Minimizing the negative log function,

$$L_{SML} = -\ln f(\mathbf{x} | \Theta) = \ln |\mathbf{R}_{xx}| + \text{tr}[\mathbf{R}_{xx}^{-1} \hat{\mathbf{R}}_{xx}], \quad (13)$$

where $\hat{\mathbf{R}}_{xx}$ is the sample covariance matrix. The ML function can be minimized with respect to σ^2 and \mathbf{S} as shown

$$\hat{\sigma}^2 = \frac{\text{tr}(\mathbf{P}_A^\perp \hat{\mathbf{R}}_{xx})}{M - N}, \quad (14)$$

$$\hat{\mathbf{S}} = \mathbf{A}^+ [\hat{\mathbf{R}}_{xx} - \hat{\sigma}^2 \mathbf{I}] \mathbf{A}^{+H}. \quad (15)$$

Again, substituting Eq. (14) and (15) to Eq. (12) shows the SML estimation as a minimization problem

$$\hat{\Theta} = \arg \min_{\Theta} \ln |\mathbf{A}\hat{\mathbf{S}}\mathbf{A}^H + \hat{\sigma}^2\mathbf{I}|, \quad (16)$$

where $|\bullet|$ denotes determinant.

3.3 Parameter estimation combined with SA optimization

In the localization stage, because parameter estimation methods mentioned in previous section require multidimensional nonlinear optimization, the analytical solutions are impossible to get. Therefore, the SA optimization method is considered again to solve the parameter estimation problem. Figure 5 shows the flowchart of SA optimization process. At first, the acoustic sources are localized with DAS method and the main-lobes are denoted as \mathbf{x}_i . The cost function of DML method is calculate as $\theta_i = tr(\mathbf{P}_A^{-1}\hat{\mathbf{R}}_{xx})$ while the SML is calculated as $\theta_i = \ln |\mathbf{A}\hat{\mathbf{S}}\mathbf{A}^H + \hat{\sigma}^2\mathbf{I}|$. Then, set up the corresponding parameter for SA algorithm mentioned in previous section. The searching regions are constrained near the main-lobes of DAS result in order to speed up the SA convergence time.

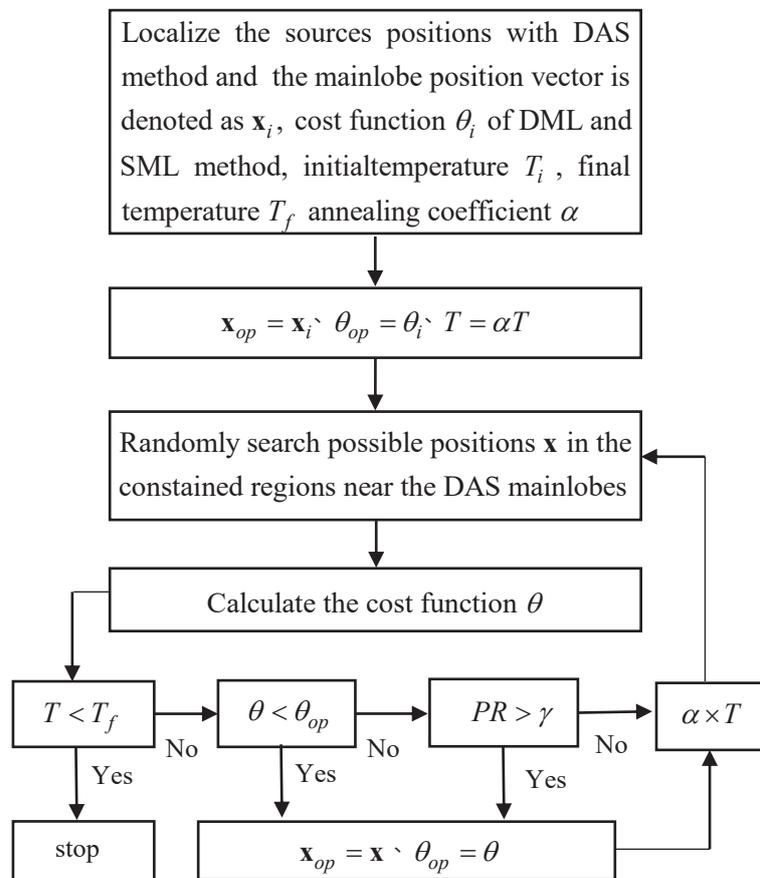


Figure 5 –The flowchart of parameter estimation optimization combined with SA method

4. Stage 2: acoustic source separation

4.1 Tikhonov regularization

Spherical wave is considered as wave propagation model here. If the noise is ignored, the pressure vector can be expressed as

$$\mathbf{p}(\omega) = \mathbf{A}(\mathbf{k})\mathbf{s}(\omega), \quad (17)$$

where $\mathbf{A} = [\mathbf{a}_1(\mathbf{k}_1) \cdots \mathbf{a}_N(\mathbf{k}_N)]$ is the steering matrix and $\mathbf{s} = [s_1(\omega_1) \cdots s_N(\omega_N)]^T$ is the waveform magnitudes and N denotes the number of spherical wave received by microphone array. Consider number of N acoustic sources is less than the number of M microphones. The acoustic separation is

turned to be an overdetermined inverse problem. Taking advantage of the Tikhonov regularization method, the algebraic equation is solved by regularized problem:

$$\min_{\mathbf{s}} \left(\|\mathbf{A}\mathbf{s} - \mathbf{p}\|_2^2 + \beta \|\mathbf{s}\|_2^2 \right), \quad (18)$$

where β is a regularization parameter. Finally, the solution of waveform amplitudes \mathbf{s} can be derived as

$$\hat{\mathbf{s}} = \left(\mathbf{A}^H \mathbf{A} + \beta \mathbf{I} \right)^{-1} \mathbf{A}^H \mathbf{p}. \quad (20)$$

4.2 Maximum likelihood estimation

Consider the array data given as follows contains noise term $\mathbf{n}(\omega)$.

$$\mathbf{p}(\omega) = \mathbf{A}(\mathbf{k})\mathbf{s}(\omega) + \mathbf{n}(\omega). \quad (21)$$

The steering matrix \mathbf{A} is known as a spherical wave propagation model and the signal waveform \mathbf{s} is unknown. With the assumption of deterministic maximum likelihood model, the probability function is obtained as^{15,21}

$$f(\mathbf{p} | \Theta) = \left| \pi \sigma^2 \mathbf{C} \right|^{-N} e^{-\frac{1}{\sigma^2} \sum_{n=1}^N (\mathbf{p} - \mathbf{A}\mathbf{s})^H \mathbf{C}^{-1} (\mathbf{p} - \mathbf{A}\mathbf{s})}. \quad (22)$$

Assume that the noise is spherically isotropic and its covariance matrix can be expressed as Eq. (23) for random microphone array.

$$C_{mn} = \text{sinc} \left[k |\mathbf{r}_m - \mathbf{r}_n| \right], \quad (23)$$

where k is the wave number and $|\mathbf{r}_m - \mathbf{r}_n|$ is the distance between m th and n th microphone position.

With the algebraic effort, the estimation of waveform signal can be derived as^{15,21}

$$\hat{\mathbf{s}} = \left(\mathbf{A}^T \mathbf{C}^{-1} \mathbf{A} \right)^{-1} \mathbf{A}^H \mathbf{C}^{-1} \mathbf{p}. \quad (24)$$

5. Performance evaluation

5.1 Simulation performance of source localization

Consider two uncorrelated audio source located at coordinate (0.36 m, 0.9 m, 1 m) and (1.08 m, 0.6 m, 1 m) received by the array located at $z = 0$ with spherical wave propagation. The DAS result is shown in Figure 6 Two audio acoustic sources are localized in roughly two regions. Then, the main-lobe peaks are chosen as initial position vector for optimization process. The multidimensional searching in SA method is constrained in regions near the main-lobes of DAS method.

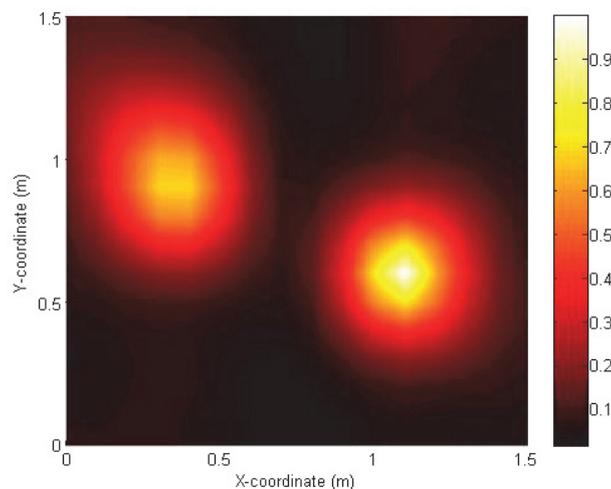


Figure 6 –DAS result of two audio acoustic sources

Figure 7 shows the optimization result of the parameter estimation. Figure 7(a) and (b) are the

DML and SML estimation, respectively, which indicates correct locations of the sources.

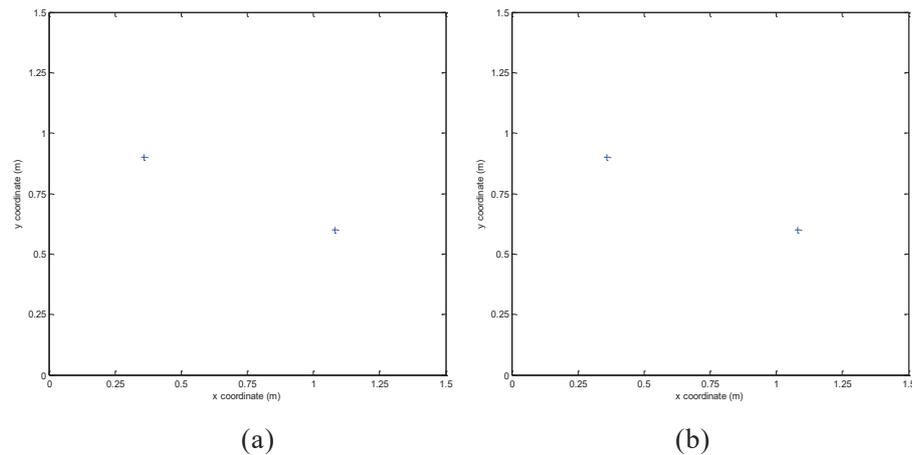


Figure 7 –Optimization result of DML and SML method

5.2 Performance of source signal separation

In the separation stage, the acoustic source positions localized in previous stage are used in constructing the propagation model. The objective test, Perceptual Evaluation of Audio Quality (PEAQ),¹⁶⁻¹⁷ is undertaken to evaluate the performance of separation algorithm. The performance index, Objective Difference Grade (ODG), ranging from 0 to -4 is shown in Table 1. The test objects are compared with their original sources file as reference and the ODG is then evaluated. Table 2 shows the ODG value of TIKR and MLE method. Both methods are feasible for acoustic separation processes.

Table 1 – Definition of ODG

Impairment description	ODG
Imperceptible	0
Perceptible, but not annoying	-1
Slightly annoying	-2
Annoying	-3
Very annoying	-4

Table 2 –ODG value of TIKR and MLE method

Methods	ODG
MLE method (source 1)	-0.13
MLE method (source 2)	-0.29
TIKR method (source 1)	-0.12
TIKR method (source 2)	-0.30

6. CONCLUSIONS

Microphone positions are optimized with SA method in designing a far-field beam-former. Maximum side-lobe level is minimized for optimal design of planar array. Then, the optimal array is

utilized to perform two stage noise identification processes. In localization stage, rough regions of audio sources are initially localized by DAS method. Next, SA method is applied to parameter estimation problem to determine much more precise locations. In separation stage, spherical wave propagation model is built based on previous localization stage. Under case of overdetermined problem, TIKR and MLE methods are utilized to solve the source waveform amplitudes. The objective test (PEAQ) shows that these two methods perform well.

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REFERENCES

1. Nakadi K, Nakajima H, Ince G, and Haseawa Y. Sound source separation and automatic speech recognition for moving sources. The 2010 IEEE/RSJ International Conference on Intelligent Robots and Systems; 18-22 October 2010; Taiwan 2010. p. 976-981.
2. Kazuhiro Nakadai. Keisuke Nakamura. Sound Source Localization and Separation. Wiley Encyclopedia of Electrical and Electronics Engineering. John Wiley & Sons 2000.
3. Friis H T and Feldman C B. A multiple unit steerable antenna for short-wave reception. Proceedings of the Institute of Radio Engineers, July 1937; New York 1937. Vol. 25, p. 841-917.
4. Bai M R, Ih J G, and Benesty J. Acoustic Array Systems: Theory, Implementation and Application. Wiley, 2013. p. 250-270.
5. Christensen J J and Hald J. Beamforming. Brüel & Kjær Technical Review No.1, 2004.
6. Li Bing, Yang DianGe, Lian XiaoMin. An Acoustic Holography Method with Random Sparse Microphone Array to Locate Moving Sound Sources. IEEE International Conference on Signal Processing; 26-29 October 2008; Beijing 2008. p. 187-190.
7. Roig E T, and Jacobsen F. Deconvolution for the localization of sound sources using a circular microphone array. J Acoust Soc Am. September 2013; 134(3): 2078-2089.
8. Le Cadre J P. Parametric Methods for Spatial Signal Processing in the Presence of Unknown Colored Noise Fields. IEEE Transactions on Acoustics, Speech, and Signal Processing; July 1989; Vol. 37, p. 965-983.
9. Krim H, Viberg M. Two decades of array signal processing research. IEEE Signal Processing Magazine; July 1996; Vol. 13, p. 67-94.
10. Ziskind I, Wax M. Maximum Likelihood Localization of Multiple Sources by Alternating Projection. IEEE Transactions on Acoustics, Speech, and Signal Processing; October 1988; Vol. 36, p. 1553-1559.
11. Shaman K. C. Maximum likelihood parameter estimation by simulated annealing. Acoustics, Speech Signal Processing; 11-14 April 1988; New York 1988; p. 2741-2744.
12. Ziskind I, Wax M. Maximum Likelihood Localization of Diversely Polarized Sources by Simulated Annealing. IEEE Transactions on Acoustics, Speech, and Signal Processing; July 1990; Vol. 38, p. 1111-1113 (1990)
13. Kokkinakis K and Loizou P C. Using blind source separation techniques to improve speech recognition in bilateral cochlear implant patients. J Acoust Soc Am. April 2008; 123(4): 2379-2390.
14. Bai M R, Kuo Chia-Hao. Acoustic Source Localization and Deconvolution-Based Separation. J Comp Acous. June 2015; Vol 23, No 02.
15. Fevotte C, Cardoso J -F. Maximum likelihood approach for blind audio source separation using time-frequency Gaussian source models. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics; 16-19 October 2005; p. 78-81.
16. Rix A W, Beerends J G, Kim D S, Kroon P, and Ghitza O. Objective Assessment of Speech and Audio Quality—Technology and Applications. IEEE Transactions on Audio, Speech, and Language Processing; November 2006; Vol. 14, p. 1890-1901.
17. Thiede, Treurniet W C, Bitto R, Schmidmer C, Sporer T, Beerends J G, Colomes C, Keyhl M, Stoll G, Brandenburg K, and Feiten B. PEAQ-The ITU standard for objective measurement of perceived audio quality. J Audio Eng Soc; February 2000; Vol. 48, p. 3-29.
18. Kirkpatrick S, Gelatt, C D, Vecchi, M P. Optimization by simulated annealing. Science; 13 May 1983; Vol. 220, p. 671-680.
19. Eduardo R T, Hao, J K, and Jose T J. An effective two-stage simulated annealing algorithm for the minimum linear arrangement problem. Computer and Operation Research; October 2008; Vol. 35, p.

- 3331-3346.
20. Murino V, Trucco A, and Regazzoni C S. Synthesis of unequally spaced arrays by simulated annealing. IEEE Transactions Signal Processing; January 1996; Vol. 44, p. 119-123.
 21. Haykin S, Litva J and Shepherd T J. Radar Array Processing. Springer-Verlag; 1993, Chap. 2-4.