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Effect of effective length of the tube on transmission loss of reactive muffler

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ABSTRACT

Reactive mufflers consist typically of ducts segments used to interconnect acoustic chambers. One-dimensional analytical models for these devices require introduction of additional terms in the modeling to account for multidimensional effects at the junctions. This term, known as "end correction", is an additional length of the considered tube or orifice that is represented mathematically in the model. It accounts the entrained mass of fluid that moves with the same velocity as that of the fluid inside the tube. On the other hand, numerical modeling, as finite element method, only requires the physical dimensions of the system, once end correction factors are automatically incorporated. In this paper, one-dimensional analytical approach (transfer matrix method) was used to predict the transmission loss of a quarter-wave-tube, a simple expansion chamber and a double-tuned expansion chamber using different estimates of end corrections factors. The pertinent effects of effective length of the tubes were shown. In order to overcome the limitations of the analytical method and to compare the results, numerical modeling of the mufflers was conducted using finite element analysis. The results show that effective length affects the performance of silencers so the end correction factor should be chosen appropriately in one-dimension models.

Keywords: Reactive mufflers, End Correction, Transfer Matrix Method, Finite Element Method
I-INCE Classification of Subjects Number(s): 34.2

1. INTRODUCTION

Reactive muffler is a type of passive noise control device which consist typically of ducts segments that are used to interconnect acoustic chambers. At lower frequencies, the one-dimensional (1D) analytical approach may be used to predict the acoustic attenuation performance of silencers since only the planar mode propagates below the cut-off frequency of the main pipe (1).

However, when two tubes of different circular cross section are joined together to form an acoustical transmission system, an additional impedance is introduced owing to the abrupt change of circular cross section at the tube junction, introducing three-dimensional (3D) effects. In order to improve the accuracy of 1D prediction, the introduction of an additional term in the model is required (2). This term, known as "end correction" is an acoustic length correction of the tubes, which accounts the multidimensional wave effects associated with the generation of evanescent modes. It accounts the entrained mass of fluid that moves with the same velocity as that of the fluid inside the tube.

The acoustic length correction can depends on the structure geometry or frequency. Several expressions for account the end corrector factor are documented in the literature. Rayleigh (*) obtained an approximate value for the acoustic length correction of duct exposure to an infinite hemisphere space. Karal (1) investigated the acoustic inductance for sudden discontinuities in the infinite circular ducts and provided an approximated expression for the acoustic inductance in terms of correction factor. Davies (3) presents an empirical end correction expression for expansion chambers based on phase changing at discontinuities. Torregrosa et al. (4) explored the possibility of obtaining relatively accurate prediction of the acoustic performance of extended duct and perforated duct mufflers by introducing a suitable end correction into simple one-dimensional models. They introduced a general correlation in terms of a non-dimensional parameter such that multidimensional effects associated with the evanescent higher-order modes at a sudden area change are considered. Ji (5) presents a numerical approach based on the three-dimensional boundary element method (BEM) to determine the acoustic length correction of closed cylindrical side-branched tube mounted

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perpendicular to a cylindrical main pipe. Kang and Ji (2) developed a two-dimensional axisymmetric analytical approach and three-dimensional finite element method (FEM) to determine the acoustic length correction of duct extension into cylindrical chamber. The effect of chamber geometry on the acoustic length correction is examined and an approximate expression for the acoustic length correction is provided. Munjal et al. (6) used a 3D model of analysis (FEM) in order to obtain information on these end corrections, and applied the same to the concentric tube resonators as well as extended tube resonators in order to design tuned extended-tube chambers with or without a perforated bridge for better attenuation.

The acoustical performance of silencers can be evaluated by different parameters, such as Transmission Loss (TL). The TL is defined as the acoustic power-level difference between the incident and transmitted waves assuming an anechoic termination (7). It is an exclusive characteristic of the muffler, independent of source impedance.

This work presents an evaluation of the effects of different acoustic length corrections on one-dimensional analytical approach to predict the acoustic attenuation performance of silencers. The TL of three different kinds of silencers (quarter-wave-tube, simple expansion chamber and double-tuned expansion chamber) were estimated by transfer matrix method adopting different end correction expressions available in the literature.

In order to overcome the limitations of the 1D analysis and to compare the results, the transmission loss was predicted numerically, using finite element analysis. The numerical model only requires the physical dimensions of the system, since the effectives lengths are automatically incorporated. It was used a commercial software (Ansys) and the TL was estimated using the three-point method.

2. THEORY BACKGROUND

2.1 Transfer Matrix Method (TMM)

The transfer matrix (also called transmission matrix or four-pole parameter representation) has been known for a long time and has been extensively explored by other researchers for analysis of acoustic systems. The TMM is based on the analogy to the electrical filters used in the transmission line theory made use for the first time by Sreenath and Munjal (8). The state variables of acoustic pressure (p) and mass velocity (v) were described as analogous to electromotive force (voltage) and current. And due consideration was given to the radiation impedance (Z_0) and noise source characteristics (p_e and Z_e) analogous to the load impedance, open circuit voltage and internal impedance of the source, respectively. Figure 1 illustrates the equivalent circuit.

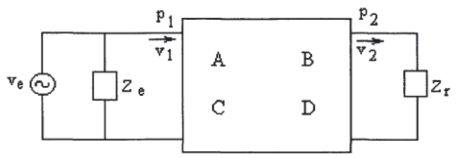


Figure 1 – Electro-acoustic analogy circuit

The two state variables (p and v) are taking at the input and output sides of an element and relating them by defining a 2x2 matrix:

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} \tag{1}$$

[p_1 , v_1] is called the state vector at the upstream point 1 and [p_2 , v_2] is called the state vector at the downstream point 2. The four-polo parameters of the transfer matrix A, B, C and D are determined by applying the following relationships to the pressure and mass velocity to the element of interest (9):

$$A = \left.\frac{p_1}{p_2}\right|_{v_2=0,\, v_1=1} \tag{2}$$

$$B = \left.\frac{p_1}{v_2}\right|_{p_2=0,\, v_1=1} \tag{3}$$

$$C = \left.\frac{v_1}{p_2}\right|_{v_2=0,\, v_1=1} \tag{4}$$

$$D = \frac{v_1}{v_2}\Big|_{p_2=0,\, v_1=1} \quad (5)$$

The advantage of the TMM is that every silencer can be divided into a number of segments or elements, as the system elements can be assumed to be linear and passive. So, each element is characterized by one transfer matrix, which depends on its geometry and flow conditions. The individual transfer matrices can then be combined through matrix multiplication to determine the global four-pole of the system. For example, the muffler shown in Figure 2 includes six elements: a straight extended tube, sudden expansion and extended inlet, uniform tube, sudden contraction, concentric resonator with perforated tube and a straight tail pipe.

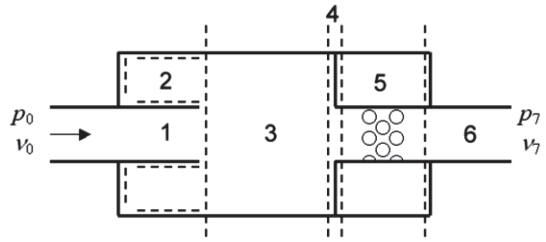


Figure 2 – Example of real muffler composed by several elements

Then, for this particular muffler, the total transfer matrix of the system can be written as:

$$\begin{bmatrix} p_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix}\begin{bmatrix} A_4 & B_4 \\ C_4 & D_4 \end{bmatrix}\begin{bmatrix} A_5 & B_5 \\ C_5 & D_5 \end{bmatrix}\begin{bmatrix} A_6 & B_6 \\ C_6 & D_6 \end{bmatrix}\begin{bmatrix} p_7 \\ v_7 \end{bmatrix} \quad (6)$$

Such that:

$$\begin{bmatrix} p_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} p_7 \\ v_7 \end{bmatrix} \quad (7)$$

where T_{11}, T_{12}, T_{21} and T_{22} are the four-pole parameters of the total system transfer matrix, given by the multiplication of transfer matrices from each element. Transfer matrices for different kinds of elements have been derived over the last three decades by different researchers and are given in detail in references (7,9)

The transfer matrix method is valid only for propagation of planar waves. However, it has been widely accepted as a tool to analyze complex systems due to its computational efficiency and flexibility. The TMM is very useful in evaluating the effectiveness of acoustic filters used for noise control applications by estimating their acoustic performance parameters, such as the transmission loss. In terms of the corresponding four-pole parameters, the TL is given by (7):

$$TL = 20\ \log\left|\frac{T_{11}+(S/c)T_{12}+(c/S)T_{21}+T_{22}}{2}\right| \quad (8)$$

where c is the speed of sound, S is the cross-sectional area of the reference pipe. This expression assumes that the area of the exhaust pipe upstream the silencer is the same as that of the tailpipe, that is *S*.

2.2 Finite Element Method

The governing formulation for the sound field inside the silencer is given by Helmholtz equation:

$$\nabla^2 p - k^2 p = 0 \quad (9)$$

where k = ω/c is the acoustic wave-number (rad/m), ω is the angular frequency (rad/s).

Galerkin's method of weighted residuals is applied to (9) and the acoustic domain of the fluid is discretized into a number of finite elements, and then use of the variational formulation for the acoustic system yields:

$$([M] - k^2[P])\{p\} = -j\rho\omega\{F\} \quad (10)$$

where $[M]$ and $[P]$ ate the inertia and stiffness matrices of the element, $\{p\}$ is the vector of acoustic pressures at all nodes of the system, $\{F\}$ is the forcing vector at all nodes of the system, ρ is the fluid density (kg/m^3), and j is the square root of - 1. Applying the correct boundary conditions and solving (10) may obtain the sound pressure at all nodes of the system. In numerical simulations is feasible to simulate an anechoic termination to compute the transmission loss.

The acoustic wave in the inlet tube contains an incoming wave (p_i) as well as a reflective wave (p_r). The acoustic wave in the outlet tube contains only an outgoing wave (p_t) due to the anechoic end, as shown in Figure 2.

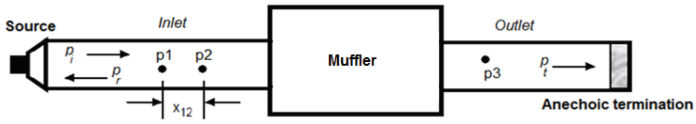


Figure 2 – Three-points Method.

Among several TL calculation formulas, the three-point method has been widely used in numerical simulations of acoustical problems because its compactness – it uses only a single run to compute the TL at each frequency. The three-point method requires the root-mean-square acoustic pressures at three points: two points in the inlet (p_1 and p_2) and one point in the outlet (p_3). As long as the inlet and outlet regions of the silencer are of the same cross section, and the properties of the fluid (density, temperature) do not change, then the TL can be expressed as (10):

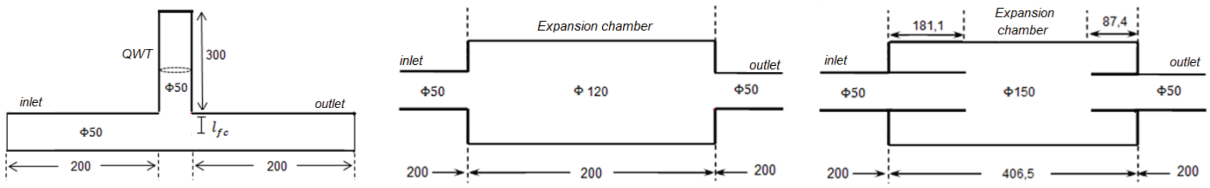
$$TL = 20 \cdot log_{10} \left| \frac{1}{p_3} \frac{p_1 - p_2 \cdot e^{-jkx_{12}}}{1 - e^{-jk2x_{12}}} \right| \quad (11)$$

where x_{12} is the distance between the two points in the inlet.

3. METHODOLOGY

3.1 1D Analysis

Figure 3 shows the dimensions of the mufflers adopted in this study: a quarter-wave-tube (QWT), a simple expansion chamber and a double-tuned expansion chamber. For all configurations it was considered d = 50 mm and l = 200 mm for the inlet/outlet ducts diameters and length, respectively.



(a) (b) (c)

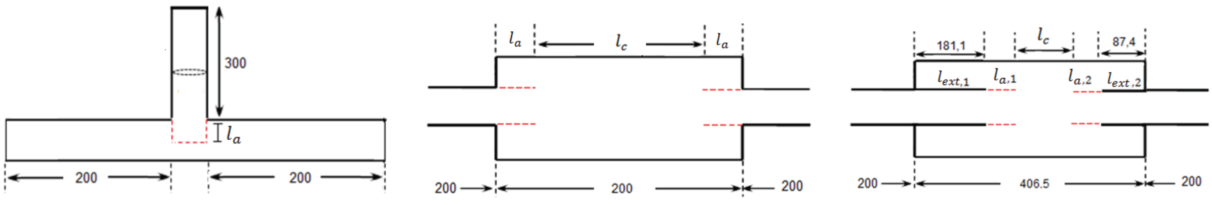
Figure 3 – Dimensions of the mufflers (a) quarter-wave-tube, (b) simple expansion chamber, (c) double-tuned expansion chamber

As showing in the last section, each silencer can be divided into a number of elements characterized by one transfer matrix, which depends on its geometry and flow conditions. Table 1 shows the elements that composed the silencers adopted in this work and the total number of transfer matrix of the system. The transfer matrix formulation for each element can be found in specific literature (**).

Table 1 – Elements for each muffler

Silencer	Elements	TM
Quarter-wave-tube	straight tube (inlet), QWT, straight tube (outlet)	3
Simple expansion chamber	straight tube (inlet); sudden expansion; straight tube (expansion chamber); sudden contraction; straight tube (outlet)	5
Double-tuned expansion chamber	straight tube (inlet); sudden expansion and extended inlet; uniform tube (expansion chamber); sudden expansion and extended outlet; straight tube (outlet)	5

In the transfer matrix of the elements cited in Table 1, to consider the effect of non-planar waves the physical length l_p of the ducts near the junctions and discontinuity areas must be replaced by the effective length $l_{eff} = l_p + l_a$, where l_a is the suitable acoustic length correction. Figure 4 indicates, in red, the acoustic length corrections necessary to take account in the models adopted.



(a) (b) (c)

Figure 4 – Acoustic length correction (a) quarter-wave-tube, (b) simple expansion chamber, (c) double-tuned expansion chamber

Table 2 presents some acoustics end corrections factors for the quarter-wave-tube resonator. Table 3 presents some end corrections factors expressions for expansion chambers silencers.

Table 2 – End correction factor expressions for quarter-wave-tube resonator

Proposed by	**End Correction Factor**	**Applies to**
Rayleigh (11)	$l_a = 0.85 r_2$	$500 < f < 1500$
Onorati (12)	$l_a = 0.3 r_2$	$f < 500$
	$l_a = r_2$	$1500 < f$
Ji (5)	$l_a = 0.9326 r_2 - 0.6196 \left(\frac{{r_2}^2}{r_1}\right)$	$\frac{r_2}{r_1} > 0{,}4$
	$l_a = 0.8216 r_2 - 0.0644 \left(\frac{r_2^2}{r_1}\right) - 0.694 \left(\frac{{r_2}^3}{r_1}\right)$	$\frac{r_2}{r_1} \leq 0{,}4$

Table 3 – End correction factor expressions for expansion chamber

Proposed by	**End Correction Factor**	**Applies to**
Karal (1)	$l_a = \frac{8 r_1}{3\pi}\left[1 - 1.238 \frac{r_1}{r_2}\right]$	$0 < \frac{r_1}{r_2} < 0{,}5$
	$l_a = \frac{8 r_1}{3\pi}\left[0.875\left(1 - \frac{r_1}{r_2}\right)\left(1.371 - \frac{r_1}{r_2}\right)\right]$	$0{,}5 < \frac{r_1}{r_2} < 1$
Davies (3)	$l_a = 0.63 r_1 \left[1 - e^{-\left(\frac{r_2/r_1 - 1}{1.5}\right)}\right]$	$0{,}5 < \frac{r_1}{r_2} < 1$
Torregrosa (4)	$l_a = 2 r_1 \left(0.26148 - e^{-(1.31906 r_2/r_1)}\right)$	$0 < \frac{r_1}{r_2} < 1$
Kang & Ji (2) (*)	$l_a = r_1 \left(0.6165 - \frac{0.7046 r_1}{r_2} + 0.2051 e^{\left(\frac{-3.4453 l_{ext}}{2 r_1}\right)} - \frac{0.3749 r_1}{r_2} e^{\left(\frac{-2.6023 l_{ext}}{2 r_1}\right)}\right)$	$\frac{r_1}{r_2} < 0.5$

(*) applies for expansion chamber with duct extension only

where $r_1$and r_2 are the radii of the small and large ducts respectively and l_{ext} is the extend duct inside de expansion chamber. For the double-tuned expansion chamber it was also adopted $l_{ext,1} = 14.1\ mm$ and $l_{ext,2} = 14.25\ mm$, as experimental calculated by Chaitanya & Munjal (13).

It important to note that as a result of the end corrections, the length of both expansion chambers is reduced to the effective length l_c, as shown in Figure 4, where $l_c = 200 - 2l_a$ to the simple expansion chamber and $l_c = 406.5 - (l_{ext,1} + l_{a,1}) - (l_{ext,2} + l_{a,2})$ to the double-tuned expansion chamber.

In order to satisfy the plane wave, the frequency range of analysis needs to be lower than the first circumferential higher order mode cut-off frequency of the main duct of the system, given by:

$$f_c = 1.84c/\pi d \qquad (12)$$

where d is the duct diameter. In this case, the cut-off frequency is 4017 Hz to the QWT, 1647 Hz to the expansion chamber and 1339 Hz, to the double-tuned expansion chamber. Respecting these limits, the frequency ranges of analysis were: 0 to 2000 Hz, 0 to 1500 Hz and 0 to 1300 Hz, respectively.

For each muffler, the transfer matrices for each element (as describe in Table 1) were identified. The effective lengths of the tubes were considerate applying the suitable acoustic end factor (Table 2 and 3).The four-pole parameters of the total system were calculated by the multiplication of the individual transfer matrices. So, the TL was estimated according to Eq. (8). This method was evaluated by means of the software Matlab.

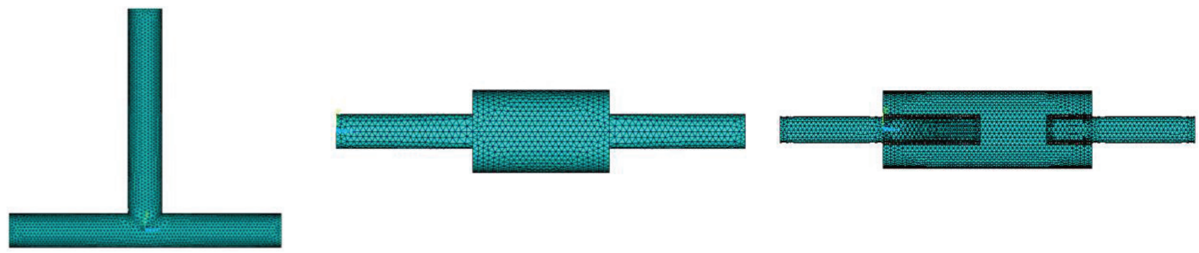
3.2 3D Analysis

A three-dimensional element model of each muffler was developed and solved for the pressure distribution over the frequency of interest using the commercial software Ansys. This step was done to provide a reliable verification reference for the one-dimensional results under investigation, since the finite element method eliminates the need to account for the end corrections.

Element FLUID30 was used for modeling the fluid medium. This element has eight nodes and four degrees of freedom per node. The following acoustical properties were used: reference pressure: 20 x 10^{-6} N/m², density = 1.21 m/s² and speed of sound = 344 m/s.

Pressure of unit magnitude was imposed in the inlet and the anechoic termination condition is imposed in the outlet. The rigid wall boundary condition is imposed on the other boundaries. Harmonic acoustical analysis is done in the frequency range of interest and the acoustic pressure in the nodes of interest (p_1, p_2, p_3), as shown in Figure 2, is obtained. So, the TL is calculated, according to Eq. (11), using the software Matlab.

Figure 5 shows the finite element mesh for each silencer. The elements were twelve times smaller than the acoustic wavelength. Table 4 summarize the mesh attributes (number of elements and nodes), frequency range and the computational time to run each analysis.



(a) (b) (c)

Figure 4 - Finite element mesh (a) quarter-wave-tube, (b) simple expansion chamber, (c) double-tuned expansion chamber

Table – 4 Mesh attributes, analysis frequency range and computational time for each model

Model	Elements	Nodes	Frequency range	Computational time
quarter-wave-tube	89910	17305	0 to 2000 Hz	04:56:37
simple expansion chamber	37454	7425	0 to 1500 Hz	00:51:08
double-tuned expansion chamber	102072	19980	0 to 1300 Hz	05:45:20

4. RESULTS AND DISCUSION

Figure 5, 6 and 7 show the transmission loss results from the 1D approach (TMM) with and without the appropriate acoustic length corrections for the quarter-wave-tube resonator, simple expansion chamber, and double-tuned expansion chamber, respectively. The names in the legend of the graphs correspond to the end correction factor applied, according to Table 2 and Table 3. For comparison purpose, the figures also include the results obtained by 3D (FEM) approach.

For the three silencers evaluated, it is observed that the shape of the TL curve obtained by 1D model with no end correction is similar to that obtained by FEM. However, differences in frequency and amplitude can be clearly noticed . For the QWT and double-tuned expansion chamber it is seen that the resonance peak in the 3D model had moved to the left of the corresponding resonance peak in the 1D model. For the simple expansion chamber the differences in amplitude becomes bigger as the frequency increase. Thereby, those factors indicates that the effective extended length really needs to be taking account in acoustic performance analysis of silencers.

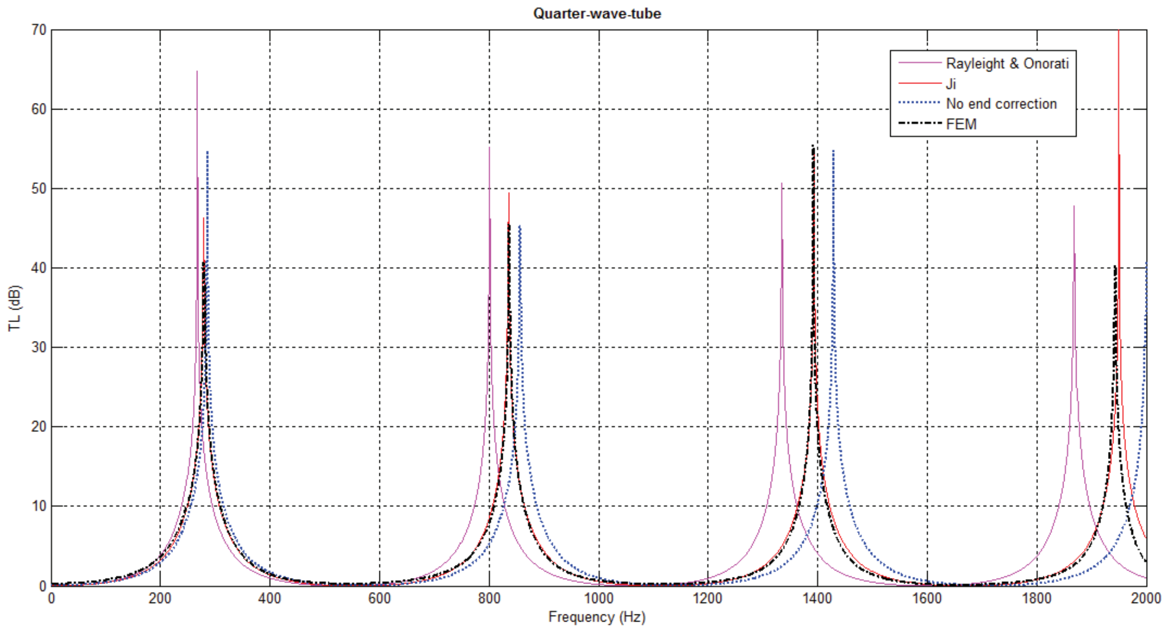


Figure 5 – Transmission loss comparison between 1D analytical model (with and without end corrections factor) and 3D FEM model for the quarter-wave-tube resonator

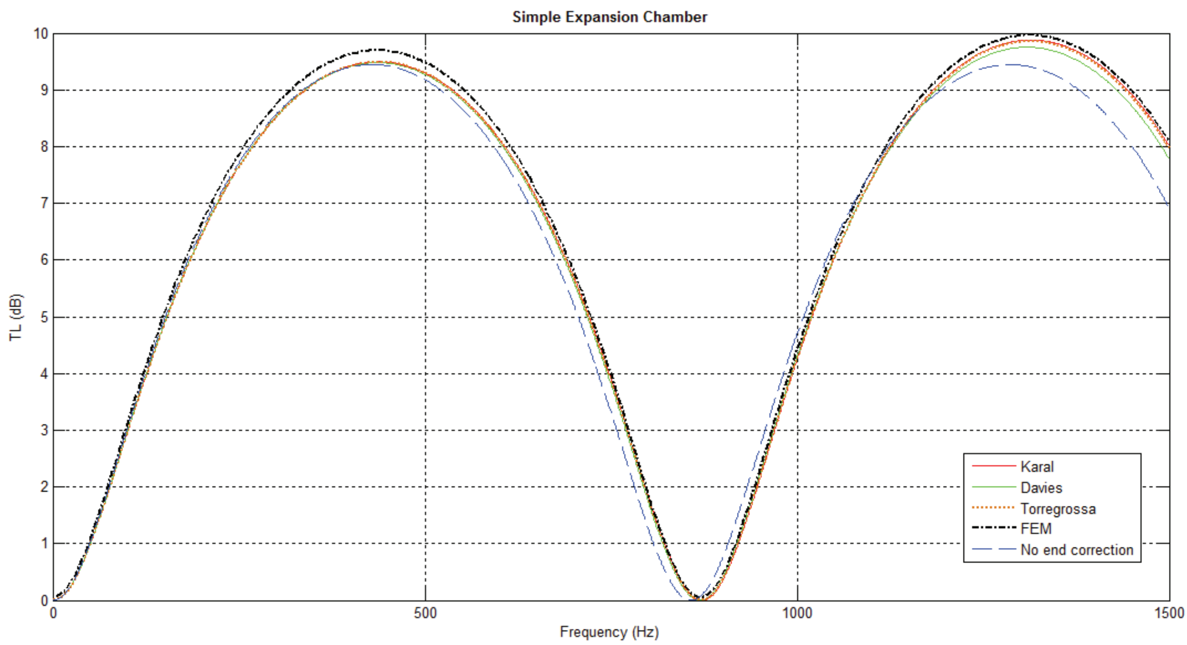


Figure 6 – Transmission loss comparison between 1D analytical model (with and without end corrections factor) and 3D FEM model for the simple expansion chamber

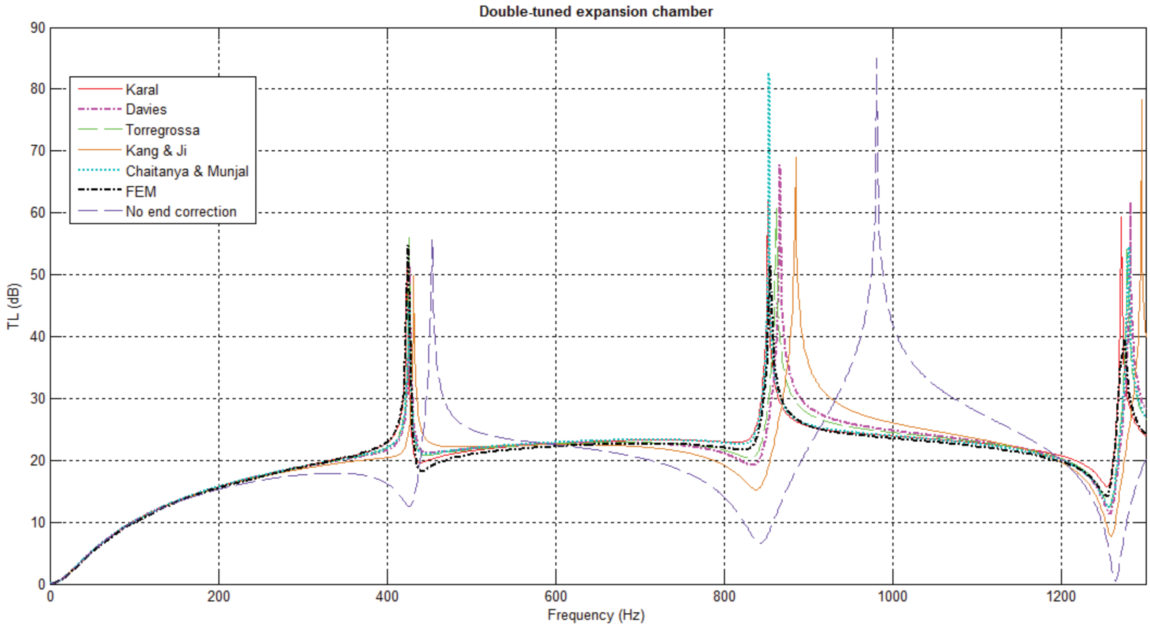


Figure 7 – Transmission loss comparison between 1D analytical model (with and without end corrections factor) and 3D FEM model for the double-tuned expansion chamber

In general, the transmission loss predictions from the corrected 1D analytical approach with the acoustic length corrections agree well with FEM models. However, for the QWT, the curve with Rayleight & Onorati end correction factor did not present good agreement. On the other hand, the Ji's end correction curve matches very well with the 3D FEM analysis, which means the acoustic correction is very accurate, as can be observed in Figure 6.

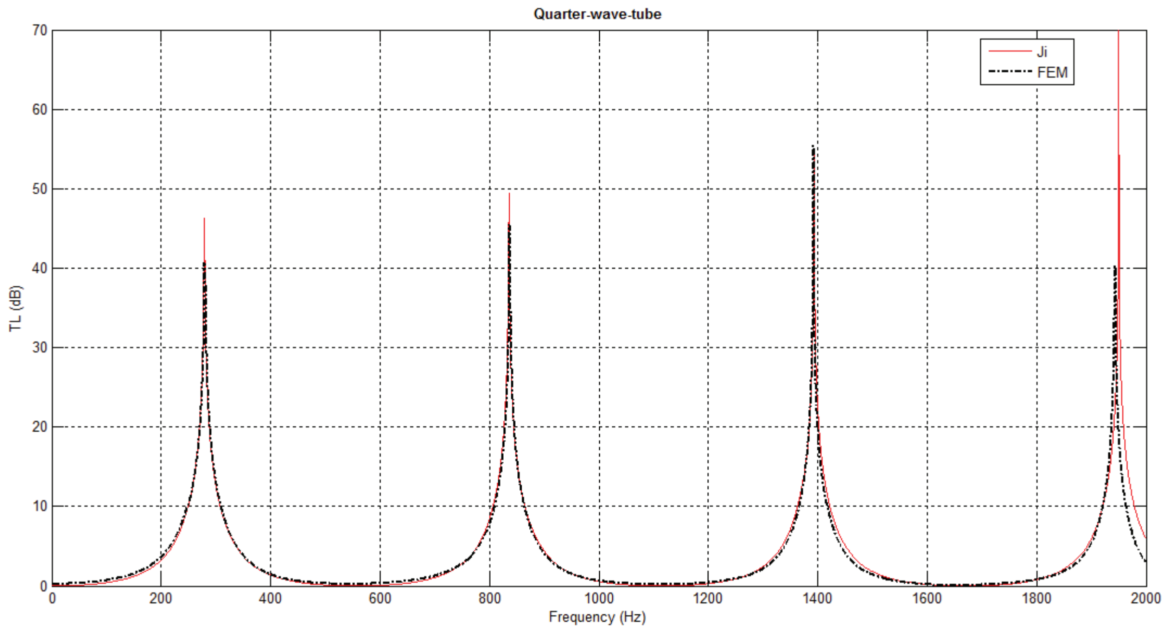


Figure 8 – Transmission loss comparison between 1D analytical model with Ji's end corrections factor and 3D FEM model for the quarter-wave-tube resonator

For the simple expansion chamber, the TL curves predicted using Karal's and Torregrossa's end corrections factor are the same, as can be observed in Figure 6. Their approximation to the FEM curve was better than Davies' curve, especially at highest frequencies.

For the double-tuned expansion chamber, it can be noted that each end correction factor results in a different TL curve. However, all of them present a better agreement to the FEM analysis than the 1D analytical approach without correction. The use of Karal's acoustic length expression gives the curve with better agreement with the 3D model, as can be observed in Figure 9.

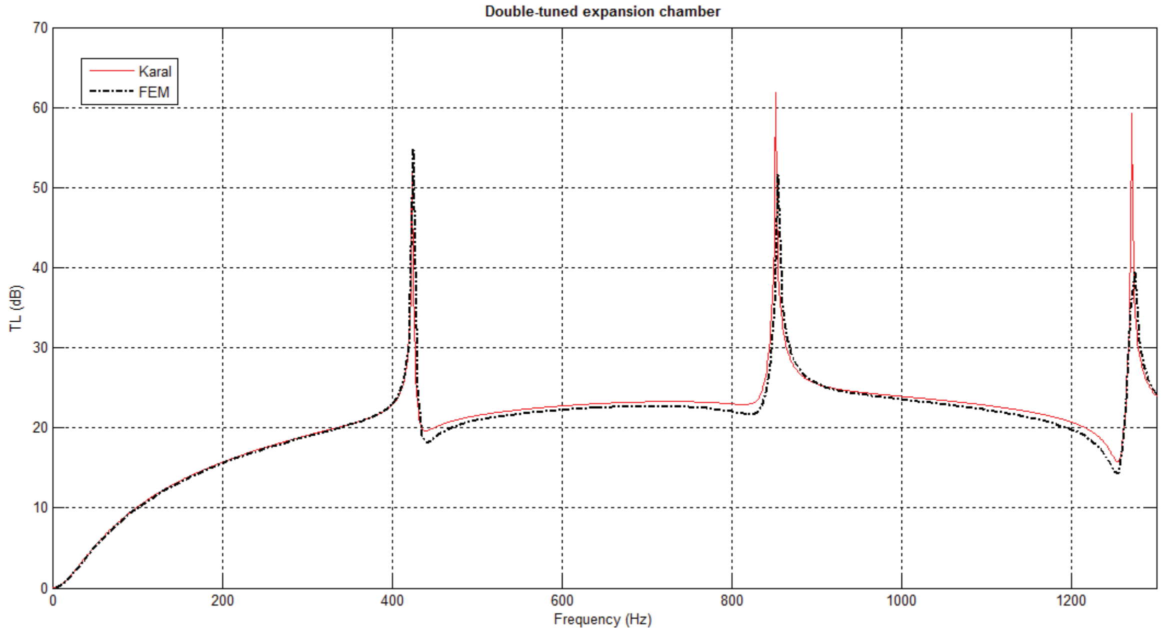


Figure 9 – Transmission loss comparison between 1D analytical model with Karal's end corrections factor and 3D FEM model for the double-tuned expansion chamber

5. CONCLUSIONS

This work evaluated the effect of the effective length of the tube on transmission loss of reactive mufflers using one-dimensional analytical approach (transfer matrix method). Different end corrections factors available in the literature were applied. The results were compared with three-dimensional approach (finite element method), that account the acoustic length automatically.

It is observed that the end correction factor affects the resonance frequency and amplitude of the transmission loss of mufflers. The transmission loss predictions from the corrected one-dimensional approach agree with the FEM predictions. However, some of the end correction factors available in the literature provide better agreement. As the Ji's acoustic length for the quarter-wave-tube resonator, Karal's and Torregrossa's for the expansion chamber and Karal's end correction expression for the double-tuned expansion chamber.

It demonstrates that the introduction of the acoustic length correction improves the prediction accuracy of the 1D theory of silencers and resonators. Computationally, successive multiplication of transfer matrices is much faster as well as more convenient than the application of the 3D finite element method. So, the corrected 1D analytical approach may be used for the quick predictions of acoustic attenuation performance of silencers and resonators, especially in low-frequency range, under the first circumferential higher order mode cut-off frequency. .

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