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Active control of noise in a strongly coupled rectangular cavity

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ABSTRACT

This paper deals with a strongly coupled rectangular cavity in which both acoustic field and vibration field strongly couple with each other, hence its fundamental characteristics significantly differ from *in vacuo* structural modes and rigid wall acoustic modes. First, the eigenpairs of a rectangular cavity comprising a flexible panel placed on top and five rigid walls are derived, its fundamental properties being investigated. It is revealed that there exist two kinds of acoustic mode; standing wave acoustic mode and evanescent mode. The former is generated as a result of sound reflection in the cavity, whereas the latter is produced by coupling effect between structure and sound, and is characterized as the sound pressure localization in the vicinity of a panel. This paper therefore elaborates the intriguing properties the evanescent mode possesses. Using the eigenpairs obtained, acoustic potential energy inside the cavity is then described, and the optimal feedforward control law for minimizing the energy is obtained. Moreover, a numerical analysis is conducted in an effort to clarify the control effect for suppressing strongly coupled acoustic modes of the cavity.

Keywords: Strongly coupled rectangular cavity, Evanescent mode, Active control: 38.2

1. INTRODUCTION

Noise inside an acoustic-structural coupled cavity is a common problem in numerous practical situations including those involving aircrafts, automobiles, and trains. The term "coupling" indicates interference between a structural and an acoustic field of a cavity, resulting in the alternation of the eigenpairs of uncoupled system dynamics. Depending on the degree of coupling, cavity systems can be classified into two categories: a modally coupled cavity system and a coupled cavity system.

A modally coupled cavity system often introduced in sound transmission control problems (1-8) is based on the modal coupling theorem (9,10) established under the assumption that the fluid medium is non-dense and the cavity walls not "thin." The characteristic of this system is that the eigenfunctions of a coupled system remain the same as those of an acoustically rigid walled cavity, while only the eigenfrequencies of the cavity change.

When cavity walls become thin and the cavity gap shallow, the assumption of a modal coupling is no longer valid; thus, such a case falls into the second category, i.e., a coupled cavity system. Considerable efforts have been made in literature to derive the exact solution of coupled rectangular cavity system that comprises five rigid walls and a flexible panel. The main focus of these studies was, however, to understand sound transmission through a cavity-backed panel, and thus their concern was directed toward deriving the exact solution of a forced vibration of the cavity-backed panel subject to external sound pressure. As such, the exact eigenpairs of the coupled cavity, which are intrinsic parameters governing the system dynamics and independent of any extraneous forces, were to be found.

With a view to suppressing cavity noise generated by external noise or disturbance forces acting on the panel, and/or noise sources inside the cavity, it is common practice to introduce active control. To establish a control strategy, the eigenpairs which are the governing parameters of the system dynamics are essential because the control law for minimizing some performance index is expressed using system parameters. System parameters such as sound pressure inside the cavity, acoustic potential energy, surface velocity of the panel, or structural kinetic energy may be described using the expansion theorem in terms of the eigenpairs. Hence, the control variables may be methodically manipulated only if the eigenpairs are available.

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This paper derives explicitly the eigenpairs of a coupled rectangular cavity comprising five rigid walls and a flexible panel placed on top. For this purpose, the eigenfunctions governing the dynamics of both the sound field and the vibration field are expressed as the infinite sum of cluster eigenfunctions that possess the same attribute in common. The characteristic matrix equation is then derived, enabling one to specify the eigenpairs of the coupled cavity. In order to investigate the fundamental properties of the eigenpairs derived, a numerical analysis is conducted, revealing the presence of evanescent modes together with the conventional standing wave modes. It is shown that the evanescent modes emerge when the eigen wavenumber of the cluster eigenfunction becomes pure imaginary and the associated coefficient is large. Using the eigenpairs obtained, acoustic potential energy inside the cavity is then described, the optimal feedforward control law for minimizing the energy being obtained. Moreover, the control effect for suppressing strongly coupled acoustic modes of the cavity is shown.

2. Eigenpairs of a Coupled Rectangular Cavity

Consider a rectangular cavity with the dimensions of $L_x \times L_y \times L_z$, consisting of five rigid walls and one simply supported flexible panel *S* placed on top. Using the velocity potential ϕ, the wave equation of sound in the cavity may be written as

$$c^2\nabla^2\phi(x,y,z,t) - \ddot{\phi}(x,y,z,t) = 0 \tag{1}$$

where *c*, ∇^2, *t* and $\dot{}$ denote the sound velocity in air, the Laplacian operator, time and temporal differentiation, respectively. Furthermore, the equation of motion of a flexible panel is given by

$$D\nabla^4 v(x,y,t) + \rho h\ddot{v}(x,y,t) = \Delta\dot{p}(x,y,z,t)\big|_{z=L_z} \tag{2}$$

where *D* is the flexural rigidity, *v* the velocity of the panel, ρ the density, *h* the thickness, Δp the sound pressure in the cavity, respectively. Suppose that a coupled cavity system is oscillating at frequency ω. Equations (1) and (2) are then written as

$$\nabla^2\phi(x,y,z) + k^2\phi(x,y,z) = 0 \tag{3}$$

$$D\nabla^4 v(x,y) - \omega^2\rho h v(x,y) = \omega^2\rho_0\phi(x,y,L_z) \tag{4}$$

where $k = \omega / c$ denotes the wavenumber of air and ρ_0 the air density. Assume that the velocity potential is described as a separable function of the three space variables

$$\phi(x,y,z) = X(x)Y(y)Z(z) \tag{5}$$

Substituting Eq.(5) into Eq.(3) yields

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} + \frac{Z''(z)}{Z(z)} = -k^2 \tag{6}$$

where X'', for instance, expresses the second derivative of *X* with respect to *x*. Now that the wavenumber k is independent of space, Eq.(6) may then be expressed as

$${k_x}^2 + {k_y}^2 + {k_z}^2 = k^2 \tag{7}$$

where

$${k_x}^2 = -\frac{X''}{X},\ {k_y}^2 = -\frac{Y''}{Y},\ {k_z}^2 = -\frac{Z''}{Z}. \tag{8}$$

The solution of Eq.(8) may be given by

$$X(x) = \alpha_x \cos k_x x + \beta_x \sin k_x x, \tag{9}$$

$$Y(y) = \alpha_y \cos k_y y + \beta_y \sin k_y y \tag{10}$$

$$Z(z) = \alpha_z \cos k_z z + \beta_z \sin k_z z \tag{11}$$

where $\alpha_i, \beta_i (i = x, y, z)$ are the coefficients determined by the boundary conditions of the cavity. Taking into consideration the boundary condition that the particle velocity at a rigid wall ($x=0, x=L_x; y=0, y=L_y$) is zero, Eqs.(9) and (10) hence become

$$X_\kappa(x) = \alpha_{x\kappa} \cos\frac{l_\kappa \pi}{L_x} x \quad (\kappa = 1, 2, 3\cdots) \tag{12}$$

$$Y_\kappa(y) = \alpha_{y\kappa} \cos\frac{m_\kappa \pi}{L_y} y \quad (\kappa = 1, 2, 3\cdots) \tag{13}$$

where $\alpha_{x\kappa}$ and $\alpha_{y\kappa}$ are the modal coefficients, and l_κ and m_κ the modal indices. Regarding $Z(z)$, the particle velocity at $z=0$ is zero, however non-zero at $z=L_z$ because of the flexible panel vibrating with the surface velocity $v(x,y)$, and hence

$$Z(z) = \alpha_z \cos k_z z \tag{14}$$

where k_z is the eigen wavenumber in terms of the z direction, which has yet to be specified. Therefore, the fundamental form of the velocity potential is written as

$$\phi_\kappa(x,y,z) = \cos\frac{l_\kappa \pi}{L_x} x \cos\frac{m_\kappa \pi}{L_y} y \cos k_z z \quad (\kappa = 1, 2, 3\cdots). \tag{15}$$

Substituting Eq.(15) into (6) yields

$$\left(\frac{l_\kappa \pi}{L_x}\right)^2 + \left(\frac{m_\kappa \pi}{L_y}\right)^2 + k_z^{\,2} = \bar{k}^{\,2} \tag{16}$$

where $\bar{k}^{\,2}$ (or $\bar{\omega}$) implies the ith eigen wavenumber (or eigenfrequency) of the coupled cavity. It should be noted that the eigen wavenumber $\bar{k}$ is dependent on modal indices; l_κ and m_κ $(\kappa = 1, 2, 3\cdots)$.

Given that $\bar{k}$ is specified using the characteristic matrix equation of the coupled cavity as will be shown later, depending on the modal indices l_κ and m_κ, k_z is determined such that Eq.(16) holds. In order to emphasize this fact, let k_z be written as $k_{l_\kappa m_\kappa}$. Also, in order to distinguish the corresponding eigenfunction ϕ_κ in Eq.(15) from ordinary eigenfunctions, let ϕ_κ be expressed as $\tilde{\phi}_\kappa$. Hence

$$\tilde{\phi}_\kappa(x,y,z) = \cos\frac{l_\kappa \pi}{L_x} x \cos\frac{m_\kappa \pi}{L_y} y \cos k_{l_\kappa m_\kappa} z \quad (\kappa = 1, 2, 3\cdots). \tag{17}$$

Note that depending on the modal indices l_κ and m_κ, the wavenumber component $k_{l_\kappa m_\kappa}$ becomes real or pure imaginary as will be shown in Section III. Moreover, because of the constraint in Eq.(16), a set of $\tilde{\phi}_\kappa$ $(\kappa = 1, 2, 3\cdots)$ has the same eigenfrequency $\bar{\omega}$ in common and thus $\tilde{\phi}_\kappa$ $(\kappa = 1, 2, 3\cdots)$ is termed the degenerate eigenfunctions in the article.

As such, the acoustic eigenfunction $\bar{\phi}$ of the coupled cavity satisfies the following sound equation

$$\nabla^2 \bar{\phi}(x,y,z) + \bar{k}^{\,2} \bar{\phi}(x,y,z) = 0 \tag{18}$$

where

$$\bar{\phi}(x,y,z) = \sum_{\kappa=1}^{\infty} a_\kappa \tilde{\phi}_\kappa(x,y,z) \approx \mathbf{a}^T \tilde{\boldsymbol{\phi}}(x,y,z) \tag{19}$$

where

$$\mathbf{a} = \begin{pmatrix} a_1 & a_2 & a_3 & \cdots \end{pmatrix}^T, \tag{20}$$

$$\tilde{\boldsymbol{\phi}} = \begin{pmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 & \tilde{\phi}_3 & \cdots \end{pmatrix}^T \tag{21}$$

and where a_κ is the κ th amplitude of the degenerated eigenfunction of the coupled cavity. Note that the acoustic eigenfunction $\bar{\phi}$ of the coupled cavity is expressed as the infinite sum of the degenerate eigenfunctions $\tilde{\phi}_\kappa$, whereas the eigenfunction of a rigid walled cavity ϕ_i is described using a single term as

$$\phi_i(x,y,z)=\cos\frac{l_i\pi}{L_x}x\cos\frac{m_i\pi}{L_y}y\cos\frac{n_i\pi}{L_z}z\,. \tag{22}$$

More important, the sum of normal eigenfunctions does not satisfy the homogeneous Helmholtz wave equation in Eq.(3), while the sum of the degenerate eigenfunctions in Eq.(19) does because of the attribute the degenerate eigenfunctions possess as shown in Eq.(16).

Suppose that the coupled cavity is oscillating with the eigenfrequency $\bar{\varpi}$. Then the equation of motion of a flexible panel is given by

$$D\nabla^4\bar{\varphi}\,(x,y)-\bar{\varpi}^{\,2}\rho h\bar{\varphi}\,(x,y)=\bar{\varpi}^{\,2}\rho_a\bar{\phi}\,(x,y,z)\big|_{z=L_z} \tag{23}$$

where $\bar{\varphi}$ denotes the structural eigenfunction of the coupled cavity, which may be expressed as the infinite sum of the *in vacuo* structural eigenfunctions of the panel φ_κ $(\kappa=1,2,3\cdots)$ satisfying the following homogeneous equation of motion of a panel,

$$D\nabla^4\varphi_\kappa(x,y)-\omega_\kappa^2\rho h\,\varphi_\kappa(x,y)=0\,. \tag{24}$$

Hence,

$$\bar{\varphi}\,(x,y)=\sum_{\kappa=1}^{\infty}b_\kappa\varphi_\kappa(x,y)\approx\mathbf{b}^T\boldsymbol{\varphi}(x,y)\,, \tag{25}$$

$$\mathbf{b}=\begin{pmatrix}b_1 & b_2 & b_3 & \cdots\end{pmatrix}^T, \tag{26}$$

$$\boldsymbol{\varphi}=\begin{pmatrix}\varphi_1 & \varphi_2 & \varphi_3 & \cdots\end{pmatrix}^T \tag{27}$$

where b_κ is the κ th modal coefficient of the structural eigenfunction of the coupled cavity. It should be noted that the sum of the *in vacuo* structural eigenfunctions does not satisfy the homogeneous equation of motion in Eq.(24), however satisfies the expression in Eq.(23) because of the non-homogeneous equation. Because of the boundary condition of a simply supported panel, the structural eigenfunction φ_κ is then given by

$$\varphi_\kappa(x,y)=\sin\frac{l'_\kappa\pi}{L_x}x\sin\frac{m'_\kappa\pi}{L_y}y \tag{28}$$

where l'_κ and m'_κ are modal indices. Substituting Eqs.(19) and (25) in Eq.(23) results in

$$\sum_{\kappa=1}^{\infty}\left(D\nabla^4-\bar{\varpi}^{\,2}\rho h\right)b_\kappa\varphi_\kappa(x,y)=\bar{\varpi}^{\,2}\rho_a\sum_{\kappa=1}^{\infty}a_\kappa\tilde{\phi}_\kappa(x,y,z)\big|_{z=L_z}\,. \tag{29}$$

Moreover, substituting Eqs.(17) and (28) in Eq.(29) leads to

$$\begin{aligned}&\sum_{\kappa=1}^{\infty}\left(\omega_\kappa^2-\bar{\varpi}^{\,2}\right)\rho h b_\kappa\sin\frac{l'_\kappa\pi}{L_x}x\sin\frac{m'_\kappa\pi}{L_y}y\\&=\bar{\varpi}^{\,2}\rho_a\sum_{\kappa=1}^{\infty}a_\kappa\cos\frac{l_\kappa\pi}{L_x}x\cos\frac{m_\kappa\pi}{L_y}y\cos k_{l_\kappa m_\kappa}L_z\end{aligned}\,. \tag{30}$$

Multiplying Eq.(30) by $\varphi_s(x,y)=\sin\frac{l'_s\pi}{L_x}x\sin\frac{m'_s\pi}{L_y}y$ and integrating over the panel S produces

$$\left(\omega_s^2-\bar{\omega}^2\right)\rho h\frac{L_xL_y}{4}b_s=\bar{\omega}^2\rho_a\sum_{\kappa=1}^{\infty}a_\kappa\beta_{s\kappa}\cos k_{l_\kappa m_\kappa}L_z \tag{31}$$

where $\beta_{s\kappa}$ denotes the coupling coefficient defined as

$$\beta_{s\kappa}=\int_0^{L_y}\int_0^{L_x}\sin\frac{l'_s\pi}{L_x}x\sin\frac{m'_s\pi}{L_y}y\cos\frac{l_\kappa\pi}{L_x}x\cos\frac{m_\kappa\pi}{L_y}ydxdy\,. \tag{32}$$

Equation (31) may be expressed in a matrix form as

$$\mathbf{B}_{couple}\mathbf{\Lambda}_c\mathbf{a}=\mathbf{\Lambda}_\omega\mathbf{b} \tag{33}$$

where

$$\mathbf{\Lambda}_c=\bar{\omega}^2\rho_a\begin{pmatrix}\cos k_{l_1m_1}L_z & & \mathbf{0}\\ & \cos k_{l_2m_2}L_z & \\ \mathbf{0} & & \ddots\end{pmatrix}, \tag{34}$$

$$\mathbf{\Lambda}_\omega=\frac{\rho hL_xL_y}{4}\begin{pmatrix}\omega_1^2-\bar{\omega}^2 & & \mathbf{0}\\ & \omega_2^2-\bar{\omega}^2 & \\ \mathbf{0} & & \ddots\end{pmatrix}, \tag{35}$$

$$\mathbf{B}_{couple}=\begin{pmatrix}\beta_{11} & \beta_{12} & \cdots\\ \beta_{21} & \beta_{22} & \cdots\\ \vdots & \vdots & \end{pmatrix}. \tag{36}$$

Next, the boundary condition in which the surface velocity of the panel spatially meets the particle velocity is expressed as

$$\bar{\varphi}(x,y)=\frac{\partial}{\partial z}\bar{\phi}(x,y,z)\Big|_{z=L_z}\,. \tag{37}$$

Substituting Eqs.(19) and (25) into Eq.(37) leads to

$$\sum_{\kappa=1}^{\infty}b_\kappa\sin\frac{l'_\kappa\pi}{L_x}x\sin\frac{m'_\kappa\pi}{L_y}y=-\sum_{\kappa=1}^{\infty}k_{l_\kappa m_\kappa}a_\kappa\cos\frac{l_\kappa\pi}{L_x}x\cos\frac{m_\kappa\pi}{L_y}y\sin k_{l_\kappa m_\kappa}L_z\,. \tag{38}$$

Furthermore, multiplying Eq.(38) by $\varphi_s(x,y)=\sin\frac{l'_s\pi}{L_x}x\sin\frac{m'_s\pi}{L_y}y$ and integrating over the panel S produces

$$b_s\frac{L_xL_y}{4}=-\sum_{\kappa=1}^{\infty}k_{l_\kappa m_\kappa}a_\kappa\beta_{s\kappa}\sin k_{l_\kappa m_\kappa}L_z\,. \tag{39}$$

Equation (39) may be expressed in a form of matrix as

$$\frac{L_x L_y}{4}\mathbf{b} = -\mathbf{B}_{couple}\mathbf{\Lambda}_s\mathbf{a} \tag{40}$$

where

$$\mathbf{\Lambda}_s = \begin{pmatrix} k_{l_1m_1}\sin k_{l_1m_1}L_z & & \mathbf{0} \\ & k_{l_2m_2}\sin k_{l_2m_2}L_z & \\ \mathbf{0} & & \ddots \end{pmatrix}. \tag{41}$$

Using Eqs.(33) and (40), the characteristic matrix equation of the coupled cavity is then given (11,12) by

$$\bar{\mathbf{A}}\bar{\mathbf{x}} = \mathbf{0} \tag{42}$$

where

$$\bar{\mathbf{A}} = \begin{pmatrix} \mathbf{B}_{couple}\mathbf{\Lambda}_c & -\mathbf{\Lambda}_\omega \\ \mathbf{B}_{couple}\mathbf{\Lambda}_s & \frac{L_x L_y}{4}\mathbf{I} \end{pmatrix}, \tag{43}$$

$$\bar{\mathbf{x}} = \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} \tag{44}$$

and where **I** denotes the identity matrix. Note that the frequency nullifying the determinant of the matrix $\bar{\mathbf{A}}$ in Eq.(43) yields the eigenfrequencies ϖ_κ $(\kappa = 1,2,3\cdots)$ of the coupled rectangular cavity system. When the eigenfrequency ϖ_κ is specified, the associated eigenvector $\bar{\mathbf{x}}_\kappa$ is then obtained. Consequently, the eigenpairs of the coupled rectangular cavity are acquired: the eigenfunctions in terms of the sound field $\bar{\phi}_\kappa(x,y,z)$ in Eq.(19), those in terms of the vibration field $\bar{\varphi}_\kappa$ in Eq.(25).

3. MINIMIZATION OF ACOUSTIC POTENTIAL ENERGY

3.1 Acoustic potential energy

Acoustic potential energy inside the cavity oscillating at ω may be expressed as

$$E_p = \frac{1}{4\rho_a c^2}\int_0^{L_z}\int_0^{L_y}\int_0^{L_x} \left|\Delta p(x,y,z)\right|^2 dxdydz \tag{45}$$

where the sound pressure is given by

$$\begin{aligned}\Delta p(x,y,z) &= -j\omega\rho_a\phi(x,y,z) \\ &= j\frac{\rho_a c^2}{\omega}\nabla^2\phi(x,y,z) \\ &= j\frac{\rho_a c^2}{\omega}\sum_{\chi=x,y,z}\frac{\partial u_\chi(x,y,z)}{\partial\chi}\end{aligned} \tag{46}$$

Furthermore, Eq.(46) leads to

$$\Delta p(x,y,z) = j\frac{\rho_a c^2}{\omega}\sum_{i=1}^{\infty}\sum_{\chi=x,y,z}\frac{\partial\bar{\phi}_{\chi,i}(x,y,z)}{\partial\chi}\eta_i \tag{47}$$

where η_i denotes the modal amplitude obtained from the modal equation. Moreover, the eigenfunction $\bar{\phi}_{\chi,i}$ in terms of the particle velocity of the coupled cavity may be expressed as

$$\begin{aligned}\overline{\phi}_{\chi,i}(x,y,z) &= \frac{\partial}{\partial \chi}\overline{\phi}_i(x,y,z) \\ &= \frac{\partial}{\partial \chi}\sum_{\kappa=1}^{\infty} a_\kappa^i \tilde{\phi}_\kappa^i(x,y,z) \\ &= \frac{\partial}{\partial \chi}\sum_{\kappa=1}^{\infty} a_\kappa^i \cos\frac{l_i\pi}{L_x}x\cos\frac{m_i\pi}{L_y}y\cos k_{l_i m_i}^i z\end{aligned} \tag{48}$$

Hence

$$\Delta p(x,y,z) = \\ j\frac{\rho_a c^2}{\omega}\sum_{i=1}^{\infty}\sum_{\kappa=1}^{\infty}\left(\left(\frac{l_\kappa\pi}{L_x}\right)^2+\left(\frac{m_\kappa\pi}{L_y}\right)^2+\left(k_{l_\kappa m_\kappa}^i\right)^2\right)a_\kappa^i\cos\frac{l_\kappa\pi}{L_x}x\cos\frac{m_\kappa\pi}{L_y}y\cos k_{l_\kappa m_\kappa}^i z\ \eta_i \tag{49}$$

Using Eqs.(16) and (19), the sound pressure in Eq.(49) is further expanded to

$$\begin{aligned}&\Delta p(x,y,z) \\ &= j\frac{\rho_a c^2}{\omega}\sum_{i=1}^{\infty}\overline{k}_i^2\eta_i\sum_{\kappa=1}^{\infty}a_\kappa^i\cos\frac{l_\kappa\pi}{L_x}x\cos\frac{m_\kappa\pi}{L_y}y\cos k_{l_\kappa m_\kappa}^i z \\ &= \sum_{i=1}^{\infty}\overline{\phi}_i(x,y,z)\, j\frac{\rho_a\overline{\omega}_i^2}{\omega}\eta_i\end{aligned} \tag{50}$$

Furthermore, the sound pressure may be expressed using the expansion theorem in terms of the eigenfunction vector as

$$\Delta p(x,y,z) = \sum_{i=1}^{\infty} p_i\overline{\phi}_i(x,y,z) \approx \mathbf{p}^T\overline{\boldsymbol{\phi}} \tag{51}$$

where

$$\mathbf{p} = \begin{pmatrix} p_1 & p_2 & p_3 & \cdots \end{pmatrix}^T \tag{52}$$

$$\begin{aligned}\overline{\boldsymbol{\phi}} &= \begin{pmatrix}\overline{\phi}_1 & \overline{\phi}_2 & \overline{\phi}_3 & \cdots\end{pmatrix}^T \\ &= \begin{pmatrix}\mathbf{a}_1^T\tilde{\boldsymbol{\phi}}_1 & \mathbf{a}_2^T\tilde{\boldsymbol{\phi}}_2 & \mathbf{a}_3^T\tilde{\boldsymbol{\phi}}_3 & \cdots\end{pmatrix}^T,\end{aligned} \tag{53}$$

so that the modal amplitude of the sound pressure is given by

$$p_i = j\frac{\rho_a\overline{\omega}_i^2}{\omega}\eta_i. \tag{54}$$

Hence the acoustic potential energy inside the coupled cavity may be expressed as

$$E_p = \mathbf{p}^H\overline{\mathbf{V}}\mathbf{p} \tag{55}$$

where ^{H} denotes the Hermitian (conjugate transpose) and the matrix $\overline{\mathbf{V}}$ is defined as

$$\overline{\mathbf{V}} = \frac{1}{4\rho_a c^2}\int_0^{L_z}\int_0^{L_y}\int_0^{L_x}\overline{\boldsymbol{\phi}}(x,y,z)\,\overline{\boldsymbol{\phi}}^T(x,y,z)dxdydz\,. \tag{56}$$

Thus, the acoustic potential energy inside the coupled cavity may be expressed using the eigenfunction vector $\bar{\boldsymbol{\phi}}$ of the coupled cavity.

3.2 Optimal feedforward control law for minimizing acoustic potential energy

Suppose that D point disturbance forces and C point control forces act on act on a panel. The acoustic potential energy in a strongly coupled rectangular cavity is then given by

$$E_p = \left(\mathbf{f}_c^H \mathbf{\Psi}_c^H + \mathbf{f}_d^H \mathbf{\Psi}_d^H\right) \mathbf{\Lambda}^H \bar{\mathbf{V}} \mathbf{\Lambda} \left(\mathbf{\Psi}_d \mathbf{f}_d + \mathbf{\Psi}_c \mathbf{f}_c\right) \tag{57}$$

where

$$\mathbf{\Lambda} = \rho_a c^2 \begin{pmatrix} \ddots & & \mathbf{0} \\ & \dfrac{k_i^2}{\omega^2 - \omega_i^2} & \\ \mathbf{0} & & \ddots \end{pmatrix} \in \mathbb{C}^{N \times N} \tag{58}$$

$$\mathbf{\Psi}_d = [\boldsymbol{\varphi}(r_{d1})\ \boldsymbol{\varphi}(r_{d2}) \cdots \boldsymbol{\varphi}(r_D)] \in \mathbb{R}^{N \times D} \tag{59}$$

$$\mathbf{\Psi}_c = [\boldsymbol{\varphi}(r_{c1})\ \boldsymbol{\varphi}(r_{c2}) \cdots \boldsymbol{\varphi}(r_C)] \in \mathbb{R}^{N \times C} \tag{60}$$

$$\boldsymbol{\varphi}(r) = [\varphi_1(r)\ \ \varphi_1(r) \cdots \varphi_N(r)]^T \in \mathbb{R}^{N \times 1} \tag{61}$$

$$\mathbf{f}_d = [f_{d1}\ f_{d2} \cdots f_D]^T \in \mathbb{C}^{D \times 1} \tag{62}$$

$$\mathbf{f}_c = [f_{c1}\ f_{c2} \cdots f_C]^T \in \mathbb{C}^{C \times 1} \tag{63}$$

The acoustic potential energy is further developed to

$$E_p = \mathbf{f}_c^H \mathbf{A}_{E_p} \mathbf{f}_c + \mathbf{f}_c^H \mathbf{B}_{E_p} + \mathbf{B}_{E_p}^H \mathbf{f}_c + C_{E_p} \tag{64}$$

where

$$\mathbf{A}_{E_p} = \mathbf{\Psi}_c^H \mathbf{\Lambda}^H \bar{\mathbf{V}} \mathbf{\Lambda} \mathbf{\Psi}_c \tag{65}$$

$$\mathbf{B}_{E_p} = \mathbf{\Psi}_c^H \mathbf{\Lambda}^H \bar{\mathbf{V}} \mathbf{\Lambda} \mathbf{\Psi}_d \mathbf{f}_d \tag{66}$$

$$C_{E_p} = \mathbf{f}_d^H \mathbf{\Psi}_d^H \mathbf{\Lambda}^H \bar{\mathbf{V}} \mathbf{\Lambda} \mathbf{\Psi}_d \mathbf{f}_d \tag{67}$$

The optimal feedforward control law for minimizing the acoustic potential energy in a strongly coupled potential energy is given by

$$\mathbf{f}_c = -\mathbf{A}_{E_p}{}^{-1} \mathbf{B}_{E_p} \tag{68}$$

4. NUMERICAL ANALYSIS

Using the specifications of a rectangular cavity comprising a simply supported flexible panel and five rigid walls, a numerical analysis is conducted. For this purpose, 15 acoustic modes and 15 structural modes of the panel are taken into consideration, the frequency of interest being set up to 250 Hz. The frequency of the 15th acoustic mode; (0,2,2) mode, and the structural mode; (1,7) mode, are, respectively, 977 Hz and 716 Hz, thereby covering enough the frequency of interest. In accordance with the procedure described above, the eigenpairs of the coupled rectangular cavity are acquired, the results of which (11,12) are shown in Table 1. After coupling, the acoustic eigenfunction may not be expressed as an (l,m,n) mode, so that the corresponding column is left blank. Observe that the (0,0,0) and (0,0,1) acoustic modes in the cavity before coupling couples with only the (odd, odd) structural mode, thereby affecting the (1,1) and (1,3) structural mode by shifting the resonant frequency. As with the (1,2) mode, it couples with the (0,1,0) acoustic mode which is not enlisted in the table though.

Illustrated in Fig.1 are the normalized velocity mode shapes of a flexible panel of the coupled cavity (11,12), which are depicted using the expression: $\overline{\varphi}_i(x,y)\approx \mathbf{b}_i^T\boldsymbol{\varphi}(x,y)$ $(i=1\sim 4)$ in Eq.(25). Observe that the structural modal behaviors from the 1st through 3rd mode are dominated by the *in vacuo* mode shape; (1,1), (1,2) and (1,3) mode, respectively. As for the 4th mode, however, the original *in vacuo* structural (2,1) mode is replaced by the deformed (1,3) mode because of the coupling effect. Figure 1 also shows the corresponding acoustic mode shapes along the z direction depicted using the expression: $\overline{\phi}_i(x,y,z)\approx \mathbf{a}_i^T\tilde{\boldsymbol{\phi}}(x,y,z)$ in Eq.(19) in the vicinity of $x=L_x/2$ and $y=L_y/2$. Unlike the acoustic mode shapes observed in a rigid wall cavity in Eq.(22), the mode shapes in the z direction appear different. Regarding the first mode at 77 Hz in Fig.1(a), although the rigid wall mode shape is depicted by two parallel straight lines along the z axis, the acoustic mode shape after coupling shrinks as z increases, leading to that of an open ended cavity at $z=L_z$ as an extreme case. Acoustic mode shapes in Figs.1(c) and (d) are similar to each other, albeit the 4th mode is considerably affected by the coupling effect. Note that among the four in Fig.1, the 2nd acoustic mode shape is different from the ordinary mode shapes. The acoustic mode shape in the z direction is described as $\cos k_z z$, implying that the amplitude at $z=0$ reaches the maximum; however, the maximum appears at $z=L_z$ and minimum at $z=0$, and therefore the fundamental properties of such an unusual mode needs investigating.

Figure 2 shows the frequency response characteristics of the acoustic potential energy (11,12) inside the cavity. In the frequency range up to 250 Hz, four peaks are noticeable, with each resonant frequency corresponding to that listed in Table 1. A solid line in the figure indicates the acoustic potential energy before control, whereas the dashed line shows that as a result of applying the optimal control law in Eq.(68). Note that the control effect depicted in the figure is theoretically possibly achievable goal which is only achieved with the knowledge of explicitly obtained eigenpairs of a strongly coupled cavity.

Table 1 Eigenpairs of the coupled rectangular cavity before and after coupling

	in vacuo		*coupled*	
Mode number	Modal indices (l,m,n)	Frequency Hz	Modal indices (l,m,n)	Frequency Hz
1	(0,0,0)	0	-	77
2	(1,1)	73	-	113
3	(1,2)	113	-	179
4	(1,3)	180	-	201
5	(0,0,1)	196	-	251
6	(2,1)	252	-	273
7	(1,4)	274	-	291

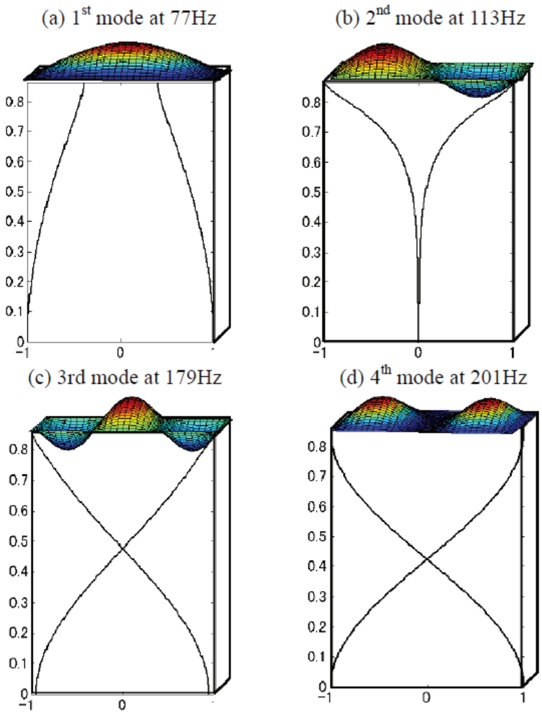

Fig.1 Acoustic mode of a strongly coupled rectangular cavity

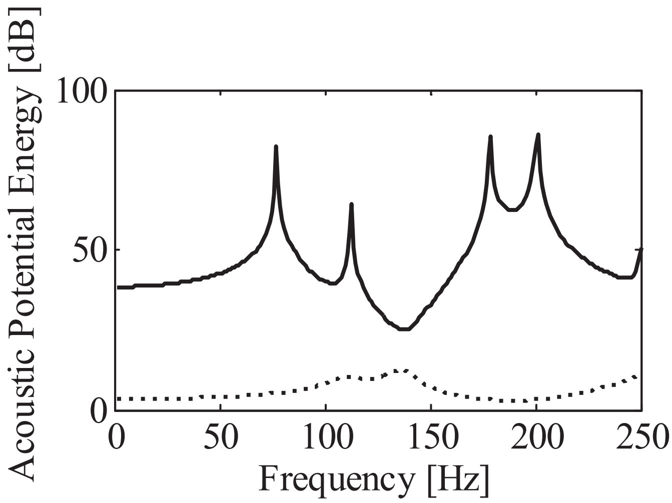

Fig.2 Frequency response of acoustic potential energy inside the coupled cavity: solid line; before control and dashed line; after control

4. CONCLUSIONS

The eigenpairs of a coupled rectangular cavity comprising a flexible panel and five rigid walls were derived. Taking into consideration the coupling effect between the sound field and the vibration filed, the eigenfunctions of a coupled cavity were expressed as the infinite sum of degenerate eigenfunctions possessing the same eigenfrequency in common. Combining the dominant equations of both the sound field and the vibration field, the characteristic matrix equation was then produced, thereby enabling the derivation of the eigenpairs of the coupled cavity. Moreover, in order to investigate the fundamental properties of the eigenpairs derived, a numerical analysis was conducted. It is found that in addition to conventional standing wave modes, evanescent modes emerge in the coupled cavity when the eigen wavenumber is pure imaginary and the associated coefficient large. Finally, the acoustic potential energy inside the cavity was expressed using the expansion theorem in terms of the eigenpairs obtained, and the optimal feedforward control law for minimizing the acoustic potential energy is derived. From a viewpoint of a numerical analysis, the fundamental properties of the acoustic potential energy comprising both the standing wave modes and evanescent modes were discussed, and the control effect as a result of minimizing the acoustic potential energy was demonstrated.

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