Improvement of the vibro-acoustic behaviour of vibratory feeders for pasta by modelling and experimental techniques

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ABSTRACT
Nowadays, the reduction of vibration and noise in mechanical systems is pivotal. In this scenario, this paper addresses the vibro-acoustical improvement of a pre-drying vibratory feeder for pasta by means of experimental and numerical vibration analysis. The moving parts of the pre-drying feeder are coupled with the structural base by means of leaf springs and the swinging motion is achieved by an eccentric-and-rod mechanism driven by an asynchronous motor. The oscillating motion leads to an unacceptable vibration level that often induces failures and extreme loudness. Operational vibrations have been measured as well as torsional oscillation of the main shaft by means of laser probe and zebra tape. Furthermore, a modal analysis of key components of the system has been carried out. In order to foresee the effects of design changes, an analytical dynamic model was developed and validated by experimental measurements. Exploiting the experimental and numerical results, several modifications have been carried out for vibration reduction. The noise and vibration level of the system has been significantly reduced and the effectiveness of vibration improvement has been experimentally verified.

Keywords: Vibratory feeder, Modal analysis, Dynamic modelling
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1. INTRODUCTION
Vibratory feeders are employed over a wide range of applications, like the motion of pasta during the pre-drying process for example. Regarding high performance vibratory feeders, great levels of noise and vibration are expected due to the high working speed and the huge inertia forces involved. Such levels of noise and vibration have to be controlled since they produce malfunctioning and unhealthy work environment due to loudness and vibrations transmitted to the ground. Therefore, the study of vibratory feeders by means of numerical model in order to improve the vibro-acoustic behaviour is pivotal for a well-designed system. In the last years, many studies address to the modelling of vibratory feeders from different standpoints. Some of these researches concern with dynamic analysis from a theoretical and experimental points of view (1–4). In the other hand, some studies deal with the modelling of the spare part motion (5,6).

The mechanical system under investigation is a pre-drying vibratory feeder for pasta composed by three horizontal parallel frames as depicted in Figure 1 and Figure 2a. The upper frame and the lower frame are rigidly joined and are named as frame A whereas the remaining frame is named frame B. Frames A and B are connected to the structural base by means of 12 leaf springs respectively, six each side. The oscillating motion is achieved by an eccentric-and-rod mechanism driven by an induction motor. In this paper, the feeder has been modelled as a single degree of freedom system considering the leaf springs as flexible and the other components as rigid. The equation of motion has been estimated exploiting the power equation that ensues from the principle of virtual work. The model has been validated with experimental measurements in different operational conditions. The results of simulations and experimental measurements have allowed the identification of the main causes of the abnormal vibro-acoustic behaviour of the feeder. The proposed model has been used for the prediction of the effects of several design changes and operational conditions in order to improve the

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2. LUMPED PARAMETER ANALYSIS

In this section, the lumped parameter model of the feeder is presented. Such a model allows the estimation of the kinematic and dynamic behaviour of the feeder in order to predict the effects of design changes. For a better understanding of the mechanical system, a 3D descriptive drawing is depicted in Figure 2a. For the sake of simplicity, only four leaf springs are represented for each frame instead of 12. The components coupled with frame A (eccentrics, connecting rod, leaf springs and frames) are coloured in green whereas the components linked to frame B are coloured in blue. The shaft that connects the engine with the frames is coloured in red.

Now, considering the Figure 2b, the lumped-parameter model is composed as follows:

- each frame presents 12 leaf springs and each set of 12 springs has been modelled as a single clamped-guided beam with an equivalent stiffness corresponding to 12 springs in parallel;
- frames A and B have been considered as two points with mass (namely $m_A$ and $m_B$) placed between the connecting rod and the leaf springs. The masses $m_A$ and $m_B$ correspond to the global translational masses of the frames A and B, respectively;
- the connecting rods, the eccentrics and the shaft have been considered as perfectly rigid bodies; each couple of connecting rods are counted as a single component as well as the eccentrics.

Some further considerations have to be made on the leaf spring modelling. In operational conditions, the leaf springs exhibit slight axial deformation with respect to bending deflection. Furthermore, the layout of the actual leaf springs impedes the rotation of the frames. Therefore, the actual leaf springs have been modelled as a clamped-guided beam since only the displacement orthogonal to leaf spring axis is allowed. In the first subsection, the kinematic analysis is presented while in the second subsection the dynamic analysis is described.

2.1 Kinematic analysis

Figure 3 depicts the lumped-parameter model of the feeder at the time instant where the leaf springs are undeformed. In this time instant, the position of $P_A$ and $P_B$ (the point mass related to frame A and B) corresponds to the origin of the $xy$ plane. Considering the previous hypothesis, $P_A$ and $P_B$ have
a planar motion and they can move only along the directions orthogonal to the leaf spring axis with displacements \( u_A \) and \( u_B \). It has to be noted that in the actual working condition, the eccentric shaft rotates in negative direction (i.e. clockwise) with reference to Figure 3. Thus, \( u_A \) and \( u_B \) indicate the actual direction of the displacements of \( P_A \) and \( P_B \), respectively.

Taken with reference \( P_A \), the relationship between displacement \( u_A \) and coordinates \( x_{P_A} \) and \( y_{P_A} \) in the \( xy \) plane is:

\[
\begin{align*}
    x_{P_A} &= u_A \cos \left( \gamma_A - \frac{\pi}{2} \right) \\
    y_{P_A} &= u_A \sin \left( \gamma_A - \frac{\pi}{2} \right)
\end{align*}
\]  

(1)

where \( \gamma_A \) is the angle between the \( x \) axis and the longitudinal axis of the leaf spring in the undeformed condition. Equation 1 implicitly states the coordinates \( x_{P_A} \) and \( y_{P_A} \) are linked by a constant term.

Thus, by a simple manipulation of Equation 1:

\[
\begin{align*}
    x_{P_A} &= \frac{\cos \left( \gamma_A - \frac{\pi}{2} \right)}{u_A} \\
    y_{P_A} &= \frac{\sin \left( \gamma_A - \frac{\pi}{2} \right)}{u_A}
\end{align*}
\]  

(2)

The motion of the frames (corresponding to point mass \( P_A \) and \( P_B \)) along directions \( u_A \) and \( u_B \) is linked with the rotation \( \varphi \) of the shaft by a kinematic relationship. Considering frame A, the following vectorial identity can be drawn:
and, after some algebraic rearrangements, such an identity leads to:

\[
\begin{align*}
\bar{P}_A + (Q_A - P_A) = \bar{C} + (\bar{Q}_A - \bar{C}) \\
\begin{cases}
x_p = x_{Q_A} - L_2 \cos(\alpha_A) \\
y_p = y_{Q_A} - L_2 \sin(\alpha_A)
\end{cases}
\end{align*}
\]

where angle \( \alpha_A \) is the angle between the \( x \) axis and the connecting rod of length \( L_2 \), \( x_{Q_A} \) and \( y_{Q_A} \) are the coordinates of point \( Q_A \) (i.e. the eccentric of length \( L_3 \)) in the \( xy \) plane. Coordinates \( x_{Q_A} \) and \( y_{Q_A} \) of the eccentric are defined as follows:

\[
\begin{align*}
x_{Q_A} &= \sqrt{L_2^2 - L_3^2} + L_3 \cos(\varphi) \\
y_{Q_A} &= L_3 \sin(\varphi)
\end{align*}
\]

Equation 4 is a non-linear underdetermined system since it presents two equations in three variables \( (x_{P_A}, y_{P_A}, \alpha_A) \). In order to obtain a determined system, Equation 2 is added to Equation 4:

\[
\begin{align*}
x_{P_A} &= x_{Q_A} - L_2 \cos(\alpha_A) \\
y_{P_A} &= y_{Q_A} - L_2 \sin(\alpha_A) \\
\frac{x_{P_A}}{y_{P_A}} &= \cot\left(\gamma_A - \frac{\pi}{2}\right)
\end{align*}
\]

The results of the equation’s system (Equation 6) gives the position of frame A \( (x_{P_A}, y_{P_A}) \) and angle \( \alpha_A \) as a function of angle \( \varphi \). The position of frame B \( (x_{P_B}, y_{P_B}) \) can be evaluated with the same mathematical procedure used for frame A.

2.2 Dynamic analysis

The equation of motion of the dynamic model can be carried out by means of the power equation that ensues from the principle of virtual work. The power equation can be briefly written as follows:
\[ \sum W + \sum W_{in} = 0 \]  

(7)

where \( W \) is the power of the forces and torques acting on the system while \( W_{in} \) is the power related to the inertial components. Assumed \( \varphi(t) \) as the independent parameter, Equation 7 can be explicitly written as:

\[ M^* = J^* \varphi + \frac{1}{2} J^* \dot{\varphi}^2 \]  

(8)

where \( M^* \) is the generalized torque acting on the eccentric shaft (equivalent to all the torques and forces in the system) and \( J^* \) is the equivalent mass moment of inertia of the mechanism. Term \( M^* \) takes into account the elastic torque due to the leaf springs \( (M_e) \), the motor torque \( (M_m) \) and the torque due to the weight force of the frames \( (M_w) \); friction is neglected. Torques \( M_e \) and \( M_w \) are evaluated by means of the first order kinematic coefficients \( r' \) as follows:

\[ M_e = F_e r'_e \]  

\[ M_w = F_w r'_w \]  

(9)

where \( F_e \) is the equivalent elastic force of the leaf springs, \( F_w \) is the frame weight, \( r'_e \) and \( r'_w \) are the first order kinematic coefficients related to \( F_e \) and \( F_w \) respectively. The equivalent mass moment of inertia \( J^* \) of the shaft takes into account:

- the inertia of the rotating components (shaft, engine, eccentrics and pulleys);
- the inertia of the frames evaluated by means of the kinematic coefficients;
- the inertia of connecting rods considered by dividing the mass of connecting rods into equivalent masses: one portion of the mass is concentrated in the frame and the remaining portion in the eccentric.

Equation 8 provides position \( \varphi \), velocity \( \dot{\varphi} \) and acceleration \( \ddot{\varphi} \) of the shaft as a function of time. Therefore, the motion of the frames is given by using the results from Equation 8 in Equations 4 and 5.

3. EXPERIMENTAL SETUP

Several experimental measurements have been carried out in order to identify the vibration sources of the system: operational acceleration measurements, measure of the torsional vibration and modal analysis. Concerning the operational vibration measurements, the signals have been acquired by means of three miniature piezoelectric accelerometers (PCB 356B21). Figure 4 depicts the layout of the accelerometers: accelerometers 1 and 2 have been placed on the frame A and B, respectively, while accelerometer 3 has been glued on the structural base. The accelerations have been recorded at several constant speed and during a run-up with a sampling frequency of 512 Hz.

![Figure 4: Accelerometer layout](image-url)
Torsional vibration measurements on the shaft were carried out by using an optical sensor (Optel-Thevon 152G7), acquiring TTL signals from zebra tapes with line width of 2 mm. The probe was fixed to the structural base with a stiff bracket whereas the zebra tape were mounted on the shaft as depicted in Figure 5. This setup provides the instantaneous angular speed of the shaft.

An experimental modal analysis has been performed in order to estimate the natural frequencies of the frames. The modal analysis has been carried out by means of an impulse hammer (PCB 086D20) and miniature piezoelectric accelerometers (PCB 356B21). The frames have been excited with the impulse hammer in several points and the response has been measured with the accelerometer. The signals have been acquired with a sampling frequency of 512 Hz.

4. MODEL VALIDATION AND DESIGN IMPROVEMENTS

The lumped-parameter model has been verified by means of experimental measurements. Several test conditions have been considered in order to improve the consistency of the validation. Since this model does not take into account the dynamic behaviour of the frames, some minor discrepancies are expected between model and actual system.

Table 1 collects the RMS values of the frame accelerations in operational condition assessed by means of experimental and numerical analysis. The last column shows the percentage difference between the RMS values of simulated accelerations (column 3) and the RMS values of measured accelerations (column 4). These results demonstrate that the difference between model and experimental data is small at 300 rpm and 330 rpm (close to 10%). At 360 rpm and 440 rpm, the differences became slightly greater (close to 20%) than the other test conditions. The results of the experimental modal analysis (Table 2) reveal the presence of experimental resonances at about 13 Hz, 20 Hz and 30 Hz. The difference between the model and the actual feeder (last column of Table 1) is due to the presence of such experimental resonances, as highlighted by modal analysis and run-ups analysis. Figure 6 depicts the acceleration signal of frame A in x direction at 360 rpm in time and frequency domain. The components of the measured acceleration at 12 Hz, 18 Hz and 30 Hz are magnified due to the resonance of the frames. This behaviour does not occur in the simulation results since the dynamic of the frames is not taken into account in the model. Nevertheless, the model can be considered validated since its goal is to globally describe the vibrational behaviour of the feeder.

The simulation results (third column of Table 1) and the experimental measurements (fourth column of Table 1) highlight an abnormal vibration level of the vibratory feeder at the working speed of 360 rpm. The excitation of some natural frequencies of the frames at the working speed contributes to the high vibration level of the feeder. The results of the torsional oscillation measurements are depicted in Figure 7. The upper diagrams of Figure 7 highlights that at the nominal speed of 300 rpm the speed variation is lower than the speed variation at 360 rpm (lower diagrams). Therefore, the
measurement of the torsional oscillation highlighted a huge speed variation due to the irregularity of the motion, which can lead to malfunctioning.

The proposed model has been exploited in order to identify the parameters linked to the high vibration level. Specifically, the model results demonstrated a relationship between the vibration level and the irregularity of the motion. In other words, the harmonics of the frame acceleration increase with the increase of the speed variation. Therefore, the reduction of the speed variation can contribute to the vibrational improvement of the feeder.

Several design changes have been simulated in order to avoid the excitation of the natural frequencies in operational condition and to reduce the torsional oscillations. With the purpose of avoiding the natural frequencies at 13 Hz, 20 Hz and 30 Hz, the working speed has been decreased from 360 rpm to 300 rpm. In order to guarantee the proper transport of the product, the eccentrics have been enlarged. The torsional oscillation of the mechanical system can be reduced by means of a

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<th>Direction</th>
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<th>Experimental</th>
<th>Model vs Experimental</th>
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<tr>
<td></td>
<td>( x_{PA} )</td>
<td>( x_{PA} )</td>
<td>( y_{PA} )</td>
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<td>300 rpm</td>
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<td>2,99</td>
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<td>330 rpm</td>
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<td>6,87</td>
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<tr>
<td>Frame A</td>
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<tr>
<td>3</td>
<td>27,9</td>
</tr>
<tr>
<td>1</td>
<td>13,2</td>
</tr>
<tr>
<td>Frame B</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>30,1</td>
</tr>
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</table>
flywheel or by using a more powerful motor. In this case, both the previous technical solutions have been adopted: the dimension of the pulley on the main shaft has been increased and a new asynchronous engine has been equipped. All these modifications have been designed by means of the model results.

The effect of such changes are also verified comparing the resultant RMS values of the acceleration measured in the structural base (see Figure 8), namely $RMS_{\text{tot}}$. The resultant RMS is evaluated by composing the RMS value of the acceleration in x and y directions as follows:

$$RMS_{\text{tot}} = \sqrt{RMS_x^2 + RMS_y^2}$$  \hspace{1cm} (9)

The results depicted in Figure 8 in the one hand confirm the effectiveness of the modifications designed by using the proposed model; in the other hand, they highlight that the vibration transmitted to the ground and the global loudness of the machine have been significantly decreased by virtue of
the design changes adopted. Therefore, it has been demonstrate the capability of the proposed model in order to properly predict the global vibrational behaviour of the feeder.

Figure 8: Resultant RMS values of the acceleration measured on the structural base

5. CONCLUSIONS

A lumped-parameter model of the feeder has been developed by using actual data of the vibratory feeder for pasta and the validation of the model has been successfully assessed with experimental measurements. The results of the dynamic model and the vibrational measurements have been exploited in order to improve the vibro-acoustic behaviour of the feeder. The vibrational measurements highlight that at the working speed the vibration level rises due to resonances of the frames. Furthermore, the measure of the torsional oscillation shows a high speed variation of the shaft that contributes to the abnormal vibration level of the feeder. Several design changes have been investigated by means of simulation and the following modifications have been carried out:

- the working speed has been decreased from 360 rpm to 300 rpm;
- the moment of inertia of the pulley has been enlarged;
- a more powerful motor has been used;

The vibro-acoustic behaviour of the actual feeder has been improved as expected from the simulation results. Therefore, the proposed model turned out to be effective on the prediction of the global vibrational behaviour of the feeder. The effects of such design changes have been experimentally verified and the RMS of the acceleration of the structural base has been more than halved. In addition, the model has been effective on the identification of the proper design changes for the vibro-acoustic improvement of the feeder. This kind of approach contributes to reducing costs and time with respect to an entirely experimental approach.

REFERENCES