



## A projection and condensation method for aerospace noise and vibration engineering

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### ABSTRACT

A condensation method based on the U-vectors from Singular Value Decomposition is applied on aeroacoustics and structural dynamics data. The aim of the condensation is to make the process of cabin noise levels estimates and noise control design more efficient. The suggested method is allowing a reduction of the number of load cases used for vibro-acoustic analyses, and hence significantly reduces the time and efforts for vibration predictions and subsequent cabin noise estimates. Application of the method is made to aeroacoustic predictions for a Counter Rotating Open Rotor propulsion system, and for structural response data for an aircraft test section available at AIRBUS GmbH, Hamburg, Germany.

### 1. INTRODUCTION

Condensation methods are common in acoustics and structural dynamics. Often Eigenmodes and the associated Eigenvectors are used in order to simplify dynamics systems and for comparisons between models and experimental results. However, in some situations the use of Eigenmodes is not feasible due to e.g. high modal density. For experimental data difficulties may be faced if the modal overlap is high. High modal overlap is a result of high modal density, high damping, or a combination of the two. A similar situation occurs in numerical analyses when the models are getting large. The number of modes in the frequency range of interest, say 60-500 Hz, increases dramatically when larger geometries are to be analyzed and the efficiency improvement for responses analyses using a modal base will no longer appear.

The concept of using an orthogonal set of vectors from a Singular Value Decomposition of response data has been used at least since the early 1990's. Halvorsen et. al. [1] and Gustavsson [2] [3] applied the concept to experimental data for structural dynamics and acoustic data for SAAB aircraft. In [3] it was shown that numerical models based on the U-vectors from a set of measured Frequency Response Functions, FRFs, could be used to expand experimental data and get good estimates for excitation points not included in the test. A direct advantage of this is the significantly reduced test time to establish a full system description by means of the Frequency Response Functions, as the number of excitation points can be reduced. For development of noise control solutions the access to aircraft is often very limited due to the large amount of other tests to be made, and the general cost of aircraft access and testing. Further, the use of a projection method such as modeling based on U-vectors reduces the influence of local effects and general noise in the measurement results, which often can be an advantage compared to directly using the measured FRF-data.

The use of U-vectors for AeroAcoustic data is believed to be new. The basic idea is to see if there are ways to reduce the extensive computations needed to get reliable excitation data to be used for aircraft cabin noise predictions, and to improve the specifications and data exchange for propulsion system suppliers. The specific case studied is a Counter-Rotating Open Rotor propulsion system, and as far as known this is the first application of the suggested method to AeroAcoustic data.

## 2. SINGULAR VALUE DECOMPOSITION

Any matrix can be decomposed by the use of Singular Value Decomposition. The result is two matrices  $U$  and  $V$ , with orthogonal columns, a diagonal matrix  $s$ , with the so called Singular Values. The original matrix,  $A$ , is given by the expression below ( (1)).

$$[A] = [U][s][V]^H \quad (1)$$

The singular values are non-negative, but some will be zero if the matrix  $A$  is not of full rank. One common use of Singular Value Decomposition is to solve under-determined and over-determined systems of equations. In both cases the so called Pseudo-Inverse is found by (1).

$$pinv[A] = [U]^H [1/\bar{s}] [V] \quad (2)$$

where  $\bar{s}$  are the non-zero Singular Values of  $A$ . It can also be noted that Singular Value Decomposition is related to Eigen Value decomposition as the Eigenvalues of  $(A^T A)$  are the singular values of  $(A)$  squared.

More on Singular Value Decomposition can be found in [4]

## 3. AEROACOUSTICS

In cruise conditions, the cabin noise of an aircraft equipped with an Propeller Propulsion System is dominated by the engine tonal noise contribution and the broadband noise radiated by the Turbulent Boundary Layer.

The present study focuses on the engine tonal noise contribution which is aimed to be described as a linear combination of a finite and limited set of orthogonal principal components.

In principle, propeller noise in cruise condition consists of three distinct noise sources:

- For a propeller operating at transonic helical speed and exhibiting high velocity in the direction orthogonal to the blades leading edge, the dominant noise contribution is caused by strong shocks occurring on the blades leading edge, travelling as a volume source towards the blade tip and radiating in a very efficient and impulsive manner in the near and far field;

- For acoustically optimized propellers, this shock source component is significantly reduced and becomes small compared to the noise radiated by the forces generated on the blade surfaces – commonly known as steady loading noise (the term “steady” referring to the steady character of the forces acting on the blades for an observer attached to the rotor rotating frame);

- For a Contra-Rotating Open Rotor propulsion system, and in addition to the two aforementioned noise generation phenomena radiating at the front and rear rotor blade passing frequencies, interaction noise sources result from the viscous or potential interaction between the front and rear rotors and reciprocally. These interaction noise sources radiate with a high harmonic content in a wide polar angular range;

- Finally, for installed propeller configurations, an unsteady loading or shock noise source is generated when a blade sees a varying flow field in the course of its rotation as a result from the non-homogeneous flow induced by the aircraft solid boundaries.

With today’s computing capabilities, it is possible to model the full set of propeller noise sources by use of CFD simulations thanks the resolution of Navier Stokes equations and to propagate these sources towards the aircraft’s skin by use of CAA tools of various levels of accuracy and complexity. In the frame of the present study, a preliminary proof of concept is carried out based on simplified aeroacoustic models believed to embed the qualitative acoustic features of interest although not expected to capture the absolute noise levels radiated by the engines. In particular, a lifting line approach is used to model the propeller noise sources discarding the complex phenomenon of shock sources generation and is used to feed a Fowcs Williams and Hawkings solver responsible for the noise propagation in uniform flow up to the aircraft skin.

More realistic aeroacoustic excitations are being computed in a second step but are excluded from the present paper.

Below, Figure 1, is a typical time history and the related frequency spectra for propeller noise.

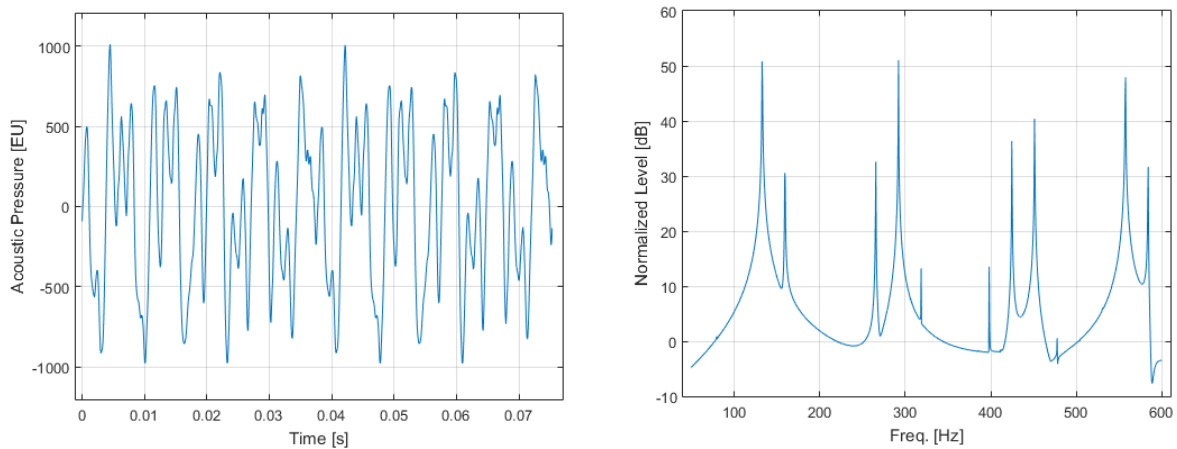


Figure 1 Pressure time history and frequency spectra for propeller noise (Schematic).

The distribution of the acoustics pressure on the fuselage is a key factor in terms of predicting and understanding the cabin noise. Example of this distribution, taken from AeroAcoustic analyses with a propeller source, are given below (Figure 2).

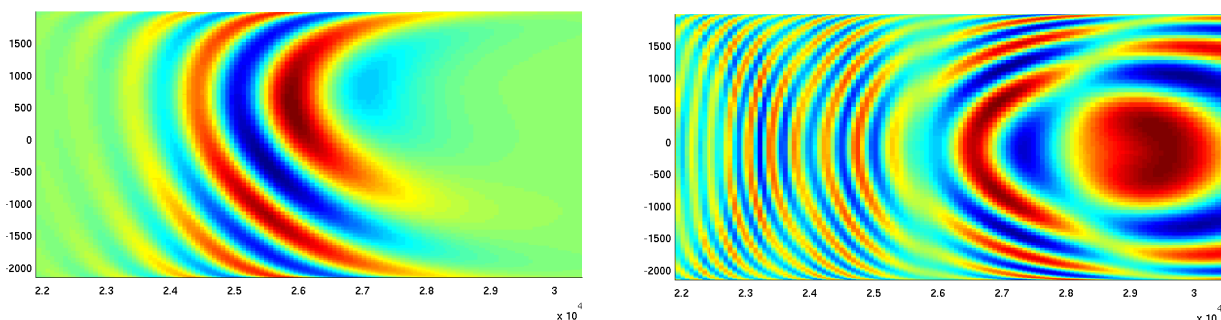


Figure 2 Instant acoustic pressure for a propeller source at an imaginary cylindrical surface in space.

In the present study results for thirty-three (33) different isolated propeller operating conditions are used as the basis for the evaluation of the projection method. The different conditions are for various propeller speeds and flight speeds. One may think this should have a significant influence on the qualitative pattern of the pressure distribution, but that is found not to be the case. Instead the application of Singular Value Decomposition show a quite rapid fall-off in the magnitude of the singular values, and an associated good quality of individual pressure distributions based on a truncated set of the U-vectors.

For the tone with the lowest frequency, the fundamental of the Rear Rotor, the three strongest U-vectors are shown below (Figure 2).

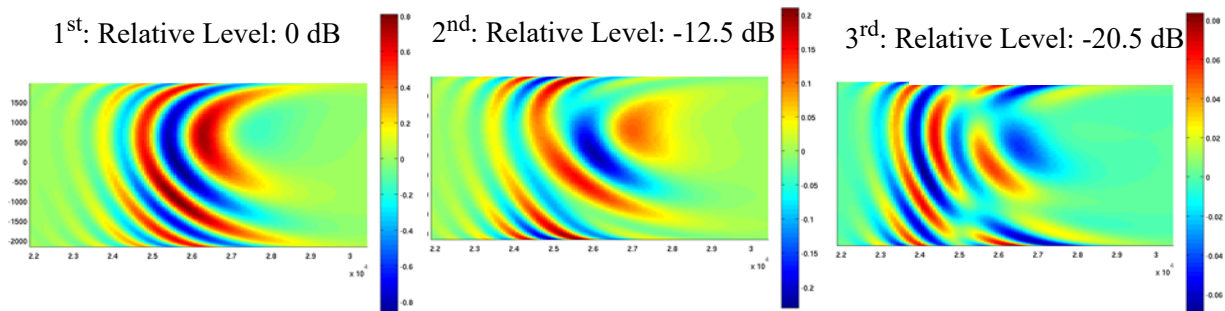


Figure 3 Pressure distribution of the three strongest U-Vectors for the Rear Rotor Fundamental frequency.

The pressure distributions are clearly showing that the Singular Value Decomposition is not only a strong mathematical tool, but also reveals that the results, at least in this case, are base vectors that can be used to better understand and describe the physics. In this case the vectors are orthogonal, complex valued, pressure distributions.

One way of evaluating the quality of a projection of the excitation pressures on a reduced set of U-vectors is to look at the level of the product of magnitude of  $s \cdot V^T$ . This quantity is the magnitude of each U-vector and is graphically given below (Figure 4).

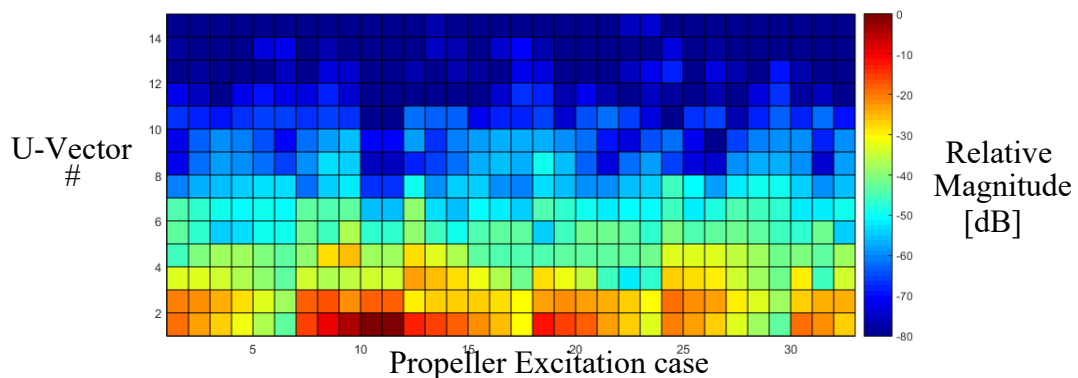


Figure 4 Magnitude of the U-vectors for each of the 33 propeller cases.

The levels are falling off quite rapidly going upwards in the picture, towards the levels of the U-vectors with lower importance (lower singular value). For the excitation cases with the highest levels, the columns with the dark red color, the relative level of U-vector number 5-6 is down by 40 dB or more compared to the U-vector with the highest level.

An alternative measure of the quality of a projection on a reduced set of U-Vectors is to compare the total level of the most significant U-vectors to the magnitude of all the other U-vectors. In the present case the total number of U-Vectors is 33, and below are results for selecting five (5) or eight (8) of them (Figure 5).

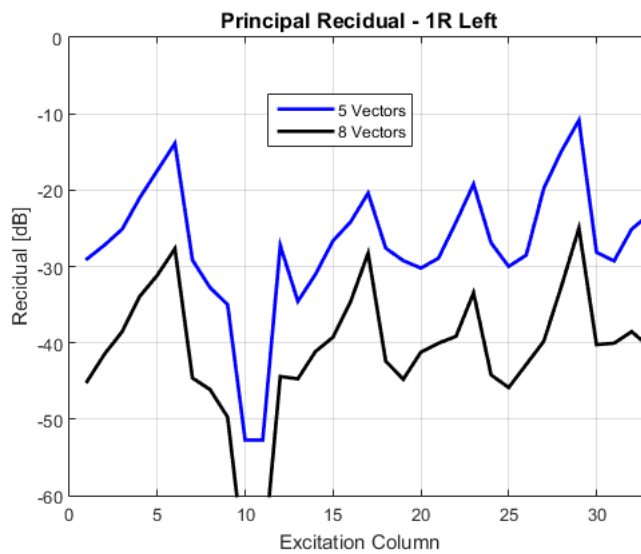


Figure 5 Relation of the level sum of the first  $N$  U-vectors to sum of the rest of the U-vectors. For  $N=5$  and  $N=8$ .

For most of the excitation vectors only five (5) U-vectors are enough to get more than 20 dB higher levels for the selected set of U-vectors compared to the non-selected U-vectors. For some cases, specifically excitation six (6) and twenty-eight (28), the low number of U-vectors will have a bit higher influence on the ability to truncate the excitation. These cases are having lower amplitude, which means that they are likely to not be the excitation cases that give the highest response in the aircraft fuselage and result in the most disturbing noise levels in the passenger cabin.

#### 4. FUSELAGE DYNAMICS

A central step in Aircraft Passenger Cabin noise predictions is the fuselage dynamics. Although there are internal noise sources inside the fuselage, the noise from the propulsion system and from the airframe dominates the cabin noise in normal flight conditions. For low frequency excitation, such as the fundamentals of the propeller propulsion system, the fuselage vibrations are of such nature that they can be analyzed with numerical tools such as The Finite Element Method (FEM). The use of FEM for vibro-acoustic analyses is well established, and even if there are several examples when models of aircraft structures and aircraft cabin acoustics do not give sufficiently good matching with experimental data, the method as such is based on solid mathematics and physics.

In the present study a model of the Acoustic Flight-LAB structure, developed by AIRBUS for studies on various analyses and testing methods, is used (Figure 6).

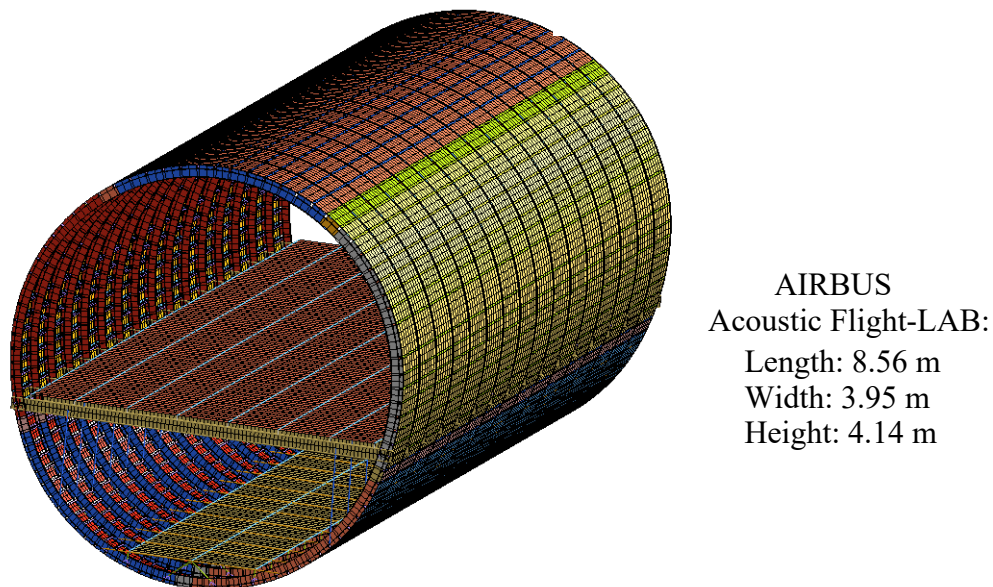


Figure 6 The model of the AIRBUS Acoustic Flight-LAB test section.

The applied excitation is the predicted field as discussed in AEROACOUSTICS (Section 3) of this paper. On the Right Hand Side the excitation pressure is transformed to represent the opposite rotation compared to the Left Hand Side, where the computed excitation is applied without any modification (Figure 7).

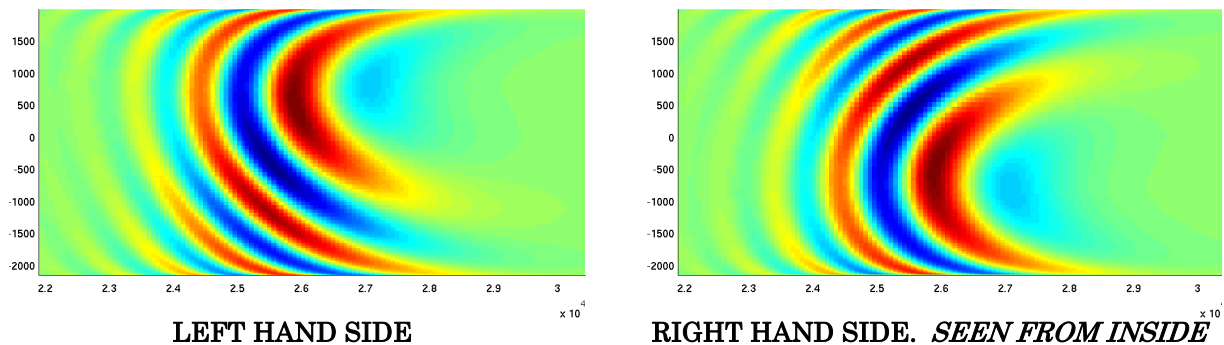


Figure 7 Instant acoustic pressure for a propeller source at an imaginary cylindrical surface in space. Left Hand fuselage Side (Left) Right Hand fuselage Side (Right)

As discussed before, the excitation cases are for a variation in propeller speed (rpm) but this is ignored and the pressure distribution for all cases are used for the same frequency range. For the fundamental rear rotor tone the operating frequency is assumed to be 105 Hz and analyses are made for 105 Hz, 103 Hz, 107 Hz, 100 Hz, 110 Hz, 96 Hz and 114 Hz (+/- 2 Hz, 5 Hz, 9 Hz).

Although the left and right responses are not identical, due to the different rotation directions of the propeller, the results are so similar that only one side (LHS) will be presented. With the seven (7) excitation frequencies and thirty-three (33) excitations there are 231 responses for each excitation tone and excitation side (LHS and RHS). Evaluations are made for the *Radial Response* of the fuselage vibrations.

Applying the Singular Value Decomposition in a similar manner as for the excitation data the dominant vibration shapes and their magnitude for each excitation case is derived. With the seven excitation frequencies the total number of U-vectors is increased from 33 to  $7 \times 33 = 231$ . With the increase from 33 to 231 U-vectors an increase of the number of U-vectors with relatively high magnitude is expected. This is also the case as can be seen in Figure 8 (below), but the fall-off for the magnitude of the U-vectors is still very clear.

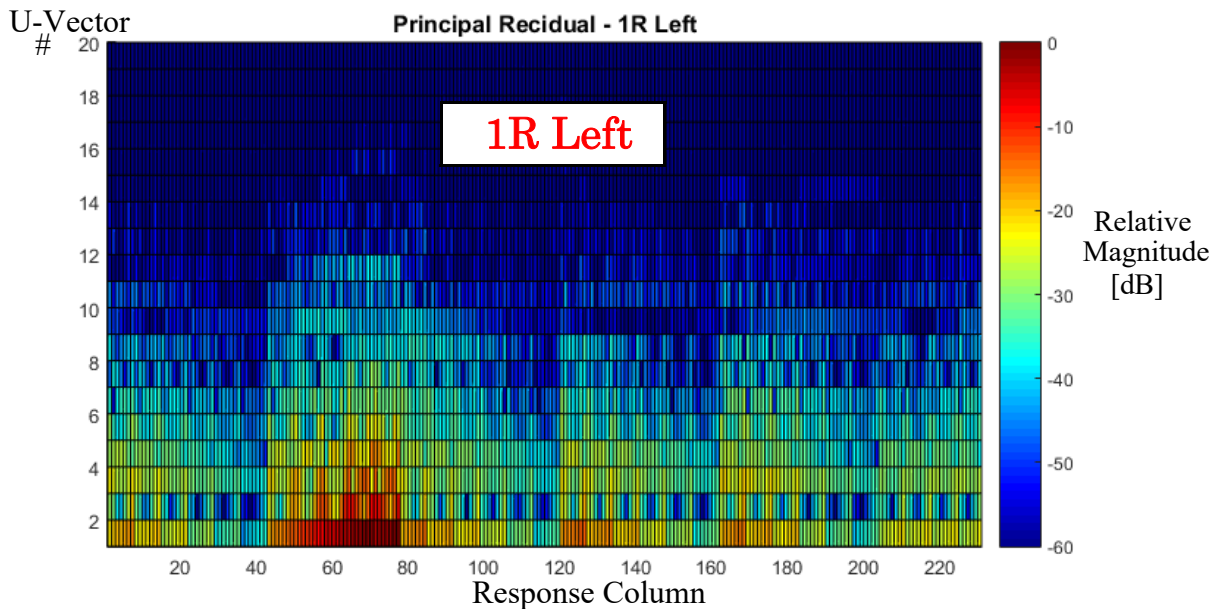


Figure 8 Magnitude of the U-vectors for each of the 33 propeller cases.  
 (Column 1 to 7: First (1) propeller excitation case, 96-114 Hz, ...  
 Column  $32 \times 7 + [1 \text{ to } 7]$ : Last (33) propeller excitation case, 96-114 Hz).

The response cases are grouped by the excitation in Figure 8, so that the first seven (7) are for propeller condition 1 and so on. Compared to the highest magnitudes, none of the U-vectors with number 12 or higher exceeds -40 dB. This is an encouraging result and suggests that a quite significant reduction of the response data can be made by means of projection to a relatively low number of U-vectors.

Figure 9, where the relative magnitude of the sum of the  $N$  first U-vectors is compared to the sum of the rest of the U-vectors, confirms the results by showing the dominance of quite few U-Vectors for each column of the response data.

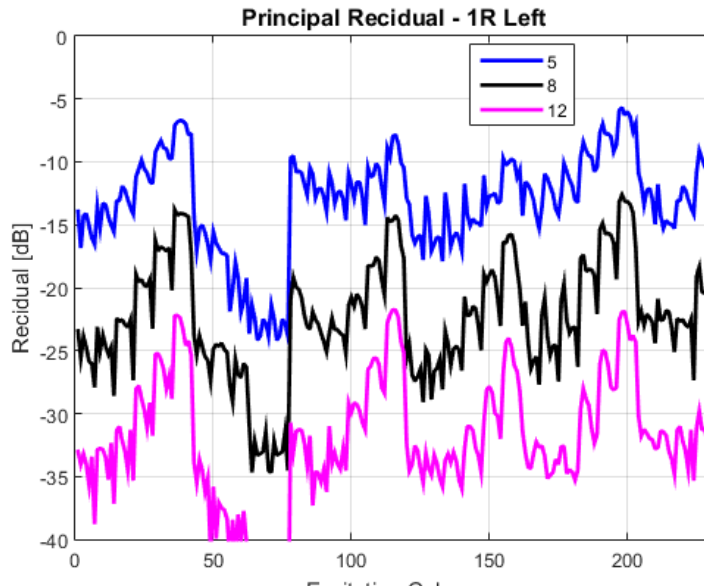


Figure 9 Relation of the level sum of the first  $N$  U-vectors to sum of the rest of the U-vectors. For  $N=5, 8$  and  $12$ .

There are clear variations of the results within each excitation, i.e. in each group of seven (7) excitation frequencies, but the general results follow the results for the excitation, Figure 4. This is clearer in the alternative graph of the magnitudes of the U-vector amplitude for one of the frequencies, e.g. for the nominal propeller rpm frequency, as given by Figure 10.

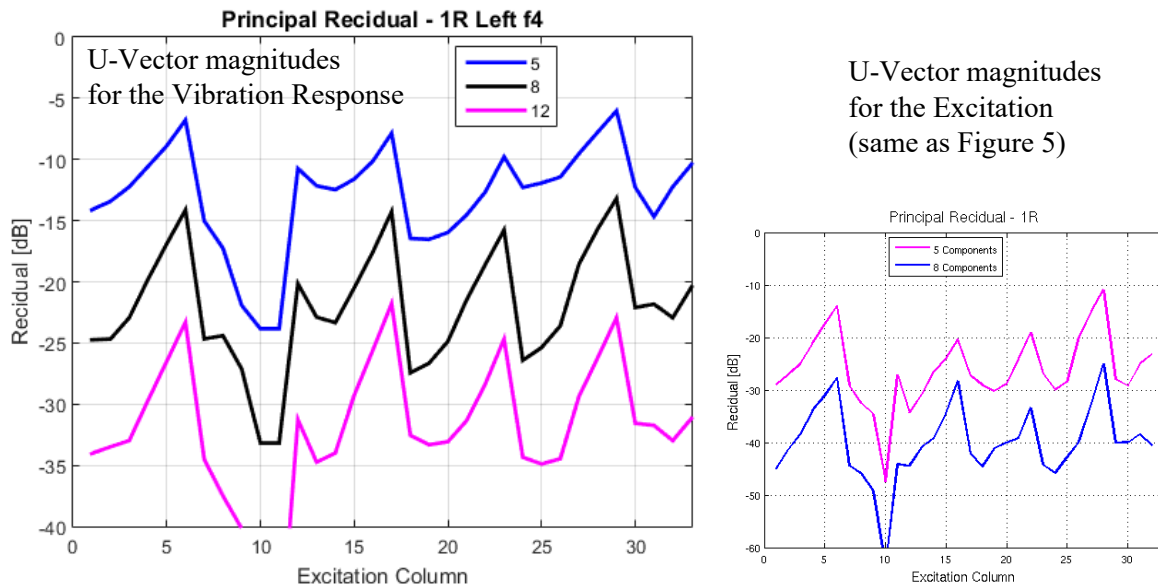


Figure 10 Relation of the level sum of the first  $N$  U-vectors to sum of the rest of the U-vectors. For the *NOMINAL* propeller rpm and  $N=5, 8$  and  $12$ .

The results for the responses show a lower dominance of the strongest U-Vectors than the excitations. This as a result of the seven (7) frequencies, giving seven times as many columns in the data matrix, and the fact that the dynamic system is of a high order in the frequency range of the analysis.



For a system with low modal density the results would have been the opposite, and a response space reduction compared to the excitation would have been noticed. It should be noted, as stated earlier, that the response data are the radial responses for the fuselage. This means, in this case, that the number of responses is about the same as the number of excitation definition points (not identical as the excitation is slightly truncated to 18576 [95%] of the 19575 response locations).

## 5. CONCLUSIONS

The suggested method for projection and reduction is shown to perform well for both acoustic excitations and structural responses. As a first result the number of load cases to be applied in vibro-acoustic analyses of aircraft can be reduced, which gives significant time savings. Complementary work is on-going to confirm the good performances of the approach when applied to more realistic engine noise excitations and higher frequencies.

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