Assessing Acoustic Black Hole performance via wavenumber transforms

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ABSTRACT

This work investigates the application of embedded grids of Acoustic Black Holes to the noise and vibration reduction of plate structures. Lighter high loss damping treatment solutions are sought by designers for efficient vehicle noise and vibration control applications. In this work measured velocity responses of plates were analyzed using wavenumber transform methods to evaluate the vibration performance of periodic grids of embedded Acoustic Black Holes. The results showed that wavenumber spectra offer useful physical insights for analyzing ABH behavior that are not available through other kinds of analysis. The wavenumber spectra were also used to investigate the structural acoustic coupling via radiation efficiency estimates. The results showed that ABHs convert acoustically fast waves into acoustically slow waves, resulting in reduced radiated sound in addition to reducing the mechanical vibration levels. The results give designers critical insight for designing and implementing treatments using Acoustic Black Holes for efficient, high performance, noise and vibration reduction of critical structures.

Keywords: Damping, Noise reduction, Transmission loss, I-INCE Classification of Subjects Number(s): 38.5, 43.2.1, 47.3

1. INTRODUCTION

Lighter high loss damping treatment solutions are sought by design engineers for efficient vehicle noise and vibration control applications. Weight efficient structures are often difficult to realize for real vehicle systems. Integral or embedded Acoustic Black Holes (ABHs) offer potential design solutions because they act as effective, passive, lightweight vibration absorbers. While the effectiveness of ABHs as a means of reducing vibration and radiated sound has been well developed in the literature, there is an ongoing need to develop methods to aid in the characterization, design, and optimization of ABHs and ABH systems. The goal of this research was to assess and develop wavenumber transform analysis methods for understanding ABH dynamics and performance. Two dimensional wavenumber spectra were computed from the measured surface velocities of plates with embedded ABHs with and without attached damping layers and compared to a baseline uniform plate. The wavenumber transforms were also used for calculation of the radiation efficiencies of the plates. These results were useful for analyzing and understanding the vibration and structural acoustic coupling of plates with grids of embedded ABHs.

2. BACKGROUND

For a thin plate the flexural bending wavenumber is given by

\[ k = \left( \frac{12 \rho (1 - \nu^2) \omega^2}{E h^2(x)} \right)^{1/4}, \]

where \( \rho \) is the density, \( \nu \) is the Poisson’s ratio, \( \omega \) is the angular frequency, \( E \) is the Young’s Modulus and \( h(x) \) is the thickness profile. Mironov (1) showed that for a thickness profile of \( h(x) = cx^n \) for \( n \geq \)

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The wavenumber becomes infinite for a zero truncation thickness. The bending wave speed vanishes within the taper and an incident bending wave is “absorbed” because the propagation time to reach the end of the taper becomes infinite. In practice, fabricated ABHs have a finite truncation thickness either by design necessity or simply due to machining limitations. Even with a finite termination thickness the ABHs can still act as effective absorbers through attached viscoelastic damping layers (2, 3) or a high inherent material loss factor (4). ABHs have been shown to be effective at reducing mechanical vibration (5) and radiated sound power (6). At high frequencies, when underlying theoretical assumptions are valid, ABHs act as a broadband absorbers (7, 8). Recent research has shown that the cut-on of modes within the ABH taper (9–12) are crucial for understanding ABH dynamics and design at low frequencies.

Wavenumber spectra show the wavenumber components of spatially varying signals in the same way that frequency spectra show the frequency components of temporally varying signals. The wavenumber transform is calculated by a two dimensional Fourier transform given by

\[
S(k_x, k_y, f) = \iint F(x, y, f) e^{-j k_x x} e^{-j k_y y} dx dy,
\]

where \(k_x\) and \(k_y\) are the wavenumbers in the \(x\) and \(y\) dimensions, respectively, \(F(x, y, f)\) is the spatial variation in physical coordinates at a given frequency \(f\). An \(e^{j\text{time}}\) time convention is assumed where \(j\) is the imaginary unit. The wavenumber spectra is useful for analyzing complex systems because it decomposes a complex vibratory field into wavenumber components that describe the direction and speed of wave propagation. This is particularly useful for ABH design since the thickness taper of the ABH changes the wavenumber at a given frequency.

Using wavenumber transforms is also useful because it allows for the calculation of the radiation efficiency of the structure. Williams and Maynard (13) developed a method for calculating the radiation efficiency of a plate using a discrete Fourier transform method. The acoustic pressure at the surface of the panel can be calculated by the wavenumber transform of the surface velocity of the plate and an averaged Green’s function developed by Williams and Maynard. The averaged Green’s function greatly reduces errors in the calculation of the Rayleigh integral in wavenumber space. Once the acoustic pressure is known, the radiated sound power and radiation efficiency can be calculated with the known surface velocities.

### 3. MEASUREMENT PROCEDURE

The test specimens used for this study consisted of a uniform plate, an undamped ABH plate, and a damped ABH plate. The plates were all made of aluminum and had dimensions of 91 cm by 61 cm by 6.6 m. The ABHs were machined into the plates in a four by five grid as shown in Figure 1. The two dimensional ABHs had a taper power of 2.2, minimum thickness of 0.86 mm and a diameter of 10 cm. A 3 mm thick free damping layer was applied to the tapers to provide energy dissipation. The plates were mounted in a massive steel frame with stable boundary conditions and excited by a point mechanical drive. The vibration shakers were attached to the plates via a stinger and impedance head to measure the drive force. Band limited white noise was used to drive the panels. The drive point was located 13.5 cm and 21.5 cm from one corner of the plates. The surface velocities of the plates were measured by a scanning Laser Doppler Vibrometer (LDV) using over 250 points. The surface velocities were normalized by the ratio of the input force spectrum for each plate to the average input force spectrum across all three plates.

The normalized surface velocities of the plates were transformed into the wavenumber domain using the discretized version of Equation 2. For analysis the wavenumber transform was squared to produce power-like wavenumber spectra, with units corresponding to \((\text{velocity/wavenumber})^2\). The data were zero padded in order to aid the visual analysis of the wavenumber spectra. The spectra were also filtered into one-third-octave frequency bands to facilitate broadband analysis and were plotted on a dB scale. Radiation efficiencies for the plates were calculated using an averaged Green’s function as derived by Williams and Maynard (13). The calculations for the radiation efficiencies assume that the plates were set in an infinite, rigid baffle while the experimental plates were un-baffled. As a result the calculated radiation efficiencies were overestimated below the critical frequency, however the relative comparisons between the plates are still useful for understanding significant trend differences.
4. RESULTS

Figures 2-5 show the wavenumber spectra of the uniform plate, undamped ABH plate, and damped ABH plate on a dB scale for several one-third-octave band frequencies. The figures also show the radiation circles at the center frequency of the corresponding one-third-octave band. The radiation circle shows the wavenumber in air at the center frequency of the one-third-octave band. Vibration within the radiation circle is supersonic (acoustically fast waves). For acoustically fast waves, the bending wave speed is higher than the wave speed in the surrounding air, and sound is efficiently radiated from the entire surface. Vibration outside of the radiation circle is subsonic (acoustically slow waves). For acoustically slow waves, the bending wave speed is slower in the plate than the surrounding air, and sound is not efficiently radiated.

Figure 2 shows the wavenumber spectra for the measured plates at 1.6 kHz. This frequency is below the critical frequency of the uniform plate (~2 kHz) and below the cut-on frequency of the first ABH mode (4.8 kHz for the undamped ABH, 2.8 kHz for the damped ABH). Most of the vibration response is outside of the radiation circle and the plates do not radiate efficiently. At this frequency the undamped ABH plate shows higher response at higher wavenumbers due to the ABH effect. The damped ABH is beginning to show some reduced vibration due to damping from the ABHs.

Figure 3 shows the wavenumber spectra for the measured plates at 2 kHz. This frequency band contains the critical frequency of the uniform plate and the effects of the damped ABH modes begin to be more significant. At the critical frequency of the plate the main band in the wavenumber spectra begins to cross the radiation circle as the bending waves in the plate transition from subsonic to
supersonic. The results show that the undamped ABH plate has a significantly higher vibration response than the uniform plate due to the ABH effect as well as the fact that the undamped ABH plate effectively behaves as a thinner, lighter plate. The reduction of the high amplitude band in wavenumber space for the damped ABH plate shows further energy dissipation from the ABHs. These trends continue in Figure 4, which shows the wavenumber spectra of the plates at the 2.5 kHz one-third-octave band. Both figures show increased response at higher wavenumbers outside of the main band in the wavenumber spectra. This is due to the ABHs reducing the bending wave speed and increasing the wavenumber. As more of the vibration response is distributed to higher, subsonic wavenumbers, the coupling between the plate and the surrounding air is reduced.

Figure 3 – Wavenumber spectra of uniform plate (A), undamped ABH plate (B), and damped ABH plate (C) at 2 kHz. The critical frequency of the uniform plate is within this frequency band and the vibration in the plate begins to transition from subsonic to supersonic. The undamped ABH plate has a higher response while the damped ABH plate shows vibration reduction.

Figure 4 – Wavenumber spectra of uniform plate (A), undamped ABH plate (B), and damped ABH plate (C) at 2.5 kHz. The undamped ABH plate shows more vibration at high wavenumbers compared to the uniform plate while the damped ABH plate is more effectively damped.

Figure 5 shows the wavenumber spectra of the plates at 3150 Hz. At this frequency range nearly all of the vibration response of the uniform plate is supersonic, the response of the undamped ABH is significantly higher than the uniform plate, and the response of the damped ABH plate has been effectively reduced.
Figure 5 – Wavenumber spectra of uniform plate (A), undamped ABH plate (B), and damped ABH plate (C) at 3150 Hz.

Figure 6 shows the radiation efficiencies of the measured plates in one-third-octave bands. The results show that the addition of ABHs and damping material affects the radiation efficiencies of the plates. The damped ABH plate has a much higher apparent critical frequency (~ 3 kHz) than the uniform and undamped ABH plates (~ 2 kHz) based on the peak in the radiation efficiency. At 2 kHz the radiation efficiency of the damped ABH plate is 6 dB less than the uniform plate. The results for the uniform plate follow a trend expected from plate theory. Below the cut-on frequency of the undamped ABH mode (4.8 kHz) the undamped ABH plate can be approximated as an equivalent uniform plate and follows a trend expected from theory with a slightly higher critical frequency. The radiation efficiency of the damped ABH plate shows pronounced difference from the uniform plate. The results deviate significantly around 2 kHz, where the first damped ABH mode begins to cut on (11). Since material damping does not affect radiation efficiency, the reduction in radiation efficiency is a result of the ABH effect acting as a structural acoustic decoupler. As the ABHs reduce the bending wave speed of the flexural waves in the plate, supersonic waves become subsonic and the radiation efficiency of the plate is reduced.

5. CONCLUSIONS
It was demonstrated that wavenumber transforms techniques can help designers understand ABH
vibration and structural-acoustic radiation characteristics. It was shown that ABHs can be used to redistribute supersonic bending waves into subsonic wavenumbers and effectively reduce the radiation efficiency of embedded ABH plates. Below the cut-on frequency of the first ABH mode the ABH plates can be approximated as uniform plates with equivalent properties. As the ABH modes begin to cut on, the structural acoustic decoupling becomes significant and the radiation efficiency of the plate can be effectively reduced along with the plate vibration amplitude.

**ACKNOWLEDGEMENTS**

This work was supported by the Walker Research Assistant program, the Applied Research Laboratory at the Pennsylvania State University.

**REFERENCES**