Symmetric Design of Multiple-Channel Active Noise Control Systems for Open Windows

Jianjun HE¹; Bhan LAM²; Tatsuya MURAO³; Rishabh RANJAN⁴; Woon Seng GAN⁵
¹²³⁴⁵ Digital Signal Processing Lab, School of Electrical & Electronic Engineering, Nanyang Technological University, SINGAPORE

ABSTRACT
There is a recent trend in deploying multiple-channel active noise control (MCANC) units to mitigate noise that propagates into urban accommodation through open windows, while preserving natural ventilation. To reduce the implementation cost, space, and complexity of the MCANC system, a symmetric configuration of the ANC units installed on window openings to control multiple channels with a single adaptive filter is proposed. The acoustic path conditions of the symmetric MCANC system are established and verified. Furthermore, we investigate the noise reduction performance of the symmetric MCANC system with respect to different secondary path variations, caused by gain, delay, and positioning differences on secondary sources. Results show that satisfying the conditions of symmetric design is critical, which suggests that proper compensation on the secondary paths need to be applied in the symmetric MCANC system.

Keywords: Multiple-channel, active noise control (ANC), symmetric design
I-INCE Classification of Subjects Number(s): 38.2

1. INTRODUCTION
Noise pollution is identified by the World Health Organization as a critical problem in increasingly dense urban cities (1, 2). Long-term exposure to high sound pressure level in dwellings poses a significant health hazard to the residents. Current passive noise mitigation approaches either affects natural ventilation (e.g., closed window) or incurs extreme high costs (e.g., noise barriers), making these approaches ineffective in shielding high-rise urban dwellings or impractical in terms of cost-effectiveness (3). Active noise control (ANC) is a potential noise mitigation solution in view of the falling costs of electronic components as well as incurring minimal obstruction when installed on apertures (3).

Based on the principle of acoustic wave superposition, ANC uses on electro-mechanical devices, such as loudspeakers to generate an anti-noise signal (equal amplitude, but out-of-phase) to cancel the primary noise (4, 5). ANC has been primarily effective at low frequencies (e.g., traffic noise) (6, 7) and in small areas (e.g., ear cannel, headrest, air ducts and exhaust pipes) (8, 9).

In recent years, ANC has been applied to tackle noise problems in larger open areas, especially through apertures. Closed-window ANC targets window fixtures and aims to reduce the residual low frequency noise from triple-glazed windows (10–13). In contrast, open-window ANC systems allow windows to retain natural ventilation for homes, especially in tropical climates. Recent advancements of open-window ANC systems can reduce noise by up to 10 dB (14–17). To enlarge the noise reduction area, multiple-channel ANC (MCANC) systems with multiple reference microphones, secondary sources and error microphones are required (8, 18). To maintain the original functionality of domestic windows, it is essential to minimize the obstruction caused by the ANC system (19). In (20), Lam et al. studied the physical limits of MCANC for open window, with a focus on secondary source positioning.

¹ jhe007@e.ntu.edu.sg
² blam002@e.ntu.edu.sg
³ mtatsuya@ntu.edu.sg
⁴ rishabh001@e.ntu.edu.sg
⁵ ewsgan@ntu.edu.sg
These previous studies on open window ANC reveal that given a typical size of a window, the number of ANC units increases dramatically, which complicates and increases the space and cost on the controller design, deployment, and maintenance.

To simplify the implementation of the MCANC controller, we introduce a symmetric configuration of the ANC units in Section 2. The aim of the symmetric design is to achieve MCANC with a single control filter. Few earlier works that employ a symmetric design of an ANC system include MCANC for fan noise (21, 22) and infant incubators (23), but no detailed study was carried out. Our theoretical analysis of symmetric MCANC is presented in Section 3. Investigation on the robustness of symmetric MCANC to secondary path variations is conducted in Section 4, which is followed by our conclusions.

2. SYMMETRIC MCANC DESIGN

To mitigate noise that propagates through open windows, Nishimura et al. proposed an active acoustic shielding (AAS) system that installs an array of AAS cells on the window opening (14, 16, 17), as shown in Fig. 1. For noise reduction to be effective in larger areas, multiple AAS cells are required. Studies in (14) have shown that the overall noise reduction performance is proportional to the density of the AAS cells. Therefore, a MCANC system needs to be developed to mitigate noise through apertures of average size.

![Figure 1 – An illustration of active acoustic shielding system](image)

Fig. 2 shows the block diagram of the feedforward MCANC system (5). The primary noise is transmitted through the primary path matrix $P$, to give $M \times 1$ vector $d(n)$ at the $M$ error microphone locations, and also through the reference path matrix $R$. The $J \times 1$ vector of reference signals $x(n)$ comprises of the primary noise through $R$ and the feedback signal that is due to the $K$ secondary source outputs $y(n)$ transmitted via feedback path $F$. The $M \times 1$ vector of error signals $e(n)$ comprises of $d(n)$ and $M \times 1$ vector of secondary output signals transmitted through secondary path matrix $S$. $W$ is the adaptive filter that is usually updated using FxLMS algorithm (24, 25), where a secondary path model $\hat{S}$ is employed to filter the reference microphone signals $x(n)$.

As illustrated in Fig. 3, a feedforward ANC system consists of three parts, namely, reference microphone, secondary source, and error microphone. Each of the three parts need to follow the same geometry of the circular symmetric design, which implies that number of units in each of the three parts are equal, i.e., $J = K = M$. Since the number of AAS cells can be varied, we propose a general symmetric design based on a circular configuration, as illustrated with secondary sources in Fig. 4. Given $M$ channels in the MCANC system, the $M$ cells are placed uniformly on the circle. The uniformly distributed AAS cells allow for swapping of the channels, while keeping the control system unchanged. Note that different aperture shapes would require a specific symmetric MCANC system with different number of units (e.g., rectangular window requires $M = 4$). Adapting the symmetric design for unconventional aperture shapes, though important, is beyond the scope of this paper. Furthermore, combining several groups of AAS cells in circles of different radii is suggested to cover a wider area. For open window applications, we consider the
reference microphones to be placed nearer (or co-located) to the secondary sources (14), and each secondary source would only require its respective reference microphone signal to derive the filter coefficients using adaptive approaches.

Figure 2 – Block diagram of feedforward MCANC system using FxLMS (adapted from (5))

Figure 3 – An illustration of symmetric MCANC system with M = 3

Figure 4 – Symmetric design of MCANC system in a circle
3. SYMMETRIC MCANC USING FXLMS

The symmetric MCANC will be formulated using FxLMS, with the following basic assumptions applied: (i) The feedback path is not considered since it can be minimized using insulation materials or cancelled with neutralization techniques (26); (ii) The adaptive filter in each channel takes only one reference signal (i.e., no cross terms), similar to the implementation in (14, 16); (iii) No other measurement or model noise is considered; (iv) Perfect secondary path modelling, i.e., \( \hat{S} = S \); (v) Identical and fixed step size \( \mu \) is applied to all channels.

According to the symmetric geometry, we can derive the following characteristics on the secondary paths:
\[
s_{i_0,i_0}(n) = s_{i_0+i_0}(n), \quad \forall (i_0+i_0 - i_0) \% M = (i_2 - i_1) \% M, \tag{1}
\]
where \( i_{31}, i_{32} \) are the indices for the secondary sources, \( i_{11}, i_{12} \) are the indices for the error microphones. For example, with \( M = 3 \) as shown in Fig. 4, we have \( s_{1,1}(n) = s_{2,3}(n) = s_{3,1}(n), \)
\( s_{1,2}(n) = s_{3,2}(n) = s_{3,1}(n), \) and \( s_{1,3}(n) = s_{3,3}(n). \) These characteristics can be easily verified by rotating the circle. Similarly, we also assume that the primary paths \( p_i(n) \) and reference paths \( r_i(n) \) are symmetrical (e.g., primary noise that comes from incident plane wave, point source in the middle), i.e.,
\[
p_i(n) = p_j(n), \quad \forall i, j \in \{1, 2, ..., M\}. \tag{2}
\]
\[
r_i(n) = r_j(n), \quad \forall i, j \in \{1, 2, ..., M\}. \tag{3}
\]
With equations (2) and (3), we have \( x_i(n) = x_j(n), \) and \( d_i(n) = d_j(n). \) Assuming at iteration \( n \), we have \( w_i(n) = w_j(n). \) Since \( y_i(n) = w_i^T(n)x_i(n), \) we get \( y_i(n) = y_j(n). \) Let index \( m, m' \in \{1, 2, ..., M\}, \) we have
\[
(m' - m) \% M = (j - i) \% M, \tag{4}
\]
where \( m' = (m - 1 + j - i) \% M + 1. \) Based on equation (1), we can rewrite the error signal as:
\[
e_i(n) = d_i(n) - \sum_{m=1}^{M} s_{m,m}(n) * y_m(n)
= d_j(n) - \sum_{m=1}^{M} s_{m',m}(n) * y_m(n)
= e_j(n). \tag{5}
\]
Finally, the adaptive filter updating can be expressed as
\[
w_i(n+1) = w_i(n) + \mu \sum_{m=1}^{M} \left[ \hat{s}_{i,m}(n) * x_i(n) \right] e_m(n)
= w_j(n) + \mu \sum_{m=1}^{M} \left[ \hat{s}_{j,m'}(n) * x_j(n) \right] e_m(n)
= w_j(n+1). \tag{6}
\]
Thus, we verify that given the symmetric characteristics stated in equations (1)-(3), all \( M \) adaptive filters are identical.

Based on the above analysis, we propose to simplify the controller with only one adaptive filter and the filtered output signal is fed to all the secondary speakers equally. For this approach, we directly average the \( M \) adaptive filters as in equation (6), and arrive at...
\[ w_{\text{single}}(n + 1) = w_{\text{single}}(n) + \mu \sum_{m=1}^{M} \left[ \hat{s}_{\text{single},m}(n) \ast x_{m}(n) \right] e_{m}(n), \quad (7) \]

where \( \hat{s}_{\text{single},m}(n) = \frac{1}{M} \sum_{i=1}^{M} \hat{s}_{i,m}(n) \) is the average of the secondary paths from all secondary sources to each error microphone \( m \).

Next, we compare the computation cost between conventional MCANC with multiple adaptive filters and MCANC with one single adaptive filter. Clearly, \( M \) sets of adaptive filters are reduced to one set as specified in equation (7). For a centralized control system, this single filter approach substantially simplifies the controller complexity (27). For a decentralized control system, the symmetric MCANC requires much less controllers. To evaluate the performance of these MCANC algorithms, we consider an overall noise reduction (NR in dB), as the ratio of the sum of the noise power at all error microphones without ANC and with ANC turned on (after convergence, \( T \) seconds at sampling rate \( f_s \)):

\[ \text{NR} = 10 \log_{10} \left( \frac{\sum_{m=1}^{M} \sum_{n=1}^{T f_s} d_{m}^2(n) \sum_{m=1}^{M} \sum_{n=1}^{T f_s} e_{m}^2(n)}{\sum_{m=1}^{M} \sum_{n=1}^{T f_s} \sum_{m=1}^{M} \sum_{n=1}^{T f_s} e_{m}^2(n)} \right), \quad (8) \]

4. SIMULATION RESULTS AND DISCUSSIONS

To investigate the performance of symmetric MCANC, we consider a 2-2-2 ANC system with the dimensions specified in Fig. 5. The ideal requirements for symmetric design include:

\[ p_1(n) = p_2(n), \quad n(n) = r(n), \quad s_{1,1}(n) = s_{2,1}(n), \quad s_{1,2}(n) = s_{2,2}(n). \]

To investigate the robustness of symmetric MCANC, variations in primary paths, reference paths, or secondary paths shall be considered. In this paper, we focus on the effect of the secondary paths. Table 1 shows the three types of variations considered in our simulations. Specifically, for positioning variations, we only consider the amplitude and delay changes on secondary paths due to the changes on their lengths. The variations on the whole secondary path impulse responses are reserved for future investigation.

Figure 5 – An illustration of 2-2-2 ANC system
<table>
<thead>
<tr>
<th>Type</th>
<th>Possible causes</th>
<th>Variations</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude only</td>
<td>Gain difference in the amplifier or secondary speaker response</td>
<td>$V_a \in (0,1,2,3)$</td>
<td>dB</td>
</tr>
<tr>
<td>Delay only</td>
<td>Temporal differences in the speaker response</td>
<td>$V_d \in (0,1,2,3,4,5)$</td>
<td>sample</td>
</tr>
<tr>
<td>Both amplitude and delay</td>
<td>Positioning differences (horizontally: Vpx; vertically: Vpy) of the secondary speaker</td>
<td>$V_{px}, V_{py} \in (-3,-2,-1,0,1,2,3)$</td>
<td>cm</td>
</tr>
</tbody>
</table>

We evaluate the noise reduction performance using adaptive filter obtained based on the three methods specified in Table 2. A low-passed white noise (cutoff at 2205 Hz at $f_s = 44100$ Hz, variance set as 1) is used as the primary noise, where the first 4 seconds are used for adaption with step size set as $\mu = 0.001$, and the fifth second is used for evaluation of noise reduction using equation (8).

<table>
<thead>
<tr>
<th>Name</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>multiple w</td>
<td>Multiple adaptive filters derived from MCANC</td>
</tr>
<tr>
<td>average w</td>
<td>Average of the multiple adaptive filters</td>
</tr>
<tr>
<td>single w</td>
<td>Single adaptive filter trained directly using (7)</td>
</tr>
</tbody>
</table>

Two simulations conducted include simple impulse responses with single delay and amplitude determined by the distance, and actual impulse response measured from a real setup (19). The NR results using simple impulse responses are plotted in Figs. 6, and 7. When there is no variation (i.e., amplitude variation = 0, delay variation = 0, and position variation = (0,0)), the NR performance among these three methods are quite close. In the cases with variations, the performance of “single w” or “average w” method degrades significantly. Specifically, we observe from Fig. 6(a) that an amplitude variation of 1 dB leads to 10 dB NR performance degradation. For delay variations shown in Fig. 6(b), one sample delay results in 20 dB NR degradation. Fig. 7 indicates that the larger the variation, the higher is the noise reduction performance degradation. For just 1 cm positioning variation, NR performance degrades by more than 10 dB. Results obtained using measured impulse responses are illustrated in Figs. 8 and 9, which exhibit similar trends of NR performance degradation. However, the NR performance degradation is less severe, because the introduced variations (from Table I) incur less relative changes to the real impulse responses as compared to the sparse simple impulse response. Nevertheless, these results indicate that it is essential to satisfy the symmetric design requirements to achieve a good performance of symmetric MCANC system.
Figure 6 – Noise reduction performance due to (a) amplitude variation, and (b) delay variation, with simple impulse responses.

Figure 7 – Noise reduction performance due to positioning variation, with simple impulse responses.
Figure 8 – Noise reduction performance due to (a) amplitude variation, and (b) delay variation, with real impulse responses.

Figure 9 – Noise reduction performance due to positioning variation, with real impulse responses.
5. CONCLUSIONS

In this paper, we investigated the multichannel ANC system for noise through apertures. A symmetric design is proposed to simplify the open-window ANC system by employing one single control filter for multiple secondary sources. The requirements for the symmetric MCANC design are specified and verified. Furthermore, we investigated the noise reduction performance affected by the different types of secondary path variations that exist in practical implementations. Our simulation results with simple impulse responses indicate that the noise reduction performance degrades by at least 10 dB when there is a slight variation of 1 dB amplitude difference, 1 sample delay, or 1 cm positioning difference on the secondary path. With real measured impulse responses, the performance degradation is less severe. These results suggest that in the actual symmetric MCANC implementation, these variations need to be avoided or compensated to achieve the benefits of the symmetric design of MCANC system. Future work includes the study of symmetric design and compensations in actual implementations.

ACKNOWLEDGEMENTS

This material is based on research/work supported by the Singapore Ministry of National Development and National Research Foundation under L2 NIC Award No. L2NICCFP1-2013-7.

REFERENCES

20. Lam B, Elliott SJ, Cheer J, Gan WS. The physical limits of active noise control of open windows, in
23. Liu L, Gujula S, Kuo SM. Multi-channel real time active noise control system for infant incubators, in Proc. 31st annual international conference of the IEEE EMBS; Minneapolis, Minnesota, Sept. 2009.