Recursive least square algorithm-based acceleration harmonics identification for an electro-hydraulic shaking table

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ABSTRACT

Hydraulic shaking tables are important mechanical environmental simulators for such fields as aerospace, nuclear industry, automobile and civil engineering to measure the specimen's reliability under vibration condition. Sinusoidal shaking tests are widely used in practical applications, but its acceleration response usually contains higher harmonics, due to the nonlinearities in the system. The harmonic distortion lowers the system control performance. A harmonic identification scheme is thus developed by utilizing a transversal filter based on recursive least square method to provide harmonic information for further harmonic cancellation. A harmonic generator generates the harmonic vector, and the parameter vector of the filter is updated by incrementing its old value by an amount equal to the product of the estimation error and the gain vector. The amplitude and phase of each harmonic including the fundamental response are then directly computed from the time-varying parameter vector. Each harmonic is also directly decomposed from the harmonic vector and the estimated parameter vector. Experimentations performed on an electro-hydraulic shaking table are carried out to validate its efficiency and accuracy.

Keywords**:** hydraulic shaking table, acceleration control, nonlinearities, harmonic identification, recursive least square method I-INCE Classification of Subjects Number(s): 48

1. INTRODUCTION

Electro-hydraulic shaking table is an important vibration test equipment for many key industrial fields, such as aerospace, nuclear, automobile, civil engineering, and so on. Due to the nonlinearities in the system, such as dead zone in servo valve, backlash and friction in joints and friction in hydraulic actuators (1, 2), the control system is generally a nonlinear system in nature. This often lowers the control precision of the system and reduces system stability margin. In the sinusoidal shaking tests, the acceleration response of the shaking table usually contains higher harmonics, resulting in serious waveform distortion, due to those nonlinearities in the system (3). How to reduce higher harmonics in its response signal has become an important topic for high performance shaking tables.

Prior knowledge about harmonics in system response is essentially required for harmonic cancellation to reduce harmonic distortion. Harmonic analysis methods have been developed to detect or estimate

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harmonics, especially in power systems. Fast Fourier transformation (FFT) is the mostly used way to analyze harmonics by transforming the samples of data in time domain into frequency domain, but its disadvantages (4, 5), such as aliasing, leakage, picket fence effects, and poor real time performance limit its accuracy and applications. To overcome the shortcomings of FFT-based harmonic identification methods, new methods and algorithms have been started to use as an alternative for harmonic estimation instead of Fourier Transform based traditional algorithms.

Neural networks (NNs), including Adaline, back propagation, multi-layer perceptron, radial basis function, Hopfield (6, 7, 8, 9), have been used to detect harmonics, but NN-based harmonic estimation method needs data samples and sometimes it needs calibration. Optimization algorithms, such as genetic algorithm, particle swarm optimization, simulated annealing (10, 11, 12), were also utilized to identify harmonics, but the initial parameters can influence the iteration and it may suffer from computation burden.

The accuracy of Kalman filter-based harmonic estimation method (13, 14) is related to the state space model, and different acceleration models result in different estimation accuracy. The performance of wavelet-based harmonic analysis methods (15, 16) varies depending on the characteristics of different wavelets.

The estimation of harmonics for a hydraulic shaking table's sinusoidal response needs real time performance and computational efficient. The simpler identification structure, the easier for applications. In this paper, a harmonic detection method is developed by using recursive least square algorithm to estimate harmonics in the sinusoidal acceleration response of an electro-hydraulic shaking table.

2. SYSTEM DESCRIPTION

The whole shaking table system is shown in Figure 1. The system can be broadly classified into two parts, the hardware system and the software system. The hardware system is mainly composed of the hydraulic shaking table, the host and target computers, the conditioner and the servo amplifier, while the software system is the real time control program running in the target computer and the user interface operated in the host computer.

The hydraulic shaking table shown in Figure 1 is a uniaxial shaker with sensors including an LVDT for measuring displacement and an accelerometer for measuring acceleration. The hydraulic cylinder is controlled by a two stage servo valve with a frequency bandwidth of 100 Hz. The parameters of the hydraulic system are shown in Table 1. The signal conditioner contains A/D conditioners for analog transducer, and the servo amplifier is applied to condition the driving signal to meet the D/A board range. The host computer is a commercial PC, which operates under Windows XP. The target computer is an industrial computer used as a real-time processing, which includes I/O boards for accessing external equipment, performing real-time execution of the system, real-time communication between the two computers and I/Os, implementation of user-performed online parameters modification, acquisition of the system internal variables and external outputs through I/O modules, and so on. The controller runs in the target computer. The system sampling time is set to 1 ms.

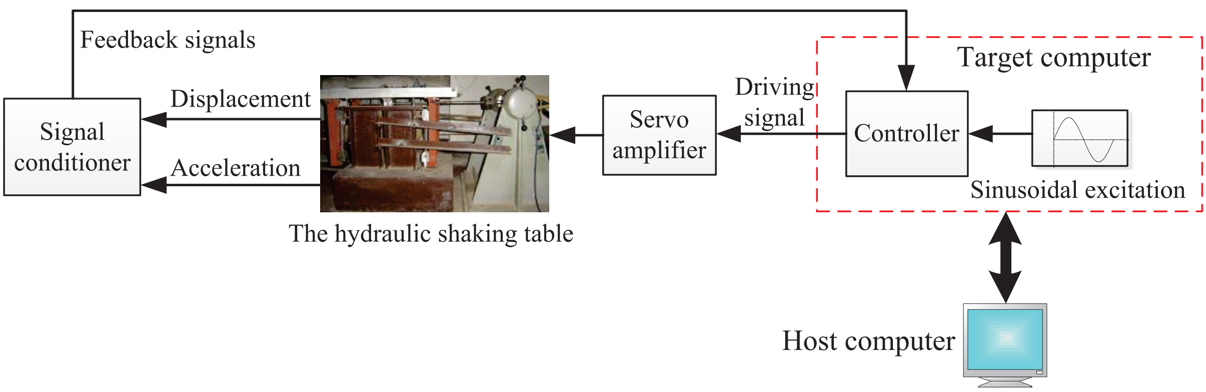


Figure 1 − The whole shaking table system

Table 1 − Parameters of electro-hydraulic servo shaking table

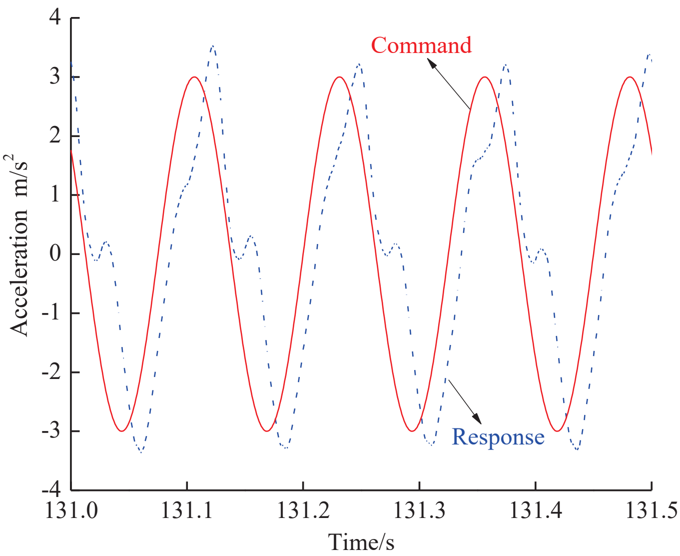
Displacement	±25mm
Cylinder piston	Φ40 mm
Cylinder rod	Φ35 mm
Maximum acceleration	$1.4 m/s^2$
Range of frequency	0~30Hz
Supply pressure	8 MPa
Table size	$900 \times 900 \times 26$ mm

The user interface is operated on the host computer, allowing users to interact the system, such as generating command signal, observing the system state, saving data, controlling the system Start/Stop sequences, and so on. The controller is mainly based on the three-variable-control (TVC, please see (17) for information in detail). The TVC scheme is the application of pole-zero assignment by using feedback displacement and acceleration, and is used to extend the system frequency bandwidth and improve system stability. The controller generates the driving signal to open or close the servo valve appropriately to port a portion of hydraulic fluid flow, which drives actuators to move in the desired direction.

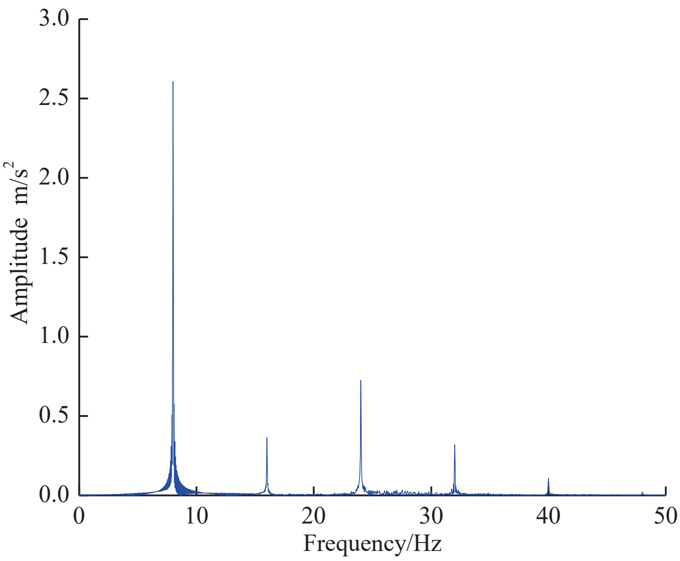
The whole control program is achieved by using xPC Target, which is a command-line interface provided by MATLAB. As a PC-compatible product installed on a host computer running a Microsoft Windows operating system, xPC target is a good solution for prototyping, testing, hardware-in-the-loop simulations and deploying real-time systems using standard PC hardware. The controller is programed by Simulink and Stateflow on the host computer, and is transformed into executable code by using the host computer with real-time workshop and a compiler. The executable code is then downloaded to the target computer running the xPC Target real-time kernel (18, 19).

3. SINUSOIDAL SHAKING TESTS

Electro-hydraulic shaking table is usually under acceleration control to reproduce the desired acceleration. Sinusoidal shaking tests are commonly employed to simulate periodic motion. When the input excitation is $3\sin(2\pi \times 8t)\,\mathrm{m/s^2}$, its response is plotted in Figure 2. It can be clearly seen that the acceleration response is seriously distorted, due to the nonlinearities in the shaking table system. Its amplitude frequency diagram computed by FFT indicates that there are second harmonic at 16 Hz, third harmonic at 24 Hz, up to sixth harmonic at 48 Hz.



(a) In time domain



(b) In frequency domain

Figure 2 − Sinusoidal acceleration response

The total harmonic distortion (THD) is an index to measure the extent of harmonic distortion which is a relative measure of the amplitudes of the harmonics as compared to the amplitude of the fundamental response. The THD can be given by (11)

$$\mathrm{THD} = \frac{\sqrt{A_2^2 + A_3^2 + A_4^2 + \cdots}}{A_1} \times 100\% \tag{1}$$

where A_1 is the amplitude of the fundamental response, A_i the ith harmonic's amplitude. The THD analysis results calculated by Equation (1) is shown in Table 2. Combined with Figure 2 (b), it can be seen that the third harmonic is in dominance among all harmonics except the fundamental response, and the sixth harmonic is the least harmonic. The second harmonic and the fourth harmonic have the similar amplitudes. The value of THD is 36.21%, which also shows high harmonic distortion.

Table 2 − THD analysis results

THD%	Harmonic components m/s^2					
	Fundamental	Second	Third	Fourth	Fifth	Sixth
36.21%	2.630	0.381	0.802	0.320	0.111	0.019

4. THE RECURSIVE LEAST SQUARE ALGORITHM

The recursive least square (RLS) algorithm is the extend use of the least square method. In contrast to other algorithms including the least mean square (LMS) which aims to reduce the mean square error, RLS recursively finds the coefficients that minimize a weighted linear least squares cost function relating to the input signals. In the derivation of the RLS, the input signals are considered deterministic, while for the LMS and similar algorithm they are considered stochastic. Compared to most of its competitors, an important feature of this adaptive transversal filter is the extremely fast convergence, due to the fact that the RLS whitens the input data by using the inverse correlation matrix of the data. For example, the RLS's convergence rate is typically an order of magnitude faster than that of the simple LMS algorithm (20).

A general single-input single-output can be described by Figure 3, where $u(k)$ is the input, $y(k)$ the output, and $n(k)$ is an unknown disturbance that accounts for various sources of system impairments including measurement noise, modeling errors and other disturbance originating from unknown sources.

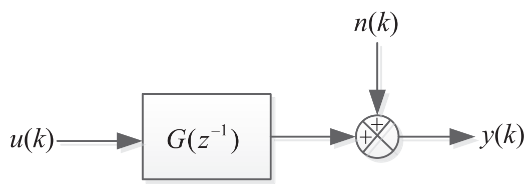


Figure 3 – A general SISO system with disturbance

Assume that the SISO system is represented as the transfer function

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}, \quad m \le n \tag{2}$$

Suppose that we have a set of observable data that fit into the multiple regression model as

$$\mathbf{Y} = \mathbf{H}_N \boldsymbol{\theta} + \mathbf{n} \tag{3}$$

where $\mathbf{Y}$ is the output vector, $\mathbf{Y} = [y(1),\ y(2),\ \cdots,\ y(N)]^{\mathrm{T}}$; $\boldsymbol{\theta}$ is the parameter vector, $\boldsymbol{\theta} = [a_1,\ a_2,\ \cdots,\ a_n,\ b_1,\ b_2,\ \cdots,\ b_m]^{\mathrm{T}}$; $\mathbf{n}$ is the disturbance vector, $\mathbf{n} = [n(1),\ n(2),\ \cdots,\ n(N)]^{\mathrm{T}}$. N is the variable length of the observable data; $\mathbf{H}_N$ is the observable data matrix

$$\mathbf{H}_N = \begin{bmatrix} \mathbf{h}^T(1) \\ \mathbf{h}^T(2) \\ \vdots \\ \mathbf{h}^T(N) \end{bmatrix} = \begin{bmatrix} -y(0) & 0 & \cdots & 0 & u(0) & \cdots & 0 \\ -y(1) & -y(0) & \cdots & 0 & u(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -y(N-1) & -y(N-2) & \cdots & -y(N-n) & u(N-1) & \cdots & u(N-m) \end{bmatrix}$$

Accordingly, the cost function is given by

$$J(\boldsymbol{\theta},\ N) = \frac{1}{2}(\mathbf{Y} - \mathbf{H}_N\boldsymbol{\theta})^{\mathrm{T}}(\mathbf{Y} - \mathbf{H}_N\boldsymbol{\theta}) = \frac{1}{2}\left\|\mathbf{Y} - \mathbf{H}_N\boldsymbol{\theta}\right\|^2 \tag{4}$$

The function of Equation (4) is minimal for parameters $\hat{\boldsymbol{\theta}}$ such that $\mathbf{H}_N{}^{\mathrm{T}}\mathbf{H}_N\hat{\boldsymbol{\theta}} = \mathbf{H}_N{}^{\mathrm{T}}\mathbf{Y}$. If the matrix $\mathbf{H}_N{}^{\mathrm{T}}\mathbf{H}_N$ is nonsingular, the minimum is only given by (20, 21)

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}_N{}^{\mathrm{T}}\mathbf{H}_N)^{-1}\mathbf{H}_N{}^{\mathrm{T}}\mathbf{Y} \tag{5}$$

The covariance matrix $\mathbf{P}(k)$ is defined as

$$\mathbf{P}^{-1}(k) = \sum_{i=1}^{k}\mathbf{h}(i)\mathbf{h}^{T}(i) = \mathbf{H}_k^T\mathbf{H}_k \tag{6}$$

The observations are obtained sequentially in real time, so it is desired to make the computations recursively to reduce the computational burden, thus to save computation time. The computation of the least-squares estimate can then be arranged in such a way that the results obtained at time *k*-1 can be used to get the estimates at time *k*, and the solution in Equation (5) to the least-squares problem will be rewritten in a recursive way.

Let $\hat{\boldsymbol{\theta}}(k-1)$ denote the least-squares estimate based on *k*-1 measurements, the least-squares estimate $\hat{\boldsymbol{\theta}}(k)$ satisfies the recursive equations (22, 23)

$$\varepsilon(k) = y(k) - \hat{y}(k) = y(k) - \mathbf{h}^{\mathrm{T}}(k)\hat{\boldsymbol{\theta}}(k-1) \tag{7}$$

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \mathbf{K}(k)\varepsilon(k) \tag{8}$$

$$\mathbf{K}(k) = \mathbf{P}(k-1)\mathbf{h}(k)[\mathbf{h}^{\mathrm{T}}(k)\mathbf{P}(k-1)\mathbf{h}(k)+1]^{-1} \tag{9}$$

$$\mathbf{P}(k) = [\mathbf{P}^{-1}(k-1) + \mathbf{h}(k)\mathbf{h}^{\mathrm{T}}(k)]^{-1} = [\mathbf{I} - \mathbf{K}(k)\mathbf{h}^{\mathrm{T}}(k)]\mathbf{P}(k-1) \tag{10}$$

Figure 4 is the block diagram of the RLS algorithm. There are two sets of state variables, $\hat{\boldsymbol{\theta}}$ and $\mathbf{P}$, which must be updated at each step. The estimation error is the difference between the measured value of $y(k)$ and its prediction based on the previous parameter estimate. It can be interpreted as the error in predicting the signal $y(k)$ one step ahead based on the estimation $\hat{\boldsymbol{\theta}}(k-1)$. The estimation $\hat{\boldsymbol{\theta}}(k)$ is obtained by adding a correction to the previous estimate $\hat{\boldsymbol{\theta}}(k-1)$. The correction is proportional to the estimation error. The components of the gain vector $\mathbf{K}(k)$ are weighting factors, which show how correction and the previous estimation should be combined. In Equation (9), a matrix inversion is necessary to compute $\mathbf{K}$. The matrix to be inverted is of the same dimension as the number of measurements. That is, for a single output system, it is a scalar.

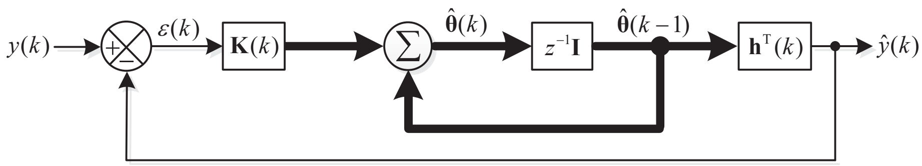


Figure 4 – Block diagram of the RLS algorithm

5. HARMONIC IDENTIFICATION SCHEME

The sinusoidal acceleration response of the shaking table at time *k* can be expressed by

$$a(k)=\sum_{i=1}^{p}A_i\sin(i\omega k+\phi_i)+n(k) \tag{11}$$

where p is the harmonic order, ω the fundamental frequency, $n(k)$ is the measurement noise. A_i and ϕ_i are the i^{th} harmonic's amplitude and phase, respectively. The i^{th} acceleration harmonic can be written as

$$a_i(k)=A_i\sin(i\omega k+\phi_i)=A_i\cos(\phi_i)\sin(i\omega k)+A_i\sin(\phi_i)\cos(i\omega k) \tag{12}$$

Let $x_{2i-1}=A_i\cos(\phi_i)$ and $x_{2i}=A_i\sin(\phi_i)$, the amplitude A_i and phase ϕ_i of the harmonic i can then be expressed by

$$\begin{cases} A_i=\sqrt{x_{2i-1}^2+x_{2i}^2} \\ \phi_i=\arctan\left(x_{2i}/x_{2i-1}\right) \end{cases} \tag{13}$$

To apply the RLS algorithm to acceleration harmonic identification, rewritten Equation (11) as

$$a(k)=\sum_{i=1}^{p}A_i\sin(i\omega k+\phi_i)+n(k)=\mathbf{h}^{\mathrm{T}}(k)\boldsymbol{\theta}+n(k) \tag{14}$$

where $\mathbf{h}(k)$ is the harmonic vector, and $\boldsymbol{\theta}$ is the actual parameter vector. They are

$$\mathbf{h}(k)=[\sin(\omega k),\ \cos(\omega k),\ \cdots,\ \sin(i\omega k),\ \cos(i\omega k),\ \cdots,\ \sin(p\omega k),\ \cos(p\omega k)]^{\mathrm{T}}$$

$$\boldsymbol{\theta}=[x_1,\ x_2,\ \cdots,\ x_{2i-1},\ x_{2i},\ \cdots,\ x_{2p-1},\ x_{2p}]^{\mathrm{T}}$$

Since the fundamental frequency is known, and the harmonic order p is also predefined, each component of the vector $\mathbf{h}(k)$ can be easily obtained. If the parameter vector $\boldsymbol{\theta}$ can be estimated, all harmonics can thus be detected. This task is finished by the RLS algorithm using Equations (7) to (10) to get the estimated parameter vector $\hat{\boldsymbol{\theta}}(k)$, $\hat{\boldsymbol{\theta}}(k)=[\hat{x}_1,\ \hat{x}_2,\ \cdots,\ \hat{x}_{2i-1},\ \hat{x}_{2i},\ \cdots,\ \hat{x}_{2p-1},\ \hat{x}_{2p}]^{\mathrm{T}}$.

The developed acceleration harmonics identification algorithm is shown in Figure 5, where s and c denote the sine and cosine functions, and the dashed box represents the RLS algorithm. The harmonic generator generates all harmonic components for the harmonic vector, $\mathbf{h}(k)$. The shaking table's sinusoidal response is the desired response, and the output of the transversal filter is the estimated acceleration. The difference between the acceleration response and the estimated signal is used as the estimation error to update parameters of RLS. The components of the estimated parameter vector $\hat{\boldsymbol{\theta}}$ are further used to compute the amplitudes and phases of harmonics. Furthermore, the i^{th} acceleration harmonic $\hat{a}_i$ can be decomposed directly from the harmonic vector and the estimated parameter vector, that is

$$\hat{a}_i(k)=\hat{x}_{2i-1}\sin(i\omega k)+\hat{x}_{2i}\cos(i\omega k) \tag{15}$$

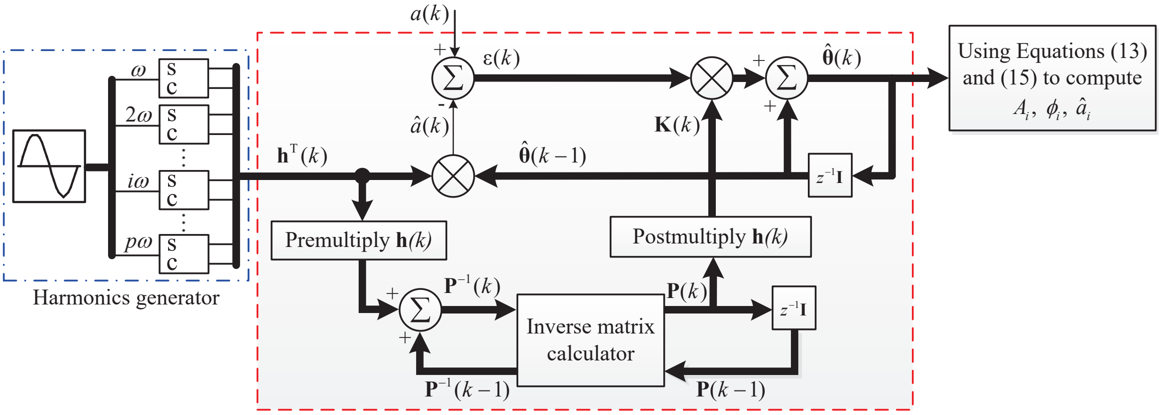


Figure 5 – Acceleration harmonics identification scheme

6. EXPERIMENTAL RESULTS

The developed acceleration harmonic identification scheme shown in Figure 5 is used to estimate the harmonics in the sinusoidal response plotted in Figure 2. The harmonics to be estimated is up to 6th harmonics, so $p = 6$. Since the recursion begins from the first group of observed data, the least-squares estimate $\hat{\boldsymbol{\theta}}(k)$ and the covariance matrix $\mathbf{P}(k)$ need to be initialised, and the customary practice is to set $\hat{\boldsymbol{\theta}}(0) = \zeta(\zeta \to 0)$ and $\mathbf{P}(0) = \sigma^2\mathbf{I}$ where σ needs to be large enough and $\mathbf{I}$ is a unit matrix with the $(2p \times 2p)$ order. The estimated amplitudes and phases are shown in Figure 6 and Figure 7. The amplitudes estimation is better than the estimation of phases. Especially, the six harmonic's phase is in larger fluctuation, but it converges to its steady state after 4s.

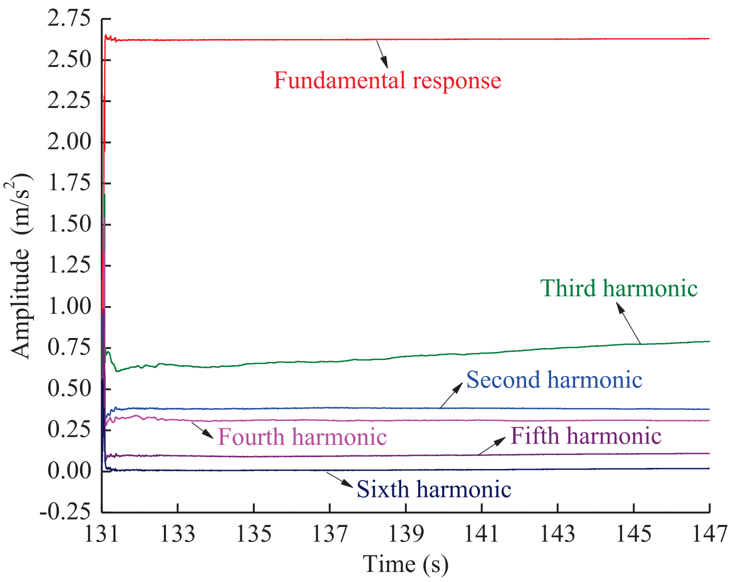


Figure 6 – The amplitudes of every harmonic components

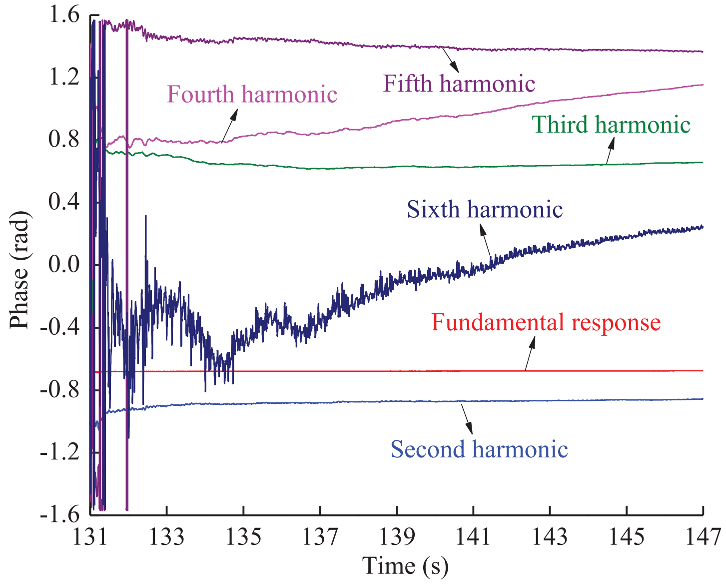


Figure 7 – The phases of every harmonic components

The estimated amplitude of each harmonic in steady state is shown in Table 3. Compared with the corresponding values in Table 2, it can be seen that the identified results are very close to the FFT-computed results.

Table 3 – Each harmonic's amplitude

A_1	A_2	A_3	A_4	A_5	A_6
2.630	0.378	0.795	0.308	0.110	0.018

The estimated harmonics are decomposed directly from the computation of the harmonic vector and the estimated parameter vector. Its results are shown in Figure 8, from which it can be seen that there are large fluctuations in the estimation of the last four harmonics at the beginning of the estimations, but converge quickly to its steady states within 0.1s.

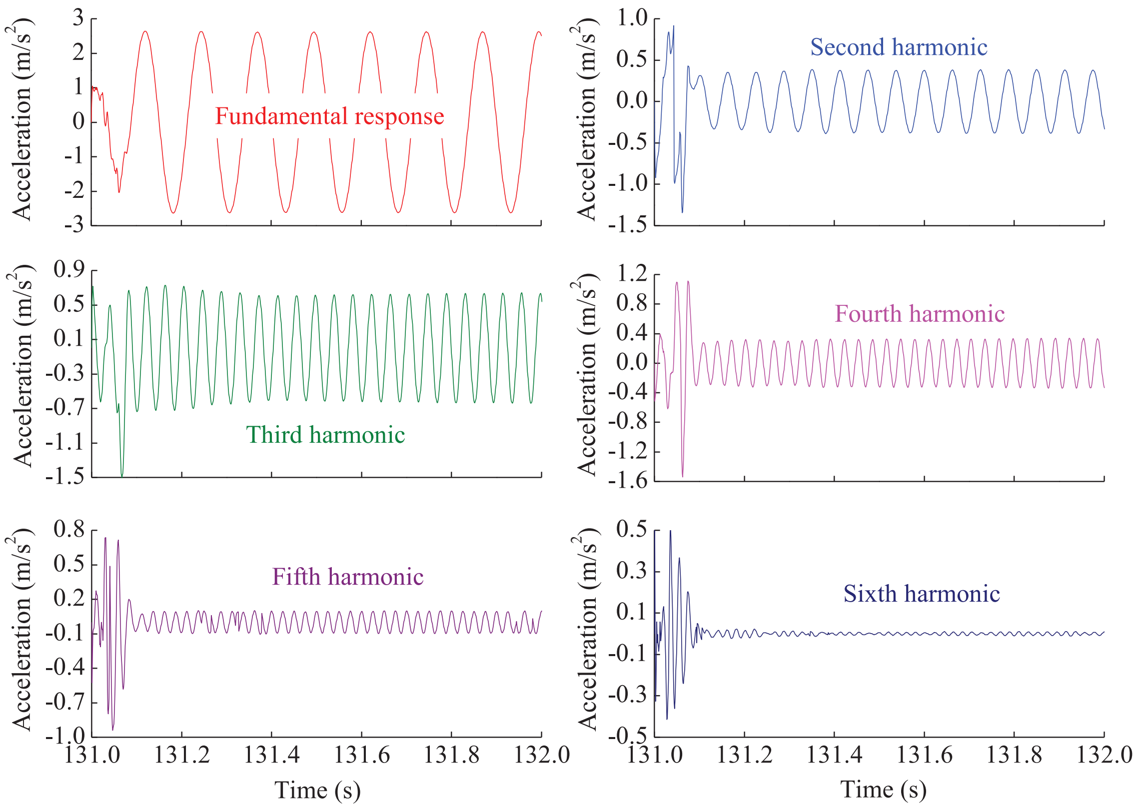


Figure 8 – The waveform of each harmonic component

The estimation error is plotted in Figure 9, which shows that the tracking error vary over a small range since the beginning of the estimation.

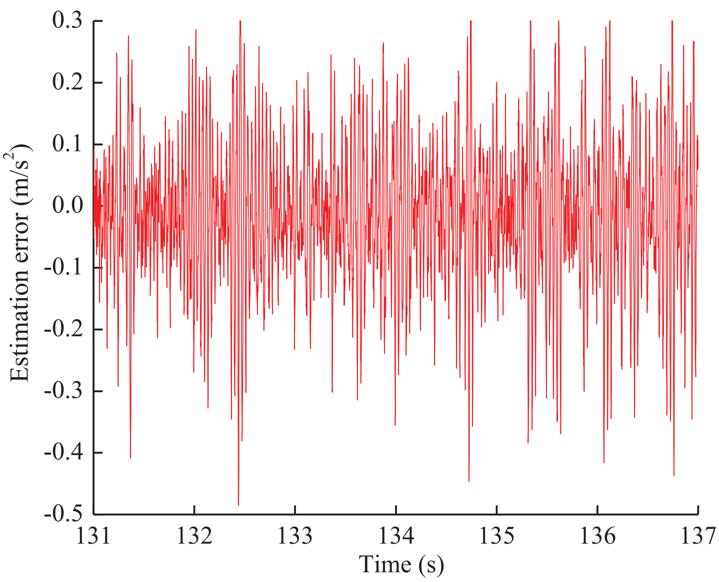


Figure 9 – Estimation error

7. CONCLUSION

Sinusoidal shaking tables are commonly applied to specimens on hydraulic shaking tables for replicating periodic shaking. However, there are usually higher harmonics in its acceleration response beside the fundamental response, due to nonlinearities in the system. This causes harmonic distortion of the system acceleration response and lowers the system control performance. To reduce the harmonic distortion, thus to improve the system replication accuracy, it is necessary to know the harmonic information, including harmonics' amplitudes and phases.

An acceleration harmonics identification is developed here by using a transversal filter, which is based on RLS algorithm to update the parameter vector according to the estimation error between the estimated acceleration and the desired acceleration, which, in practical, is the system sinusoidal response. A harmonic generator yields the harmonic vector. The adaptive identification scheme gives the amplitude and phase of each harmonic computed from the estimated parameter vector. Simultaneously, each harmonic can be directly decomposed from the harmonic vector and the estimated parameter vector.

To testify its validity and convergence, experiments are performed on a hydraulic shaking table, whose real-time control system is achieved based on the MATLAB real-time workshop, xPC target. The experimental results show that it can well track the desired acceleration both in amplitude and in phase. Since FFT is the commonly used method in the field of frequency analysis, the comparison with the results computed from FFT is carried out to demonstrate the identification accuracy.

The parameters of RLS algorithm are easy to determine, and its structure is simple and is easy to realize. It does not require a priori knowledge of the system. The performance of the identification algorithm is superior for its good learning ability, high real-time performance and high speed recognition, because it is adaptive and capable of tracking the variations of amplitude and phase angle of the harmonics. The proposed harmonics identification is easy to be embedded in the original control system. The estimated harmonic information is helpful for harmonic analysis and system nonlinearities analysis, as well as for developing harmonic cancellation strategies.

ACKNOWLEDGEMENTS

This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

This project is supported by National Natural Science Foundation of China (Grant No. 51375102), Natural Science Foundation of Heilongjiang Province of China (Grant No. QC2014C050), Supporting Plan Project for Youth Scholar Backbone of General Colleges and Universities of Heilongjiang Province (Grant No. 1253G020), Supporting Plan Project for Youth Scholar Backbone of Harbin Engineering University, and the Fundamental Research Funds for the Central Universities (Grant No. HEUCFQ20150711).

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