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DOA estimation through isosceles trapezoidal array

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ABSTRACT

The direction of arrival (DOA) estimation is the basic problem in the orientation or tracking of ships or other underwater targets, through receiving their radiated noise by hydrophone arrays. In most of the available DOA estimation algorithms, Nyquist spatial sampling theory must be followed; otherwise, the ambiguous azimuth angles will appear in DOA estimation results. In this paper, a new DOA estimation method is proposed based on the hydrophone array with isosceles trapezoidal shape, which will reduce the azimuth angle ambiguousness. Through the equation derivations as well as the numerical simulations presented in the research, it is revealed that this kind of DOA estimation can also be applied in other shapes of the receiver array, and further extended in the research field of the room impulse response analysis, room geometry inference, and so on.

Keywords: DOA estimation, Isosceles trapezoidal array, Hydrophone array
I-INCE Classification of Subjects Number(s): 13.5

1. INTRODUCTION

The direction of arrival (DOA) estimation is the basic problem in the orientation or tracking of ships or other underwater targets, through receiving their radiated noise by hydrophone arrays. So the DOA estimation, or called the spatial spectrum estimation became an important research field. Phase-comparison method (1) is a direct DOA estimation approach, where the phase difference of two equivalent array elements is measured. Generally, the resolution of the sonar array is dependent on the array aperture. So the intuitive idea of improving the estimation resolution is to expand the aperture of sonar array. On the other hand, spectrum estimation methods will work well only when the Nyquist spatial sampling theorem satisfied; otherwise, the ambiguous azimuth angles, i.e. the multi-value problem will appear in DOA estimation results (2). That is, the spacing interval should not be greater than half of wavelength. This limitation has motivated the development of non-uniform arrays (3-5). By using non-uniform array, the limitation of spatial spectrum estimation will break. The Chinese surplus theorem (6) is a popular tool to design non-uniform antenna array, while it is rarely used in sonar array. This is partly due to the complexity of setting the increasing prime number sequence.

In the present research, a non-uniform sonar array, i.e. the isosceles trapezoidal array will be applied in the DOA estimation. Its design principle is proposed, as well as the corresponding DOA estimation method, which will reduce the multi-value problem. Through the equation derivations as well as the numerical simulations presented in the research, it is revealed that this kind of DOA estimation can also be applied in other shapes of the receiver array.

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2. BASIC THEOTY

2.1 Isosceles Trapezoidal Array

Shown in Figure 1, the isosceles trapezoidal array is designed as follows: four hydrophones are located as the vertexes of the isosceles trapezoid, i.e. *a*, *b*, *c* and *d*, where the length between *a* and *b* is L_1, the length between *a* and *d* as well as between *b* and *c* is L_2, the length between *c* and *d* is $L_3 = L_1 + 2*l$, thus the height of the isosceles trapezoidal array is $H = \sqrt{L_2^2 - l^2}$, and the angle constructed by the line *ad* and the line *cd* is $\Phi = \arctan(H / l)$.

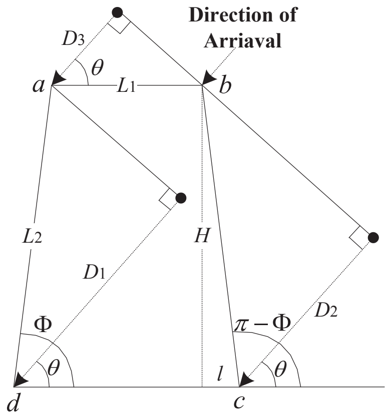

Figure 1 – The designed the isosceles trapezoidal array

2.2 Formula Derivation for DOA

It is supposed that the angle is θ between the horizontal direction and the DOA, thus in Figure 1

$$D_1 = L_2 * \cos(\Phi - \theta) = \tau_1 C, \quad D_2 = L_2 * \cos(\pi - \Phi - \theta) = \tau_2 C, \quad D_3 = L_1 \cos\theta = \tau_3 C \tag{1}$$

where C is the sound speed, f_0 is the signal frequency, the time delay between the sound arriving times at *a* and *d* is τ_1, and the phase differences is Ω_1, thus $\Omega_1 = 2\pi f_0 \tau_1 + 2\pi n_1$, and n_1 is an integer to satisfy $-\pi < \Omega_1 \le \pi$; the time delay between the sound arriving times at *b* and *c* is τ_2, and the phase differences is Ω_2, thus $\Omega_2 = 2\pi f_0 \tau_2 + 2\pi n_2$, and n_2 is an integer to satisfy $-\pi < \Omega_2 \le \pi$; the time delay between the sound arriving times at *a* and *b* is τ_3, and the phase differences is Ω_3, thus $\Omega_3 = 2\pi f_0 \tau_3 + 2\pi n_3$, and n_3 is an integer to satisfy $-\pi < \Omega_3 \le \pi$.

If the hydrophones *a* and *b* are applied to estimate DOA, the measured direction is

$$\tilde{\theta} = \arccos\left[\frac{\Omega_3 - 2\pi n_3}{2\pi f_0 L_1 / C}\right] \tag{2}$$

In the actual measurement process, n_3 is unknown, thus n_3 in Equation (2) should be chosen as all the integers that satisfy $\left|(\Omega_3 - 2\pi n_3)/(2\pi f_0 L_1 / C)\right| \le 1$, and then $\tilde{\theta}$ is calculated. In that way, there will be several DOA, i.e. the azimuth angle ambiguousness or the multi-value problem in DOA measurement.

If the isosceles trapezoidal array is applied to measure DOA,

$$D_1 - D_2 = 2L_2 * [\sin(\frac{\pi - 2\theta}{2})\sin(\frac{\pi - 2\Phi}{2})] = 2L_2 * \cos\theta * \cos\Phi \tag{3}$$

thus

$$D_3 - (D_1 - D_2) = L_1 \cos\theta - 2L_2 * \cos\theta * \cos\Phi = \tau_3 c - (\tau_1 - \tau_2) C \tag{4}$$

In that way, the measured direction is calculated as

$$\tilde{\theta} = \arccos\left[\frac{\Omega_3 - (\Omega_1 - \Omega_2) - 2\pi n}{2\pi f_0 (L_1 - 2L_2 * \cos\Phi) / C}\right] \tag{5}$$

where $n = n_3 - (n_1 - n_2)$ is an integer. In the actual measurement process, n is unknown, thus $\Omega_1 = 2\pi f_0 \tau_1 + 2\pi n_1$, and n in Equation (5) should be chosen as all the integers that satisfy $\left|[\Omega_3 - (\Omega_1 - \Omega_2) - 2\pi n]/[2\pi f_0 (L_1 - 2L_2 * \cos\Phi)/C]\right| \le 1$, and then $\tilde{\theta}$ is calculated. In that way, there will also be several DOA. If the conditions $L_1 > L_2 \cos\Phi$ and $L_1 \ne 2L_2 \cos\Phi$ are satisfied in the isosceles trapezoidal array, the relation $\left|2\pi f_0 (L_1 - 2L_2 * \cos\Phi)/c\right| < \left|2\pi f_0 L_1 / C\right|$ is obtained with the comparison between Equation (2) and Equation (5), and then the range of $\Omega_3\text{-}(\Omega_1\text{-}\Omega_2)\text{-}2\pi n$ is smaller than that of $\Omega_3 - 2\pi n_3$, therefore the required number of n in Equation (5) is smaller than that of n_3 in Equation (2), i.e., the multi-value problem can be reduced in DOA measurement. Specially, if $-\lambda/2 < L_1 - 2L_2 * \cos\Phi < \lambda/2$ is satisfied (λ is the signal wavelength), the number of n is only one, and then the measured DOA is unique.

It is seen that the multi-value problem in DOA estimation is reduced through the combination of the phase differences between every two elements in the hydrophone array. Based on this kind of DOA estimation, other shapes of the receiver array can also be designed to reduce the multi-value problem.

3. Estimation Procedure

In order to reduce the multi-value problem in DOA measurement, the estimation procedure based on the isosceles trapezoidal array is listed as follows:

(1) According to Figure 1, the isosceles trapezoidal array is constructed with four hydrophones, which satisfies $L_1 > L_2 \cos\Phi$ and $L_1 \ne 2L_2 \cos\Phi$. The length of the top side in the isosceles trapezoid is L_1, the length of the trapezoid waist is L_2, the length of the bottom side is $L_3 = L_1 + 2l$, the height of the isosceles trapezoid is $H = \sqrt{L_2^2 - l^2}$, and the angle between the waist and the bottom side is $\Phi = \arctan(H/l)$. This kind of isosceles trapezoidal array has the advantage to reduce the multi-value problem in DOA measurement.

(2) The phase difference Ω_1 is measured between the top and bottom elements on the left waist of the trapezoid, as well as the phase difference Ω_2 between the top and bottom elements on the right waist of the trapezoid and the phase difference Ω_3 between the right and left elements on the top side of the trapezoid.

(3) DOA is calculated as

$$\theta = \arccos\left[\frac{\Omega_3 C - (\Omega_1 - \Omega_2)C - 2\pi n}{2\pi f_0 (L_1 - 2L_2 * \cos\Phi)}\right]$$

where n should be chosen as all the integers that satisfy

$$\left|[\Omega_3 - (\Omega_1 - \Omega_2) - 2\pi n]/[2\pi f_0 (L_1 - 2L_2 * \cos\Phi)/C]\right| \le 1$$

4. Numerical Simulations

4.1 Case Study 1

In the isosceles trapezoidal array shown in Figure 1, the length between *a* and *b* is $L_1 = 2m$, the lengths between *a* and *d* (as well as *b* and *c*) is $L_2 = 2.06m$, the length between *c* and *d* is $L_3 = 3m$, the trapezoid height is $H = 2m$, and the angle constructed by the line *ad* and the line *cd* is $\Phi = 76^0$. $L_1 > L_2 \cos\Phi$ and $L_1 \ne 2L_2 \cos\Phi$ are satisfied in the isosceles trapezoidal array. The signal frequency is $f_0 = 500Hz$, and the signal speed is $C = 1500m/s$, thus the wavelength is $\lambda = 3m$. Because of $-\lambda/2 < L_1 - 2L_2 * \cos\Phi < \lambda/2$, the unique DOA can be calculated through Equation (5).

Table 1 – True and measured directions in case study 1

True Incident Direction (Degree)	**Measured Direction by elements *a* and *b* (Degree)**		**Measured Direction by elements *c* and *d* (Degree)**			**Measured Direction by Isosceles Trapezoidal Array (Degree)**
0	**1.28**	*120.02*	**0.00**	*90.00*	*180.00*	**2.56**
10	**9.77**	*120.96*	**9.94**	*90.86*		**9.25**
20	**19.61**	*123.92*	**20.03**	*93.47*		**18.29**
30	**29.72**	*129.16*	**30.00**	*97.70*		**28.84**
40	**39.69**	*136.93*	**40.00**	*103.53*		**38.74**
50	**49.72**		**50.02**	*110.95*		**48.81**
60	**59.78**		**60.00**	*120.00*		**59.14**
70	**69.86**		**70.00**	*131.15*		**69.45**
80	**79.94**		**80.01**	*145.74*		**79.72**
90	**90.00**		*0.00*	**90.00**	*180.00*	**90.00**
100	**100.06**		*34.26*	**99.99**		**100.28**
110	**110.14**		*48.85*	**110.00**		**110.55**
120	**120.22**		*60.00*	**120.00**		**120.86**
130	**130.28**		*69.05*	**129.98**		**131.19**
140	*43.07*	**140.31**	*76.47*	**140.00**		**141.26**
150	*50.84*	**150.28**	*82.30*	**150.00**		**151.16**
160	*56.08*	**160.39**	*86.53*	**159.97**		**161.71**
170	*59.04*	**170.23**	*89.14*	**170.06**		**170.75**
180	*59.98*	**178.72**	*0.00*	90.00	**180.00**	**177.44**

Shown in Table 1, the true incident directions are distributed from 0 degree to 180 degree with the same interval of 10 degree. The multi-value problem appears in the directions of 0~40 degree and 140~180 degree when the elements *a* and *b* are used to measure DOA, where the bolding values are similar to the true directions, and can be regarded as the correct results, while the italic values are the false measured results. The multi-value problem appears in all the directions of 0~180 degree when the elements *c* and *d* are used to measure DOA, and there are three values at 0, 90 and 180 degree, where the bolding values are similar to the true directions, and are regarded as the correct results, while the italic values are the false measured results. When isosceles trapezoidal array is used to measure DOA, there is no multi-value problem, and the measured values are generally in agreement with the true directions.

4.2 Case Study 2

Except for the signal frequency $f_0 = 800Hz$, the other parameters are the same as Section 4.1. The true incident directions are distributed from 0 degree to 180 degree with the same interval of 10 degree in Table 2. The multi-value problem appears when the elements *a* and *b* as well as *c* and *d* are used respectively to measure DOA, where the bolding values are similar to the true directions, and can be regarded as the correct results, while the italic values are the false measured results. It can be seen in this table, although multi-value problem also appears when isosceles trapezoidal array is used to calculate DOA, the number of the measured results are the fewest. Thus the proposed isosceles trapezoidal array and the corresponding estimation method has the ability to reduce the multi-value problem in the DOA measurement.

Table 2 – True and measured directions in case study 2

True Incident Direction (Degree)	**Measured Direction by elements *a* and *b* (Degree)**			**Measured Direction by elements *c* and *d* (Degree)**				**Measured Direction by Isosceles Trapezoidal Array (Degree)**	
0	**1.3**	86.4	151.1	**0.0**	68.0	104.5	151.0	**2.6**	151.2
10	**9.8**	87.2	152.8	**9.9**	68.9	105.4	152.9	**9.2**	152.6
20	**19.6**	89.7	158.9	**20.0**	71.7	108.1	159.3	**18.3**	157.7
30	**29.7**	94.0		**30.0**	76.1	112.6		**28.8**	177.4
40	**39.7**	99.7		**40.0**	81.9	118.9		**38.7**	
50	**49.7**	106.9		**50.0**	89.0	127.4		**48.8**	
60	**59.8**	115.7		**60.0**	97.2	138.6		**59.1**	
70	**69.9**	126.4		14.8	**70.0**	106.4	155.2	**69.5**	
80	**79.9**	139.7		37.0	**80.0**	116.8		**79.7**	
90	20.4	**90.0**	159.6	51.3	**90.0**	128.7		**90.0**	
100	40.3	**100.1**		63.2	**100.0**	143.0		**100.3**	
110	53.6	**110.1**		24.8	73.6	**110.0**	165.2	**110.5**	
120	64.3	**120.2**		41.4	82.8	**120.0**		**120.9**	
130	73.1	**130.3**		52.6	91.0	**130.0**		**131.2**	
140	80.3	**140.3**		61.1	98.1	**140.0**		**141.3**	
150	86.0	**150.3**		67.4	103.9	**150.0**		2.6	**151.2**
160	21.1	90.3	**160.4**	20.7	71.9	108.3	**160.0**	22.3	**161.7**
170	27.2	92.8	**170.2**	27.1	74.6	111.1	**170.1**	27.4	**170.8**
180	28.9	93.6	**178.7**	29.0	75.5	112.0	**180.0**	28.8	**177.4**

5. CONCLUSIONS

In this paper, a new DOA estimation method is proposed based on the hydrophone array with isosceles trapezoidal shape, which reduces the azimuth angle ambiguousness, i.e. the multi-value problem. Through the equation derivations and the numerical simulations presented in the research, it is revealed that the multi-value problem in DOA estimation is reduced through the combination of the phase differences between every two elements in the hydrophone array. Based on it, other shapes of the receiver array can also be designed to reduce the multi-value problem. Besides the orientation or tracking of ships or other underwater targets, this kind of DOA estimation has the potential to be extended in the research field of the room impulse response analysis, room geometry inference, and further the radar, seismology, biomedicine, communication, and so on .

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