Active Shielding based on implicit control: a one dimensional approach

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ABSTRACT
In order to apply an active noise control system, the sensors position is one of the most important variables to obtain a desired silent zone. This is due to the fact that the attenuation is achieved at the same location where the noise is measured. As a consequence, sensors become obstacles inside the silent zone. This issue produces several solutions which highly increase the computational cost, e.g. the estimation in virtual sensing. The aim of this article is to present a new method to generate a silent zone only controlling the noise at its boundaries for one dimensional enclosure, which eliminates the computational cost of the estimation. The principle of the method and its conditions is defined in this document as implicit control. Afterward, the new method is proven using a discrete model of the wave equation. Next, the required condition for applying implicit control is empirically proven for one dimensional duct through output error method for system identification. At the end, a simulation of the control scheme for one dimensional enclosure is shown.

Keywords: Active Shielding, Active noise control in ducts , Implicit control

1. INTRODUCTION

The active noise control (ANC) is a very known method to control the noise with energy at low frequencies. The principles of ANC were proposed by Coanda and Leug on their patents (1, 2). This theory has been implemented to several cases (3). ANC in enclosures is a special case due to the reflections and some other acoustical phenomena.

In order to attenuate the pressure that receives a sensor inside an enclosure, several algorithms have been created. Some of them are based on adaptive filters such as the filtered X Least Mean Squares (LMS) algorithm, Filtered U LMS algorithm, etc. (4). Other algorithms are based on robust control theory, e.g. (5–8). According to these references, these algorithms obtain an attenuation around 20dB at the sensor location. However, the attenuation of the sound is not only required at the sensor location. Usually, the user, who is exposed to the noise, is not located at the sensor position. Inside enclosures with high volume, the mentioned ANC systems only control the noise at locations near the sensors. In order to increase the silent zone (locations where the noise is attenuated), multichannel control has been used (9, 10). Nevertheless, sensors cannot be located at the position of the users. Then, increasing the silent zone without using microphones at the user locations is a problem mentioned for several authors. A solution was proposed in (11), where an optimization problem is set out. This mentions the possibility to optimize the sound pressure using the sensors and actuators (also called secondary sources) as optimization variables. A similar procedure is applied in (12), but it applies two consecutive optimization problems. The first problem takes into account only the sources positions, and the second the sensor locations. This can be applied using a simulation of the room. Thus, the sources and sensors are located where the results indicate.

Increasing the silent zone is possible estimating the pressure at a desired location. This is known as virtual sensing. The first algorithm was called virtual microphone arrangement (13). It takes into account the sound

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pressure measured by a sensor. Its limitation is due to the assumption that the sound pressure generated by the noise sources is equal for the virtual and real sensor locations (desired and measured locations respectively). A modification called remote microphone applies the same concept (14). However, it uses a transformation of the estimation of primary noise to avoid the assumption that the sound pressure component generated by the primary source is equal to both locations, and this transformation is not determined if it is causal or not. (15) uses a Kalman filter as the estimator of the pressure. It previously needs to identify the model of the system. Another proposal is based on the correlation of the pressure of different locations (16), but it is limited by the distance between real and virtual sensors. (17) introduces the nonlinear estimation to solve the problem of the non-causality relation between the measured and estimated pressure. The solution is a modification of the remote microphone algorithm. Other algorithms have been presented but only applied to one dimensional or not enclosures systems (18–20).

Another important methodology is active shielding. It has the aim to control the acoustic pressure inside a desired volume using the information of sensors located at the boundary of a desired silent zone. The first approach is given by (21) and (22). (23) formulates the active shielding solution based on the Kirchhoff-Helmholtz integral equation. However, this solution entails the issues associated to this equation which is unknown for characteristic frequencies of the desired silent zone (the solution is known depending on the shape of silent zone) (24). (25) shows an analytic solution based on the control of particle velocity of one dimensional wave equation and proves it experimentally. Furthermore, the same author shows a three dimensional case of the solution in (26). This theory has only been tested under laboratory conditions and implies the use of a particle velocity sensor.

This document deals with the attenuation of noise inside a microphones array. The solution is a novel active shielding method which reduces the hardware complexity respect to virtual sensing methods because it avoids the estimation process. Also, it works for all frequencies, including where the Kirchhoff-Helmholtz integral equation does not have solution. It only uses pressure sensors, instead of the particle velocity sensors. The next section defines a new concept called implicit control. This concept is used to propose a novel method for active shielding in section 3. Section 4 shows a proposal to obtain a experimental validation of the restrictions to achieve implicit control. Section 5 shows simulated results of the one dimensional system. The paper ends with concluding remarks and future works.

2. IMPLICIT CONTROL

Before understanding how the proposed active shielding system works, it is necessary to define a new concept:

Definition 2.1. "Implicit control": It is the phenomenon which controlling a variable implies controlling other variable.

For active noise control, it occurs when attenuating (controlling) the pressure at the sensor location, implies the attenuation at any other location. It is important to remark that the pressure at each location is defined as a different variable. The aim of this new concept is to be applied as a method to ensure that attenuating the boundaries of a desired silent zone, any location inside it is also attenuated.

In order to clearly show this concept, an example of an active noise control system in two dimensional free field is taken into account. See the locations in figure 1. The figure shows two point sources, one generates the noise and the other is the actuator (controlled source), located at $s_1 = [A_{1,a}, 0]$ and $s_2 = [A_{2,a}, 0]$ respectively (for the axis $[X_1, X_2]$). The sensor is located at $S_l = [s_a, s_b]$ (Location which is the pressure to be controlled) with a distance d_1 and d_2 from the noise source and actuator respectively. As additional restriction $A_{1,a} < s_a$ and $A_{2,a} > s_a$.

The total pressure $p_t(x, t)$ at $x = [x_1, x_2]$ location can be expressed as a function of the pressure components $p_1(x, t)$ and $p_2(x, t)$, which represent the pressure due to the noise source and the actuator respectively:

$$p_t(x, t) = p_1(x, t) + p_2(x, t) \tag{1}$$

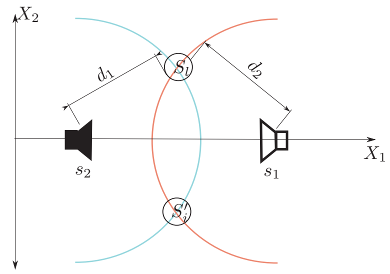

Figure 1. Scheme of acoustic system taken into account.

For the frequency analysis of this case, these two components of pressure can be expressed as functions of distance d_1 or d_2 between the receiver at location S_l and the source s_1 or s_2 respectively, i.e.:

$$p_1(S_l,t) = j\rho_0 c \frac{Q_1\eta}{4\pi d_1} e^{j(\omega t-\eta d_1)} \quad (2)$$

$$p_2(S_l,t) = j\rho_0 c \frac{Q_2\eta}{4\pi d_2} e^{j(\omega t-\eta d_2)} \quad (3)$$

Where ρ_0 is the air density; c is the speed of sound; Q_1 and Q_2 are the source strength.

Furthermore, the control of the pressure at S_l is achieved if $p_t(S_l,t) = 0$. It means $p_1(S_l,t) = -p_2(S_l,t)$. This condition is accomplished if:

$$Q_2 = -\frac{Q_1 d_2}{d_1} e^{j(\eta d_2 - \eta d_1)} \quad (4)$$

Notice that Q_2, $p_1(S_l,t)$ and $p_2(S_l,t)$ only depend on the distances d_1 and d_2 due to ρ_0, c and κ are constants. Hence, equation (4) is a solution for any other location S_l' such that the distance between the sources and S_l' are equal to S_l, eg. $S_l' = [-s_a, s_b]$ and both locations have the same value of pressure. It means that the implicit control occurs and controlling the pressure at S_l implies to control the pressure at S_l'.

As a consequence of the implicit control phenomenon, a related issue appears:

Problem 1. How to find the locations to be controlled in order to obtain implicit control in a desired silent zone?

From the definition of implicit control, the issue is to find a set of locations $x_{1,s}, x_{2,s}, \cdots, x_{N,s}$ such that they achieve the implicit control at another set of locations $x_{i,r}$, with $i = 1, \cdots, M$. And M is the number of locations where the implicit control occurs. Defining two vectors with the pressure in frequency space for these locations as $P(\omega) := [F(p_t(x_{1,r},t)), F(p_t(x_{2,r},t)), \cdots, F(p_t(x_{M,r},t))]$; $F(\cdot)$ and $\bar{P}(\omega) := [F(p_t(x_{1,s},t)), F(p_t(x_{2,s},t)), \cdots, F(p_t(x_{N,s},t))]$, it is evident that the implicit control can be proven if the next restriction is achieved:

$$\bar{P}(\omega) | \left\{ P(\omega) = G(\omega)\bar{P}(\omega) \right\} \quad (5)$$

Where $G(\omega)$ is any linear transform.

For the example in figure 1, the equation (4) ensures the restriction (5) for the $[x_{1,s} = L_s]$ and $[x_{1,r} = L_s']$ and $G(\omega) = 1$. Hence, the implicit control is accomplished in the case of the figure 1.

3. ACTIVE SHIELDING BASED ON IMPLICIT CONTROL

This section describes a novel methodology to implement an active shielding system based on implicit control. Basically, the aim is to solve the problem 1 using a set of pressure at boundaries $\bar{P}(\omega)$. For sake of simplicity, the analysis will be carried out in one dimensional system. The procedure consists in discretizing the wave equation using the finite difference method, such that, the behavior of the silent zone is described by the pressure at the boundaries. This discrete model is written as an space state model which has a transfer function $G(\omega)$.

In this section the sources are not taken into account, only the scheme shown in figure 2. The desired silent zone has green color. The aim is to obtain information about the behavior of its pressure based on the behavior of the pressure at its boundaries.

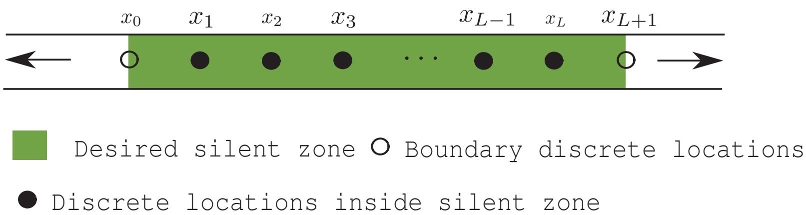

Figure 2. Scheme of acoustic system taken into account.

The equation that describes the behavior of pressure by the desired silent zone in figure 2 is the damped wave equation:

$$\frac{\partial^2 p(x,t)}{\partial x^2} - \zeta \frac{\partial p(x,t)}{\partial t} - \frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial t^2} = 0 \tag{6}$$

Where ζ is the damping coefficient. Notice that this equation assumes that the noise source is located far from the silent zone, where the analysis is carried out.

From equation (6), it is quite complex to express the pressure in a desired area based on the pressure of another location. Hence, it is is discretized using the finite difference method. Specifically, the differential operator is approximated by:

$$\frac{\partial p(x_i,t)}{\partial t} \approx \frac{p(x_i,(k+1)\Delta_t) - p(x_i,(k-1)\Delta_t)}{2\Delta_t} \tag{7}$$

$$\frac{\partial^2 p(x_i,t)}{\partial t^2} \approx \frac{p(x_i,(k+1)\Delta_t) - 2p(x_i,k\Delta_t) + p(x_{i,j},(k-1)\Delta_t)}{\Delta_t^2} \tag{8}$$

$$\frac{\partial^2 p(x_i,t)}{\partial x^2} \approx \frac{p(x_{i+1},k\Delta_t) - 2p(x_i,k\Delta_t) + p(x_{i-1},k\Delta_t)}{\Delta_x^2} \tag{9}$$

Where Δ_t is the sample frequency and Δ_x is the distance between x_i and x_{i+1} for all $i \in \mathbb{Z}$. Discrete locations and time are identified by i and k respectively.

Replacing equations (7), (8) and (9) in equation (6) and after algebraic simplifications, the term $p(x_i,(k+1)\Delta_t)$ yields:

$$p(x_i,(k+1)\Delta_t) = \gamma_1\left[p(x_{i+1},k\Delta_t) + p(x_{i-1},k\Delta_t)\right] + \gamma_2 p(x_i,k\Delta_t) + \gamma_3 p(x_i,(k-1)\Delta_t) \tag{10}$$

Where:

$$\gamma_1 = \left[\frac{2c^2\Delta_t^2}{\Delta_x^2(\zeta c^2\Delta_t + 2)}\right] \tag{11}$$

$$\gamma_2 = \left[\frac{4}{\zeta c^2\Delta_t + 2}\right] - \left[\frac{4c^2\Delta_t^2}{\Delta_x^2(\zeta c^2\Delta_t + 2)}\right] \tag{12}$$

$$\gamma_3 = \left[\frac{\zeta c^2\Delta_t - 2}{\zeta c^2\Delta_t + 2}\right] \tag{13}$$

Let consider a set of sound pressure of L successive locations of the desired silent zone:

$$z(k) = [p(x_1,k\Delta_t), \quad p(x_2,k\Delta_t), \quad \cdots, \quad p(x_L,k\Delta_t)]^T \tag{14}$$

In Fig.2, the discrete locations are shown by circles and the locations of the pressures that form $z(k)$ are in black color. On the other hand, x_0 and x_{L+1} are empty circles. Using the equation (10), $z(k)$ can be written as:

$$z(k+1) = A_1 z(k) + \gamma_3 \mathbb{I}_L z(k-1) + B \begin{bmatrix} p(x_0, k\Delta_t) \\ p(x_L, k\Delta_t) \end{bmatrix} \tag{15}$$

Where $\mathbb{I}_L$ is an identity matrix with size $L \times L$ and

$$A_1 = \begin{bmatrix} \gamma_2 & \gamma_1 & 0 & \cdots & \cdots & 0 \\ \gamma_1 & \gamma_2 & \gamma_1 & 0 & & \vdots \\ 0 & \gamma_1 & \gamma_2 & \gamma_1 & \ddots & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \gamma_1 & \gamma_2 & \gamma_1 \\ 0 & \cdots & \cdots & 0 & \gamma_1 & \gamma_2 \end{bmatrix} \tag{16}$$

$$B_1 = \begin{bmatrix} \gamma_2 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & \gamma_2 \end{bmatrix} \tag{17}$$

Eq. (15) is rewritten as an space state model and it yields:

$$\begin{bmatrix} z(k+1) \\ z(k) \end{bmatrix} = \begin{bmatrix} A_1 & \gamma_3 \mathbb{I}_L \\ \mathbb{I}_L & 0_L \end{bmatrix} \begin{bmatrix} z(k) \\ z(k-1) \end{bmatrix} + B \begin{bmatrix} p(x_0, k\Delta_t) \\ p(x_{L+1}, k\Delta_t) \end{bmatrix} \tag{18}$$

This model has a linear relation between $[p(x_0, k\Delta_t), \quad p(x_{L+1}, k\Delta_t)]^T$ and $z(k)$. Indeed, it ensures to achieve equation (5). As a consequence,if A_1 is stable, then to make $[p(x_0, k\Delta_t), \quad p(x_{L+1}, k\Delta_t)]^T = [0, \quad 0]$ is a sufficient condition to ensure $z(k) = 0$. A_1 is stable if the Courant number is less than one and $0 < \zeta < 1$. Notice that $p(x_0, (k)\Delta_t)$ and $p(x_{L+1}, (k)\Delta_t)$ are the pressures at the boundary of the desired silent zone. It means, this is an active shielding method.

4. EXPERIMENTAL VALIDATION OF THE LINEAR RELATION BETWEEN THE SILENT ZONE AND ITS BOUNDARIES

The previous section shows analytically the linear relation between the discrete location in a desired silent zone. However, there is no empiric evidence of this relation. This section shows a proposal to obtain empiric evidence that ensures the condition (5). Notice that condition (5) is a mathematical relation, it means it cannot be measured. The empirical evidence proposed to compare a simulation using mathematical model that achieve the condition (5) with the measured data.

In order to obtain the mathematical model, (27) shows that the system identification method called output error is useful for a one dimensional acoustic system. It uses measured data to estimate the coefficients $a_{1,i}$, $a_{2,i}$, $b_{1,j}$ and $b_{2,j}$ for $i = 1, \cdots, N_a$ and $j = 1, \cdots, N_b$ of the next mathematical model:

$$y_1(k) = a_{1,1} y_1(k-1) + \cdots + a_{1,N_a} y_1(k - N_a) + b_{1,1} u_1(k-1) + \cdots + b_{1,N_b} u_1(k - N_b) \tag{19}$$

$$y_2(k) = a_{2,1} y_2(k-1) + \cdots + a_{2,N_a} y_2(k - N_a) + b_{1,1} u_2(k-1) + \cdots + b_{1,N_b} u_2(k - N_b) \tag{20}$$

$$y(k) = y_1(k) + y_2(k) \tag{21}$$

Where N_a is the number of coefficients related to the actual and past input, N_b is number of coefficients related to the past outputs of the model. $u_1(k) := p(x_0, k\Delta_t)$ and $u_2(k) := p(x_{L+1}, k\Delta_t)$ are the input signals, y_1 and y_2 are state variables without physical meaning and $y(k)$ is the output. Notice that N_a and N_b are the order of the model. For this case, $y(k)$ is an estimation of $p(x_i, k\Delta_t)$, for notation $y(x_i, k)$ indicates the location that corresponds to this estimation. Notice that i has different possible values between 1 and L. It means the procedure must be carried out L times, one by each possible value. The procedure to obtain the coefficients is detailed in (28).

The system in figure 2 does not have noise sources, which makes impossible to prove its behavior. For this reason, it is measured the system in scheme 3. The difference consists of adding a sound source at the beginning and a surface at the end of the duct. Also, for this case, $L = 3$, $\Delta_x = 0.2$m. The distance between the source and x_0 is 0m. The distance between $x_L + 1$ and the surface is 0m. The sample frequency is 4410Hz. The system is identified using 22 seconds of audio. The order of the system is $N_a = 4$ and $N_b = 16$.

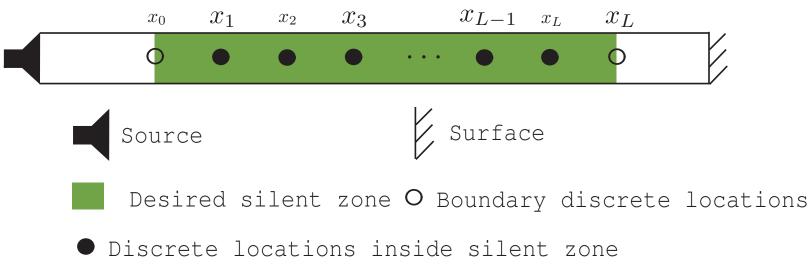

Figure 3. Scheme of acoustic measured system.

After obtaining the mathematical model using measured data, the model is compared with the measured data. Another measured data set is used to validate the condition (5). The parameter to quantify this comparison is called FIT:

$$FIT(x_i) = 100 \left[1 - \frac{||\tilde{e}(n)||_2}{\left|\left|\tilde{y}(k) - \hat{E}(\tilde{y}(k))\right|\right|_2} \right] \tag{22}$$

Where $\hat{E}(.)$ is the mean, $\tilde{e}(n) = \bar{y}(x_i, k) - \tilde{y}(x_i, k)$, $\bar{y}(x_i, k) = [y(x_i, (k)), y(x_i, (k-1)), \cdots, y(x_i, (k-K))]$, $\tilde{y}(x_i, k) = [p(x_i, (k)\Delta_t), p(x_i, (k-1)\Delta_t), \cdots, p(x_i, (k-K)\Delta_t)]$ and K is the number of samples used to evaluate the error.

Notice that if $e(n) = 0$ for all n, then $FIT = 100$. Also, $FIT = 0$ means $||e(n)||_2 = ||\bar{y}(n)||_2$

Figure 4. Estimated and measured signals for $p(x_1, t)$.

The comparison between the measured and the simulated output for the location x_1 is shown in figure 4. Both signals are very similar and is expected to achieve the condition (5). The FIT value obtained for each location is:

$$FIT(x_1) = 81.8514 \tag{23}$$
$$FIT(x_2) = 89.2338 \tag{24}$$
$$FIT(x_3) = 91.8943 \tag{25}$$

These FIT values are commonly accepted for a system identification procedure. For this reason, it can be assumed that the condition (5) is achieved.

5. SIMULATION OF THE ACTIVE SHIELDING METHOD

Towards validating the proposed conditions, a simulation of the control system is carried out. The aim is to obtain the attenuation in the silent zone if the pressure is controlled at its boundaries. According to this, the attenuation inside the silent zone must be similar to the obtained in the control sensor locations. It means, a control law is applied only to attenuate the noise at x_0 and x_{L+1}. Then, the result that confirms the theory is that the noise at x_i with $1 = 1, \cdots, L$ is also attenuated.

This simulation is based on the active sound control proposed in (29) for non-virtual sensors. The simulated system is shown in the figure 5. It consists of a duct with length equal to 6.8m and the reference ($x = 0$) is the start of the duct. Two noise sources are located at the end of both sides of the duct. Two actuators (secondary sources) are located at 0.2m and 5.7m. Also, $x_0 = 1.7$m and $x_{L+1} = 5.1$m. It is assumed the location of the control sensors and all possible location between them is the desired silent zone. The system is simulated using the ray tracing method and the reflection coefficients for the end surfaces (the surface of the noise sources) is 0.7.

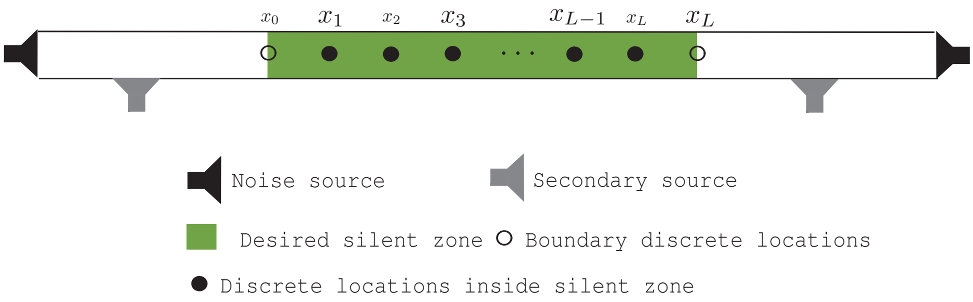

Figure 5. Scheme of acoustic simulated system.

The attenuation achieved as function of the distance is shown in the figure 6. The frequencies tested in the simulation are 63Hz, 125Hz, 250Hz, 500Hz, 1KHz. For all the frequencies, the behavior of the attenuation is the same. The silent zone is achieved for all the locations between the actuators. Hence, the attenuation inside the desired silent zone is very high (more than 100dB) and similar to the obtained at x_0 and x_{L+1}. It means that the proposed theory is coherent with the simulation results, independent of the frequency.

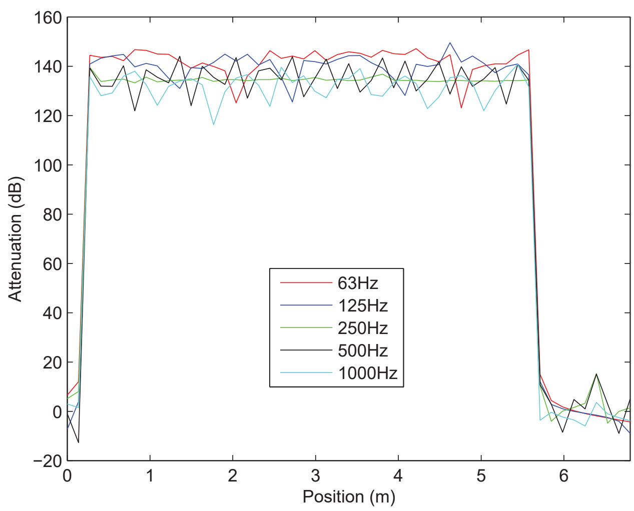

Figure 6. Estimated and measured signals for $p(x_1, t)$.

6. CONCLUSIONS

This article defined the implicit control. As a result of this definition, it was found a sufficient condition to evaluate if the implicit control can be implemented. Based on this concept, a method of active shielding was proposed. It was shown for one dimensional system that making zero the pressure at the boundaries of the desired silent zone ensures to achieve the silent zone, which is the active shielding method. A methodology to obtain a experimental validation of the condition to achieve implicit control was proposed. This methodology obtains a mathematical model that procure a FIT value higher than 80. Thus, it is inferred that the condition to obtain the implicit control is achieved. At the end, a control system was simulated showing that the attenuation over all the desired silent zone is similar to the obtained at the boundary locations. It means, the proposed method is verified not only theoretically, but also through a simulation.

For future works, the active shielding method will be expanded for two and three dimensional systems using a similar procedure. Furthermore, the number of sources and actuators is another problem to be solved. The number of sources and actuator will be minimized using sparse matrices for the control. Moreover, the influence of surfaces at the boundary of the desired silent zone will be analyzed.

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