Variability in structure-borne flanking transmission at low and mid frequencies

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ABSTRACT

Structure-borne flanking transmission in buildings depends on the vibrational energy transmission at junctions between plates. Heavyweight cavity walls are commonly used as party wall between row houses and in apartment buildings. For these structures, the assessment of the vibration transmission is complicated by the low modal density and low modal overlap in a broad frequency range. In this paper, a wave based model (WBM) is used to assess the variability of flanking transmission across rigid junctions composed of rectangular single and double walls. The wave based model accounts for the modal behaviour of the plates. Calculations for concrete wall junctions show that the specific plate dimensions and modal coupling can influence the transmission in a broad frequency range. As a result, the variability in vibration transmission with varying source position and plate dimensions is large. Ensemble average WBM results for double wall T-junctions are compared with statistical energy analysis (SEA) results. The vibration transmission between identical plates is reasonably well predicted by SEA, even when the modal overlap factor and mode count are low. SEA however strongly overestimates the vibration transmission between non-identical plates in the lower frequency range.

Keywords: Sound insulation, Flanking, Uncertainty

1. INTRODUCTION

The prediction of the sound insulation between rooms is a complex problem. The sound insulation between adjacent rooms in buildings is not only determined by airborne sound transmission through the common wall, but also by structure-borne flanking transmission. The vibration reduction index $K_{ij}$ is an important input quantity in building acoustical models for flanking transmission. It describes the transmission of bending wave vibration across a junction of plates. For single plate junctions like shown in figures 1a and 1c, empirical models based on measurements (1) and simulations (2) are available. In Belgium, heavyweight cavity walls are commonly used as party wall between row houses and in apartment buildings (3). Currently, no empirical models exist for double wall junctions as shown in figures 1b and 1d. Because laboratory measurements of $K_{ij}$ are costly and time-expensive, only a limited amount of measured $K_{ij}$-data for double wall junctions is available. Within the frame of the Belgian RaDS-project (Robust acoustic Details Standard), vibration reduction indices will therefore be measured for a number of heavyweight double wall junctions at a laboratory test set-up at the Belgian Building Research Institute. Schoenwald et al. (4,5) experimentally analyzed the vibration transmission across lightweight double leaf walls and solid wood double walls with continuous floors.

The wave approach is commonly used to predict vibration transmission across junctions composed of semi-infinite, single plates (2,6). The method has also been extended to multiple parallel junctions (7) and double wall junctions (8). The angular averaged transmission coefficients obtained with these models can be used in statistical energy analysis (SEA) models. The SEA-based predictions assume that all the plates have high modal density and overlap and thus diffuse vibration fields, which is often not the case for heavyweight floors and walls. Large fluctuations related to the modal behaviour can for example be observed in laboratory measurements of $K_{ij}$ for rigid junctions of cellular concrete and concrete block walls (9).

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To overcome the limitations of SEA-based models, the vibration transmission between finite plates has also been investigated by means of modal analysis and numerical methods. Guyader et al. (10) employed a global modal summation method to study flanking transmission between rooms in buildings. Shen and Gibbs (11) developed a similar model for plate assemblies consisting of thin finite-sized plates with different types of boundary conditions. It was applied to an L-junction of two plates and a series T-combination of five concrete plates. Kim et al. (12) used modal analysis to study bending wave transmission in box-like structures composed of multiple rectangular plates. It was shown that SEA underestimates vibration transmission across multiple junctions because it neglects direct coupling mechanisms between non-adjacent plates. Bosmans (6) extended the semi-modal decomposition method to junctions with orthotropic plates and point-connected plates. Díaz-Cereceda et al. (13) developed modal expansion models for impact sound transmission through structural connections, disregarding in-plane wave motion. The method was applied to study vibration transmission across in-line junctions and T-junctions of rectangular, simply supported plates.

While the modal approaches are restricted to problems with relatively simple geometry and boundary conditions, numerical methods like the finite element method (FEM) can be applied to more complex problems of vibration transmission. The computational effort required by numerical methods is however significantly larger, making them less applicable for parametric studies and Monte Carlo simulations. Simmons (14) and Fredö (15) used the FEM to calculate plate energies for coupled plates with the intention that these could be used to determine SEA coupling loss factors for plates with low modal density and overlap. In this approach, the coupling loss factors are determined from an inversion of the SEA power-balance equations with FEM results as input. Hopkins (16,17) improved the experimental SEA approaches of Simmons and Fredö by using the FEM results for an ensemble as input. It is advantageous to use the ensemble, rather than a single deterministic analysis, because relatively small variations in the physical properties of the plates can already cause large deviations in the coupling parameters. In (18), Hopkins compared FEM and measured data for masonry junctions, showing further that an individual deterministic analysis is of limited use to describe the modal response at low frequencies due to uncertainty in boundary conditions and material properties. The Monte Carlo method was used to assess the uncertainty related with material properties and dimensions in FEM. Calculations were performed at three frequencies per one-third octave band and the ensembles were limited to 10 cases up to 1000 Hz and 5 cases at higher frequencies. Crispin et al. (3) used the FEM to investigate vibration reduction indices of rigid single and double wall junctions. FEM results for different double wall X-junctions (figure 1d) were used to suggest an empirical model for the frequency average $K_{ij}$-values.
In this paper, structure-borne sound transmission across rigid junctions of rectangular single and double walls is investigated by means of a wave based model taking into account the modal behaviour of the structure. Section 2 gives a short overview of the wave based model. Thanks to the computational efficiency of the model, Monte Carlo simulations can be performed for large ensembles with a high frequency resolution and up to high frequencies. This allows an in-depth investigation of the variability in flanking transmission in a broad frequency range. The results of a large parametric study are given in section 3. The numerical study uses a heavyweight, concrete double wall T-junction as a reference. Ensemble average results are also compared with SEA predictions.

2. WAVE BASED MODEL

The wave based method (WBM) is an indirect Trefftz method which has been developed for the steady-state dynamic analysis of acoustic, structural and coupled vibro-acoustic problems (19). For problem geometries with a moderate complexity, the method has an enhanced computational efficiency compared to element-based approaches, such as the FEM. The field variables are expressed in terms of a set of wave functions which exactly satisfy the governing dynamic equations. The contribution factors of the wave functions are determined by the boundary and continuity conditions, which are solved by means of a weighted residual formulation.

In (20), a wave based model has been developed to predict the structure-borne sound transmission between finite-sized plates. The model of Bosmans (6) for junctions of single plates was extended to the case of double walls. The model is restricted to junctions of rectangular single or double plates connected along the entire width of a common edge. Figures 1b and 1d show a double wall T-junction and a double wall X-junction, respectively. A transverse harmonic point force is applied on one of the plates. Simply supported boundary conditions are assumed at the edges of the plates and the plates have equal width. The plates are assumed solid, homogeneous and isotropic, and are modeled using thin plate theory. Airborne coupling between the double wall leafs is neglected. For lightweight double walls, the airborne coupling can be dominant, especially below and around the mass-spring-mass resonance frequency. For heavyweight cavity walls, the structure-borne transmission is however most important in the entire frequency range of interest (8). The plate response is described by three wave types: bending, quasi-longitudinal and in-plane transverse shear waves. The incorporation of in-plane waves is needed when modeling structure-borne sound transmission across double wall junctions. While a model with bending waves only can give reasonable results for rigid, single plate junctions, it leads to unreasonably high transmission loss values for rigid, double wall junctions (8).

The vibrational energy flow through the junction is quantified by the rms velocity level difference between the source plate $i$ and the receiver plate $j$,

$$D_{ij} = 10 \log_{10} \left( \frac{\langle v_{i}^2 \rangle}{\langle v_{j}^2 \rangle} \right)$$

(1)

with

$$\langle v_{i}^2 \rangle = \frac{1}{2S_{i,j}} \int_{S_{i,j}} v_{i,j}^2 dS$$

(2)

Here, $v_{ij}$ is the transverse velocity and $S_{i,j}$ the area of plate $i/j$. The plate average squared velocities are calculated analytically over the entire surface area of the plate (6).

As a reference, the WBM results are compared with SEA predictions. The SEA model is described in detail in (8). Each plate is represented by three subsystems to incorporate bending, quasi-longitudinal, and in-plane shear waves into the SEA model. The plate connecting the two junction beams is not taken into account explicitly as a separate subsystem in the SEA model. The coupling loss factors between the different subsystems are determined from the angular average transmission coefficients calculated with wave theory for semi-infinite plates. The wave approach considers a plane wave (bending, quasi-longitudinal or in-plane transverse shear) that impinges upon the junction at a specific angle of incidence and calculates the transmitted intensities to each plate. This gives an angle-specific transmission coefficient, which is then averaged assuming a diffuse incident vibration field.
3. NUMERICAL RESULTS

In the parametric study, a double wall T-junction composed of 15 cm thick concrete floors (plates 1 and 2 in figure 1b) and a double concrete wall is considered. The double wall consists of 10 cm concrete leafs (plates 3 and 4 in figure 1b) separated by an air cavity of 5 cm. The material properties used for the concrete are a density \( \rho = 2500 \text{ kg/m}^3 \), a Young's modulus \( E = 30 \text{ GPa} \), a Poisson's ratio \( \nu = 0.17 \), and an internal loss factor \( \eta = 0.01 \). In the reference case, the junction length is 4 m, the floors have a width of 5 m and the walls have a height of 3.5 m.

3.1 Narrow band velocity level difference

Figure 2 shows the narrow band and one-third octave band velocity level differences between plate 1 and plate 2, \( D_{v,12} \), and between plate 1 and plate 4, \( D_{v,14} \), when plate 1 is excited at a position (2.25 m, 1.05 m). The WBM calculations are performed at 81 frequencies per one-third octave band. At low frequencies, the narrow band results are characterized by strong fluctuations. The amplitude of these fluctuations gradually decreases with increasing frequency and is generally larger for the transmission from floor to wall. These fluctuations are leveled out by integrating the plate response over one-third octave bands. Large dips and peaks are only visible in the one-third octave band results for the floor-wall transmission below 200 Hz.

![Figure 2](image)

Figure 2 – Double wall T-junction with 15 cm concrete floors and 10 cm concrete walls. Narrow band and one-third octave band velocity level difference (a) \( D_{v,12} \) and (b) \( D_{v,14} \).

3.2 Variability with source position

Figure 3 shows the influence of the source position on the one-third octave band velocity level differences \( D_{v,12} \) and \( D_{v,14} \) for the concrete double wall T-junction. The average values and standard deviations are shown for 20 randomly chosen excitation positions in the central part of plate 1. The areas within 0.5 m from the plate boundaries and 1 m from the junction are avoided. The variation with source position is large over a broad frequency range. Only at frequencies above 2500 Hz, the standard deviation is smaller than 1 dB. Furthermore, the influence of the source position is larger for the transmission from floor to wall. To limit the influence of the source position on the simulation results and include as many modes as possible, all subsequent results show the average velocity level difference over 20 random source positions.
Figure 3 – Double wall T-junction with 15 cm concrete floors and 10 cm concrete walls. Average one-third octave band velocity level difference and standard deviation for 20 different excitation positions on plate 1.

3.3 Ensemble average velocity level difference and comparison with SEA

The one-third octave band results are compared to SEA predictions in figure 4. The WBM results for the single, deterministic double wall T-junction are indicated by 'WBM single junction'. The SEA predictions are made with the double wall SEA model of (8) described briefly in section 2. Statistical predictions represent averages of an ensemble of similar systems with randomly distributed physical properties. Consequently, any numerical evaluation of SEA should incorporate some form of statistical behaviour. Furthermore, Hopkins (16,17) has indicated that for plates with relatively low modal densities, it is rarely appropriate to draw conclusions based on a single deterministic analysis. Relatively small variations in the physical properties can cause large differences in vibration transmission for heavyweight structures. In many practical situations, especially related to building acoustics, there is relatively large uncertainty in the material properties and dimensions. Hence, an ensemble of double wall T-junctions is created and analysed with the wave based model. Fifty similar junctions are considered by varying the junction length and plate dimensions according to a Gaussian distribution with a standard deviation of 1%. The width of each plate is varied independently. The average WBM results for the ensemble are compared with the single deterministic WBM in figure 4. Furthermore, the variation in velocity level difference in the ensemble is indicated by standard deviation intervals. For the floor-wall velocity level differences $D_{v,14}$ (figure 4b) and $D_{v,32}$ (figure 4c), the ensemble average is closely approximated by the result obtained for the single artefact. The ensemble averaged velocity level differences between floor and floor $D_{v,12}$ (figure 4a) and between wall and wall $D_{v,34}$ (figure 4d) exceed however the results for the single artifact in a wide frequency range. These observations indicate that the vibration transmission between two different plates is less sensitive to small variations of the plate dimensions than the vibration transmission between two identical plates. In the latter case, the plate eigenfrequencies and modes perfectly match for the single artifact case, leading to increased vibration transmission. Due to small variations in plate dimensions, this coupling between plate modes is weakened.
Figure 4 – Double wall T-junction with 15 cm concrete floors and 10 cm concrete walls. One-third octave band velocity level difference (a) $D_{v,12}$, (b) $D_{v,14}$, (c) $D_{v,32}$, and (d) $D_{v,34}$.

SEA models using the angular average transmission coefficient determined from wave theory tend to give good agreement with measurements and numerical modeling when the modal overlap factors is greater than or equal to unity (2,21) and when the mode count within a frequency band is at least five (22). Figure 5 shows the modal overlap factor $M$ and the mode count $N$ for the concrete floors and walls. The modal overlap factors and mode counts for the floors and walls are similar. For a loss factor $\eta = 0.01$, the modal overlap factor exceeds unity for frequencies above 1600 Hz. In this frequency range, the mode count is larger than the reference value of 5 per frequency band. For $D_{v,14}$, $D_{v,32}$, and $D_{v,34}$, good agreement between the WBM ensemble average and SEA results is found above 1600 Hz (figure 4). The SEA model overestimates however the floor-floor velocity level difference $D_{v,12}$ at high frequencies. This suggests that the conditions $M \geq 1$ and $N \geq 5$ are generally, but not always, sufficient to assure good accuracy with SEA. In the frequency range below 1600 Hz, where the modal overlap factor is less than unity, there is still good agreement between the SEA and WBM ensemble average results for $D_{v,12}$ and $D_{v,34}$. At low frequencies, dips and peaks related to the modal behaviour are visible, but the WBM ensemble average fluctuates around the SEA predictions. The variation in plate dimensions is too small to level out these fluctuations. For the floor-wall transmission, there is a large discrepancy between the WBM and SEA results below 1600 Hz. Due to the mismatch between the eigenfrequencies of floors and walls under conditions of low modal overlap, SEA overestimates the structure-borne sound transmission from floor to wall since it assumes coupled eigenmodes. The overestimation of vibration transmission between coupled systems when using SEA for plates with low modal densities and low modal overlap has also been found by other researchers (6,18,21,22).
Figure 5 – (a) Modal overlap factor and (b) mode count for the concrete floors and walls.

Figure 6 shows the velocity level differences $D_{v,12}$ and $D_{v,14}$ for the concrete double wall T-junction when the internal loss factor of the plates is increased to 0.05. The velocity level differences are on average 5 dB larger in the entire frequency range of interest compared to results for $\eta = 0.01$ (figure 4). By increasing the loss factor, the modal overlap factor is increased and exceeds unity above 315 Hz (figure 5a). Due to the higher modal overlap, the WBM results are less sensitive to small variations in plate dimensions. Hence, the results obtained for the single artifact are in close agreement with the ensemble average results. Furthermore, the agreement between SEA and WBM results is good in the broad frequency range 315-10000 Hz. The discrepancy at high frequencies between SEA and WBM results for $D_{v,12}$ is not observed any more. Below 315 Hz, the floor-wall transmission is still overestimated by the SEA model. Due to the limited number of plate modes and the mismatch in floor and wall eigenfrequencies, the velocity level difference is larger in the WBM results.

Figure 6 – Double wall T-junction with 15 cm concrete floors and 10 cm concrete walls with internal loss factor 0.05. One-third octave band velocity level difference (a) $D_{v,12}$ and (b) $D_{v,14}$.

Finally, figure 7 considers the velocity level difference for a concrete double wall T-junction built up from identical, 15 cm thick concrete plates with internal loss factor 0.01. All plates have dimensions 4 m × 5 m. Below 1600 Hz, the WBM vibration transmission from floor to wall is larger (figure 7b) than for the non-identical plate junction (figure 4b) because the eigenfrequencies and modes are perfectly matched. As a result, the SEA predictions which assume coupled eigenmodes also agree better with the WBM results.
3.4 Variability with plate dimensions

To assess the variability in velocity level differences with varying plate dimensions, the wall height, floor width and junction length of the reference double wall T-junction are varied independently between 3 and 5 m.

Figure 8 shows the ensemble averaged velocity level differences for three ensembles with average wall heights of 3, 4 and 5 m. Above 800 Hz, the floor-wall velocity level difference $D_{v,14}$ increases with the wall height (figure 8b). The vibration energy flowing from floor to wall is spread over a larger wall area leading to lower vibration levels in the wall. The floor-floor velocity level difference $D_{v,12}$ is not significantly influenced by the wall size above 800 Hz (figure 8a). In this frequency range, the vibration transmission is determined by the surface mass ratio according to wave theory for semi-infinite plates. Below 800 Hz, the wall height does have an influence on the floor-floor vibration transmission. The location of the dips and peaks associated with the modal behaviour of the floors is however not changed. Because the wall height determines the wall eigenfrequencies, it has a larger influence on the floor-wall transmission at low frequencies.

Figure 8 – Double wall T-junction with 15 cm concrete floors and 10 cm concrete walls. Influence of wall height on the one-third octave band velocity level difference (a) $D_{v,12}$ and (b) $D_{v,14}$. WBM results.

To further investigate the variability with varying wall height, an ensemble of 210 double wall T-junctions is set up where the wall height is changed randomly between 3 and 5 m. In the same time, the other plate dimensions are varied according to a Gaussian distribution with a standard deviation of 1 %. As a reference, a similar ensemble of 210 junctions has been analysed by means of SEA.
shows the average WBM and SEA results and the standard deviation intervals. The wall height has no influence on $D_{v,12}$ in the SEA model as indicated by the zero standard deviation (figure 9a). SEA predicts an increase of $D_{v,14}$ with increasing wall height, which is in agreement with WBM predictions at high frequencies (figure 8b). The variation in SEA results is of the same order of magnitude in the entire frequency range. At high frequencies, this variability is in close agreement with the WBM results. The repeated use of SEA wave theory models with varying wall dimensions does not create a realistic range for the response, however, when the modal density and modal overlap are low (16). Below 1600 Hz, the variability in WBM results increases with decreasing frequency due to the decrease in modal density and modal overlap.

Figure 9 – Double wall T-junction with 15 cm concrete floors and 10 cm concrete walls. Variability of the one-third octave band velocity level difference (a) $D_{v,12}$ and (b) $D_{v,14}$ with varying wall height.

The variability of the vibration transmission with varying floor width is shown in figure 10. A population of 210 double wall T-junctions with a floor width between 3 and 5 m has been analysed with WBM and SEA. The influence of the floor dimensions is the largest for the floor-floor transmission. SEA predicts an increase of $D_{v,12}$ with increasing floor width. Similarly, the SEA velocity level difference $D_{v,14}$ increases with increasing floor width, but the influence is smaller. Again, the variability in SEA results is almost independent of frequency and unrealistically low at low and mid frequencies. In general, the variation in WBM results is larger for the floor-wall transmission than for the floor-floor transmission, which is in contrast with SEA predictions.

Figure 10 – Double wall T-junction with 15 cm concrete floors and 10 cm concrete walls. Variability of the one-third octave band velocity level difference (a) $D_{v,12}$ and (b) $D_{v,14}$ with varying floor width.
Finally, figure 11 shows the average and standard deviation for an ensemble of double wall T-junctions where the junction length is varied randomly between 3 and 5 m. While the junction length has no influence on the SEA results, there is a significant variation in WBM results. Standard deviations of 1 dB and more are predicted up to 800 Hz. These results indicate that caution is needed when drawing conclusions from a single deterministic measurement or numerical analysis for junctions consisting of heavyweight structures. Due to the low modal density and modal overlap in a broad frequency range, the variation in velocity level difference with plate dimensions is large.

![Figure 11 – Double wall T-junction with 15 cm concrete floors and 10 cm concrete walls. Variability of the one-third octave band velocity level difference (a) $D_{v,12}$ and (b) $D_{v,14}$ with varying junction length.](image)

4. CONCLUSIONS

In this paper, a wave based model is used to assess the variability in structure-borne sound transmission across rigid junction composed of single and double walls. The model takes into account the modal interaction at the plate junctions. Due to the computational efficiency of the WBM compared to element based methods, Monte Carlo simulations are possible in a broad frequency range. Calculations for concrete double wall T-junctions show that the source position, specific plate dimensions and modal coupling can influence the structural transmission in a broad frequency range. It is thus important to average velocity level differences over a significant number of source positions. Due to the low modal density and modal overlap, the variation in velocity level difference with plate dimensions is also significant up to higher frequencies. For the concrete wall junctions, variations of 5 dB and more are predicted up to 1000 Hz when changing the plate dimensions between 3 and 5 m. Below 200 Hz the variability can increase up to 10 dB. Therefore caution is needed when drawing conclusions from a single deterministic measurement or numerical analysis for junctions consisting of heavyweight structures. The flanking transmission is generally the largest for the worst case scenario of identical plates with similar mode shapes and eigenfrequencies.

Ensemble average WBM results for junctions composed of concrete plates are compared with SEA results. Generally, there is good agreement between WBM and SEA results when the modal overlap factor is larger than one and the mode count is larger than five. For heavyweight structures with low damping, these conditions are only met above 1000 Hz. At lower frequencies, SEA strongly overestimates the vibration transmission between plates with different properties. The vibration transmission between identical plates is reasonably well predicted by SEA, even when the modal overlap factor and mode count are low.

ACKNOWLEDGEMENTS

Arne Dijckmans is a postdoctoral fellow of the Research Foundation Flanders (FWO). Part of this research was performed within a research stay at the Acoustics Research Unit of the University of Liverpool, funded by the FWO. The financial support is gratefully acknowledged.
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