The Modified Green's Function Used for the Phase Conjugation Method to Reconstruct the Sound Source

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ABSTRACT
The identification of the known strength of the sound source was calculated based on phase conjugation method by the planar array according to measure the positive sound propagation. The error was obtained by comparison of the identified strength of the sound source and the known strength and the error was caused by the Green's function of the propagation space. The modified Green's function was get by introducing the correction term. The reconstruction of the sound source was calculated by the phase conjugation formula by the modified Green's function. The acoustical contribution analysis were also studied numerically. The results showed that the proposed method achieved sub-wavelength focusing to identify the sound source and the method could get the accurately acoustical contribution analysis to identify the main source.

Keywords: Modified Green's Function, phase conjugation method, identified sound source, acoustical contribution

1. INTRODUCTION
The identification of sound sources plays an important role in the effective noise control. Time reversal can be used to focus sound and is also called phase conjugation (PC) in the frequency domain. The equivalence between phase conjugation in the frequency domain and time reversal in the time domain is established by Jackson and Dowling [1]. Due to its focusing property, the phase conjugation arrays could be used to build the image of a noise source and for source identification. However, the spatial resolution of the focused field of a classical PC array has a half wavelength limit because of diffraction. De Rosny and Fink [2] first showed that this limitation can be overcome by an acoustic sink. Fink et al [3] also showed that the subwavelength focusing can be achieved inside a microstructured medium. Conti et al [4] demonstrated the subwavelength focusing could be obtained without a priori knowledge of the source by a near-field time reversal procedure. De Rosny and Fink [5] investigated three species of the time reversal arrays in the near field of the initial source and concluded that only the dipole time-reversal array leads to subwavelength focusing. Liu Song [6] had improved the phase conjugation method could be used to identify and locate the complex sound source.

The boundary element method (BEM) has been used extensively in acoustic radiation from bodies with known velocity, pressure, or impedance distribution [7]. The main specifications in BEM was the green function. This parameter caused the accuracy of the sound source identification just like the non-free sound field. The acoustical sink method was to modify the Green function basically.

For many cases the characters of the sound source, vibration plates for example, could not be obtain for real environment and the experiment. The complex vibration environment and the complex sources make the vibration and noise control be difficult. The acoustical contribution analysis [8] could get the ratio that the element radiated to the field points took up for the whole radiation and confirm the main source.

In this paper, a modified Green function was developed for the identification of the pressure and normal velocity distribution of a vibrating plate. The acoustical contribution analysis for the vibrating plate radiated to the field point were calculated after the normal velocity was obtained.

2. Theory

2.1 The phase conjugation method and the modified Green functions

There are two types to calculate the phase conjugation field by using the discrete finite array in the sound field considering the area of the array:

The phase-conjugated field by the array made of monopole transceivers based on the pressure measurement is:

$$p_{PCM}(P) = \sum_{n=1}^{N} \left[\left(G(Q_n, P) p^*(Q_n) \right) \times S_n \right] \tag{1}$$

The phase-conjugated field by the array made of dipole transceivers based on the pressure gradient measurement is:

$$p_{PCD}(P) = \sum_{n=1}^{N} \left[\left(\frac{\partial G(Q_n, P)}{\partial n} \frac{\partial p^*(Q_n)}{\partial n} \right) \times S_n \right] \tag{2}$$

Where: r_n is the location of the array element, r_s is the location of the sound source, r is the location of the field point.

$G(r, r_n) = \dfrac{e^{ikR}}{4\pi R}$ is the green function of the free field, $R = |r - r_n|$, S_n is the area of the array, n is the normal vector of S_n. P^* and $\dfrac{\partial P^*}{\partial n}$ is the pressure and pressure gradient measured by the phase conjugation array respectively.

For sound source identification, the most important specification was the Green functions. The accurate function can give the accurate solution of the inverse problem. We provide the modified Green function as follows:

$$G(r, r_n) = \frac{e^{ikR}}{4\pi R} + A * \frac{e^{ir_1}}{r_1} \tag{3}$$

Where: A is the fitting coefficient by the measurement array whose location was known. r_1 is the normal distance between the known location array and the real measurement array. The coefficient was fitted by the error comparison of the known strength forward propagation of the vibration plate and the identification of the array.

This method was essentially carried out the measurement two times to modify the free field Green function.

2.2 The acoustical contribution analysis

The definition of the acoustic contribution analysis was calculating the source to target field point pressure contribution by means of the acoustic transfer vector to provide the basis for the reduction of vibration and noise. The sound source to target field point acoustic contribution coefficient is defined as follows:

$$E_{ACA} = \mathrm{Re}\left(\frac{p_e p^*}{|p|^2}\right) \tag{4}$$

Where: p_e is the pressure produced by the single point source at the field point location, p is the total pressure at the field point location, * represents the conjugation, Re represents taking the real parts. The sound field could be calculated by the ATV method. The ATV formula was：

$$p = ATV(\omega) \times v_n(\omega) = \sum_{i=1}^{N} ATV_i(\omega) \times v_{e,i}(\omega) = \sum_{i=1}^{N} p_{e,i}(\omega) \tag{5}$$

Where $ATV(\omega)$ is the acoustical transfer vector, $p_{e,i}(\omega)$ is the contribution vector for element i.

Firstly the source identification and separation based on the phase conjugation method was calculated to get the normal velocity of the vibrating plate, then identified the main noise source by the acoustic contribution analysis according to formula (4) and (5).

3. Numerical simulations

3.1 The identification of the vibration plate

The dimensions of the plate are $0.8m \times 0.6m \times 0.005m$, the plate material is steel ($\rho_s = 7833\,\mathrm{kg/m^3}$, $E = 210\,\mathrm{GPa}$, $\nu = 0.3$). The geometric center of the plate is the origin of the rectangular coordinate system. The plate is simply supported in a baffle. The acoustic fluid is air with density $\rho = 1.21\,\mathrm{kg/m^3}$ and speed of sound $c = 350\,\mathrm{m/s}$. The excitation is a transverse point force of magnitude $1\,\mathrm{N}$ located at (0.2, -0.1, 0). The finite element method is used to get the natural frequencies and the surface normal velocity of the plate. The plate is discretized into 192 four-node elements and 221 nodes for finite element analysis. The natural frequencies of the steel plate are listed in Table 1.

Table 1- The natural frequencies of the steel plate

Mode type	Natural frequency（Hz）
(1, 1)	52.0
(2, 1)	108.1
(3, 1)	201.5
(4, 1)	332.0
(5, 1)	499.7

The surface pressure and the field point pressure at 500Hz (corresponding to the (5, 1) mode) are calculated by the Rayleigh integral based on the surface normal velocity obtained by finite element analysis. The Rayleigh integral is made by discretisizing the plate surface into the same mesh as plate finite element mesh. The surface normal velocities calculated by FEM are shown in Figure 1. The surface pressures calculated by the Rayleigh integral are shown in Figure 2. The units used in the figures are Pa for the pressure and m/s for the normal velocity. The calculated field pressures at the array elements are used as the measurement pressures of the PC arrays.

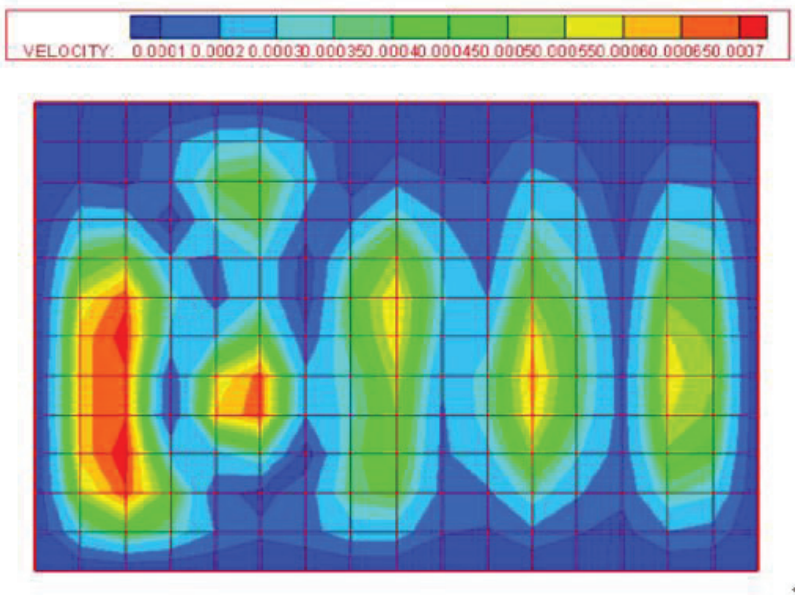

Figure 1 – The normal velocity calculated by FEM.

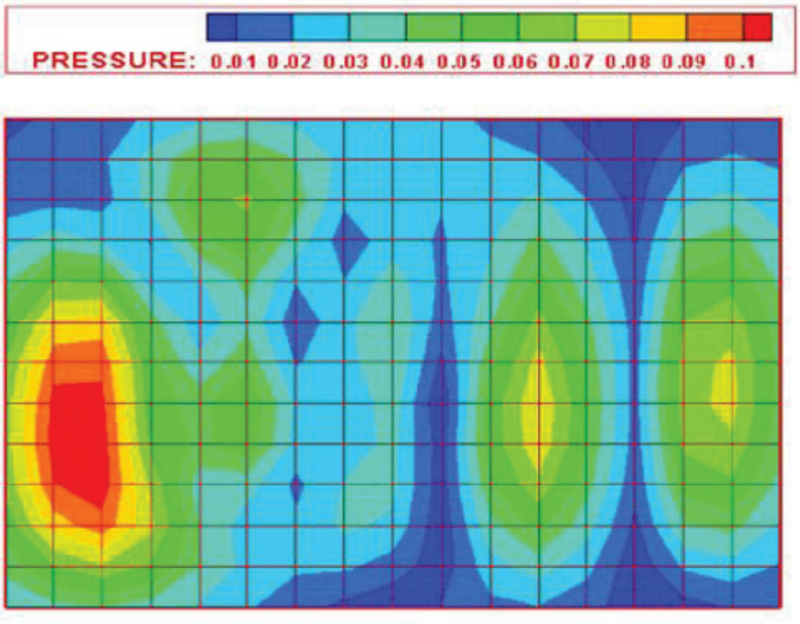

Figure 2 – The surface pressure calculated by Rayleigh integral.

(1) $d = 0.1\lambda$

The PC array is a plane array that has the same dimension as the plate surface, the number of the uniform distributed array elements is 221, the array element spacing is 0.05m = 0.07λ. The surface normal velocities obtained by PCD array are shown in Figure 3. The surface pressure and normal velocities obtained by modified Green function were shown in Figure 4 and Figure 5. The known location PC array plane were the same 16×12 of 192 elements and 221 nodes as the mesh of the vibration plate and the normal distance to the real measurement array was $0.5\lambda = 0.35m$. It can be seen that the results by the modified method clearly show the response shapes of the surface pressure and the surface normal velocity and the spatial resolution achieved is at least $0.2m = 0.29\lambda$ which is smaller than half a wavelength (The half wavelength $0.5\lambda = 0.35m$).

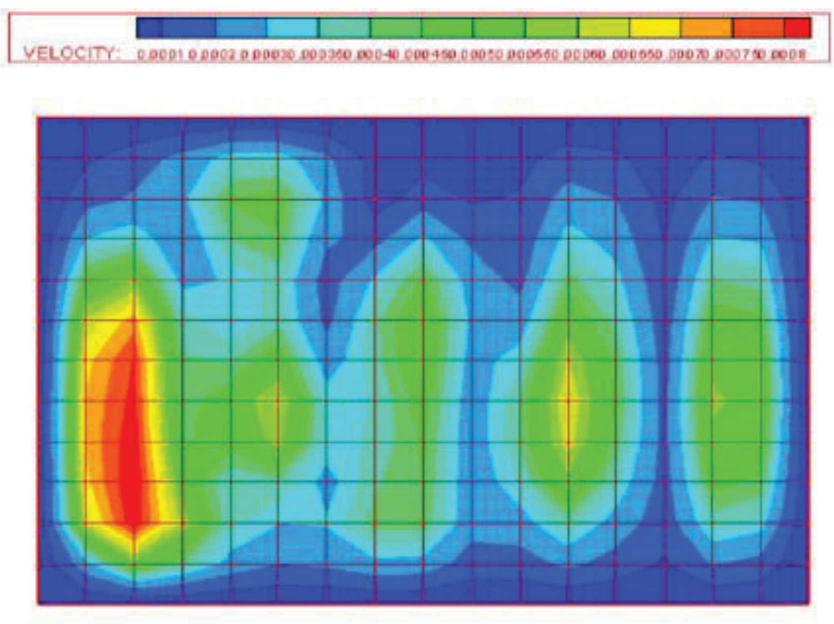

Figure 3 – The normal velocity obtained by PCD.

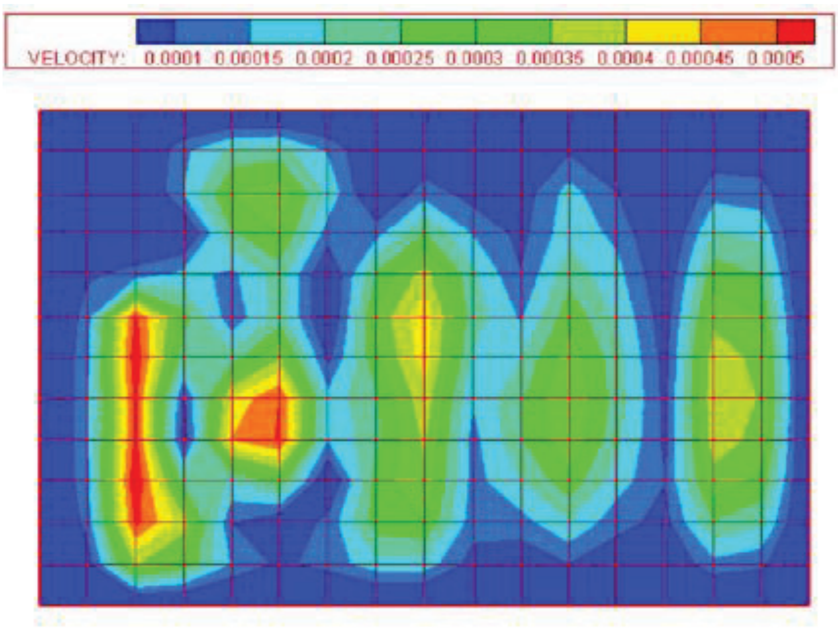

Figure 4 – The normal velocity obtained by modified Green function.

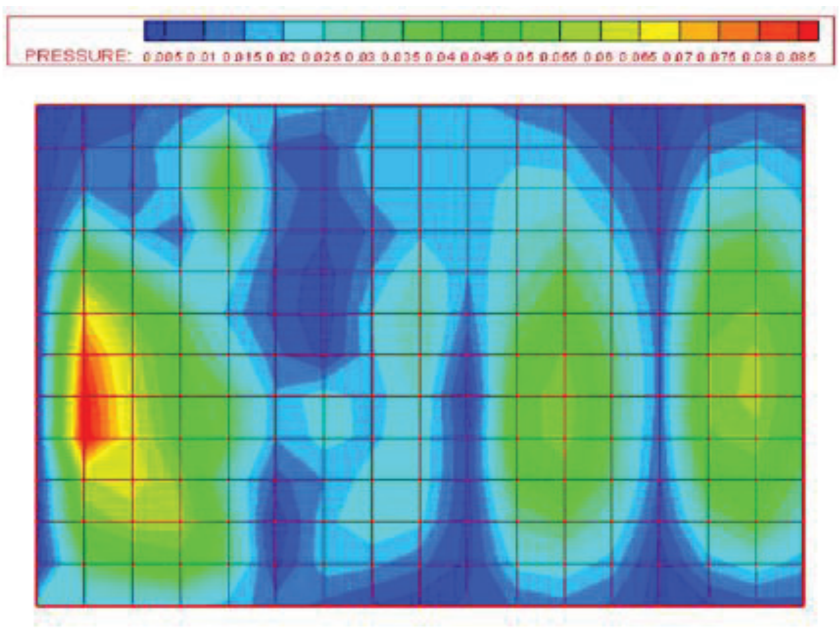

Figure 5 – The surface pressure obtained by modified Green function.

3.2 The acoustical contribution analysis

To obtain the main source for complex vibration and noise environment, the acoustical contribution analysis was calculated for the vibrating plate. According to the shape of the mode (5,1) ,we decomposed the plate into five element plate along the X direction and the spacing of the element was 0.075m which represents the five mode of the plate. The node number of these five element plate were that (plate01: 57-78, plate02:90-111, plate03:134-155, plate04:167-188, plate05:200-221). The node sketch of the five element plate was shown in Figure 6.

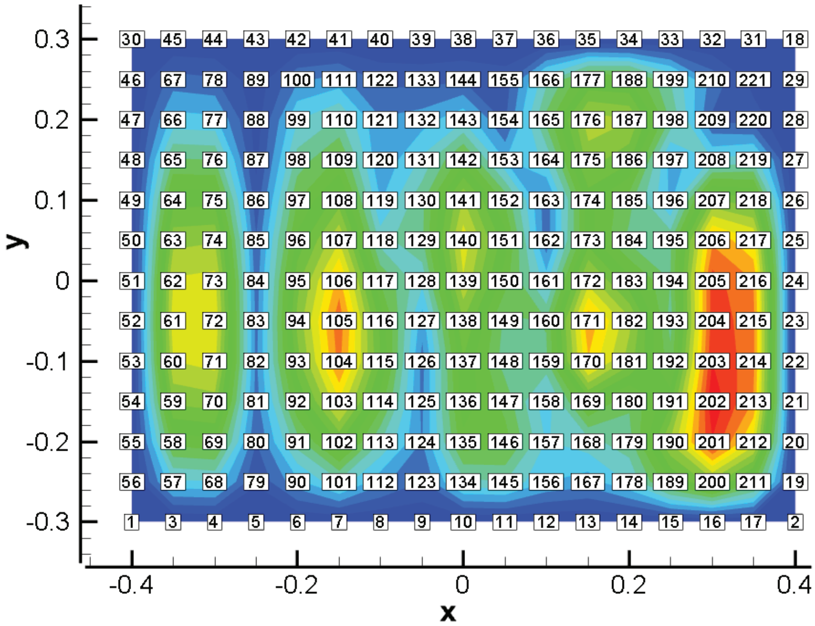

Figure 6 – The node sketch of the five element plate.

The field point was taken for (0,0,1) on the Z direction top of the plate and the distance represents the 1.43 times of the wavelength. The acoustical contribution coefficient was calculated after the normal velocity or surface pressure was obtained according to Equation (4). The numerical result was shown as follows:

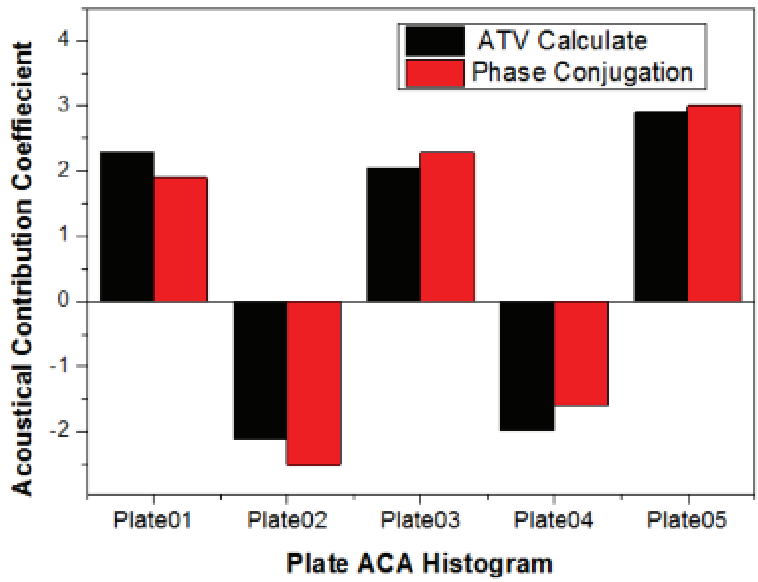

Figure 7 – The acoustical contribution coefficient of the five element plate.

According to the results in Figure 7 we can see that the phase conjugation method could achieve the same accuracy to the ATV calculated by Equation (5).These five element plates get the similar coefficient which mean that these five half-wave mode contributed the same radiation. But the main source radiated to the field point was plate05 which represents the element plate located at the force excitation cause the largest radiation sound. Plate01, plate03 and plate05 produced the positive acoustical contribution and plate02, plate04 produced the negative acoustical contribution which mean the upper vibration mode brought the main radiation sound. If these upper vibration modes were reduced than the whole radiation sound could be controlled.

4. CONCLUSIONS

The phase conjugation arrays could be used to build the image of a noise source and for source identification. The phase conjugation had been solved by the modified Green function to identify the sound sources. A plate subject to a point force was involved to verify the effectiveness of the method. An interior problem was formed by enclosing the PC array plane and the plate surface. The acoustical contribution analysis were also studied numerically and the results showed that the upper vibration mode brought the main radiation sound. If these upper vibration modes were reduced than the whole radiation sound could be controlled. The research achievement shows that the proposed method achieved sub-wavelength focusing to identify the surface pressure and normal velocity distribution with the array located in the near field and the method could get the accurately acoustical contribution analysis to identify the main source.

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