Effects of the parameters of wavelets applied in de-noising of room impulse responses

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ABSTRACT

Wavelets have commonly been applied in de-noising of various signals, especially speech and audio signals. However, to the best of knowledge of the authors, de-noising of room impulse responses (RIRs) by wavelets can hardly be found in literature. This paper presents the use of various wavelet algorithms for RIR de-noising. For this purpose, two types of synthesized RIRs are used. In order to have RIRs with different SNRs, synthetic white noise of different levels is added. The analysis is based on applying various wavelets, Daubechies, Haar, Coiflets, Symlet, Biorthogonal, Reverse biorthogonal and Mayer, either on room response to excitation or directly to extracted RIRs. The effects of wavelets' parameters, such as level and thresholding, are observed. RIRs obtained before and after the wavelets are mutually compared in both time and frequency domain, and they are compared to the corresponding noiseless RIRs. Special attention is paid to the decay curves generated by backward integration of these RIRs. The results show that a significant improvement in dynamic range of the decay curve of even more than 20 dB can be achieved by applying wavelets. However, the technique is rather sensitive to wavelets' parameters so they should be optimally set.

Keywords: De-noising, Room Impulse Response, Wavelets I-INCE Classification of Subjects Number(s): 72.6, 51.1, 74.4.

1. INTRODUCTION

Signals are often disturbed by noise, and it is a major problem in many scientific fields including room acoustics. Noise is sometimes applied on purpose to see how some signals will react on the noise impact. In room acoustic measurements, noise is always an inherent part of the results. Noise is either product from an environment (ambient noise) or it is generated by the measurement setup (equipment or electronic noise). Namely, when an excitation signal passes through connecting wires and devices, it naturally gets noise. Once a signal is polluted with noise, it is difficult to remove noise and obtain signal without noise disturbance. For this purpose, a number of different de-noising techniques have been developed, and some of them are still in the process of development and improvement.

Some methods that are specialized in the field of de-noising are: adaptive and optimal filtering, wavelets, notch filtering, Wiener filtering, etc. (1-3). These methods can be applied to different signals such as audio signals, image signals, speech, etc, where noise is generated by different sources, and where noise impact is different. Results of de-noising may vary and they depend on used method and nature of noise. In an ideal scenario, de-noising method will completely remove noise from the signal yielding the original signal (without noise). However, in most practical cases, noise will still have a certain influence on the signal after de-noising, but the results can be satisfactory at the end.

Audio signal de-noising has been very popular method recently, especially in the field of speech processing. Generally speaking, recording of audio signals is not so easy task as it sounds to be, and the recorded signals typically have to be post-processed to remove the artifacts including the most common one – noise. Consequently, one of the main problems to be solved in audio signal processing is de-noising, as it is mentioned above (4).

Noise is also one of the main disturbances in room acoustic measurements, that is, in measurements

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of room impulse responses (RIRs), as one of the fundamental measurements in this field. Noise can cause significant problems in estimating the acoustic parameters from the measured RIRs (5). Various methods have been proposed to counteract the noise problem, such as averaging, truncation of an RIR, subtraction of noise floor or multiplication of responses measured under the same conditions (6).

On the other hand, for audio signal de-noising, the wavelet method is proven to be very useful nowadays (7). Wavelets have significantly improved the digital filter methods and their usage is widespread. An advantage of wavelet transforms is that wavelet analysis allows the usage of long time segments for low-frequency information, and shorter segments for high-frequency information (8). Wavelet transform is commonly used for signal de-noising. The first step is to apply the wavelet transform to the signal with artifacts. Then, the thresholding method is implemented on the coefficients that are below some value in magnitude, and then in the third step, the inverse transforming is performed to obtain a smoother version of the original signal (8).

This work presents the usage of different wavelets algorithms for removal of the noise artifacts from RIRs. De-noising and required signal processing are performed by using software package Matlab. A number of different wavelets are used, and the effects of changing their parameters are investigated. A focus is on an increase of dynamic range of the processed RIRs, that is, dynamic range of the decay curves obtained by the Schroeder backward integration of the RIRs (9). The investigation shows how large increase of the decay curve dynamic range can be achieved, and what wavelets and their parameters lead to the largest increase in dynamic range.

2. ROOM IMPULSE RESPONSE AND WAVELETS

2.1 Room impulse response and noise compensation

RIR is defined as a function of pressure response in time on the place of the receiver in a room as a result of room response to an impulse signal such as Dirac impulse (10). Sometimes, RIR can also be called room acoustic response. This phenomenon has gained bigger and bigger importance in the field of acoustics, especially in the field of room acoustics. Figure 1 presents an RIR in time and its decay curve generated by the Schroeder backward integration.

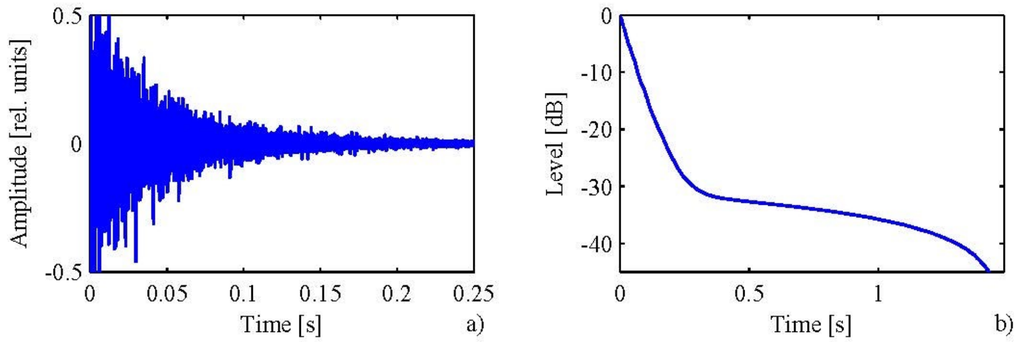

Figure 1 – RIR synthesized by the image source model (ISM) and its backward integrated decay curve

RIR consists of several components. The first of them is direct sound, which comes first to the receiver, with some small delay necessary for sound to travel between the source and receiver. The next components are early reflections with somewhat larger delay, and finally reverberation part (10). The reverberation is probably the most characteristic acoustical phenomenon in a closed room (10) and it refers to the fact that sound produced in the room will not disappear immediately after the sound source is shut off, but it remains audible for some time. Reverberation is also used for determining the quality of the room.

Over the last few decades, some advanced measurement techniques have been developed for measurement of RIRs, such as maximum length sequence (MLS), inverse repeated sequence (IRS), swept sine technique or time-stretched pulses (11). However, even measured with these more noise immune techniques, real RIRs still contain certain noise that depends on conditions found in the tested room and characteristics of the measurement system as well as applied measurement technique.

Noise occurs in every measurement of an RIR, There are two different categories of noises that contaminate measured RIRs - background and transient noise (12). While background noise is always a part of the RIR measurement, transient noise might appear in some cases adding some additional disturbance.

In order to obtain better RIRs for further analysis, noise and its effects should be reduced or compensated for. Common approaches for that purpose in RIR measurements are averaging or usage of excitation signal of higher energy. Negative effects of noise in the decay curves generated by the Schroeder backward integration (integrated impulse decay curve - IIDC) is minimized by applying techniques such as truncation of an RIR, subtraction of noise floor from an RIR before the integration, multiplication of two RIRs or by nonlinear regression approach (6). On the other hand, different techniques can be used for de-noising of audio signals representing a topic always attractive to research. The most common technique is adaptive filtering, especially when transient noise is present in signal (13). Besides, different adaptive least mean square (LMS) algorithms are often in use, but also Wiener and notch filters, various FIR and IIR filters, Kalman filter, etc. To the best of knowledge of the authors, wavelets have barely been used for de-noising of RIRs or noise compensation in RIRs. After promising results in the field of image and speech processing, we consider worthwhile to investigate potentials of applying wavelets in compensation of noise effects in calculation of room acoustical parameters from RIRs, that is, in increasing dynamic range of backward integrated RIRs.

2.2 Wavelets and their application in de-noising

During last few decades, the usage of wavelets in the field of de-noising has been becoming more and more popular. Wavelets have found their place in applications that deal with analysis and processing of different types of signals. Noise removal has become a major problem in many applications such as cellular mobile communications, speech recognition, image processing, medical signal processing, radar, sonar etc.

In 1946, Gabor found and defined time-frequency functions called wavelets as a wave function with a compact support (14). Its name comes from the oscillatory nature, and diminutive is used because of the finite domain where it is different from zero (the compact support). Scaling and translation of the basic wavelet $\psi(x)$ define the wavelet basis (14,15):

$$\psi_{a,b,}(x) = \frac{1}{\sqrt{a}}\psi(\frac{x-b}{a}), a > 0 \tag{1}$$

Small segments of a complicated form may be represented with a higher resolution, while smooth section can be represented with a lower resolution if scaling parameter a and the translation parameter b have appropriate values (14). This is an important feature of the wavelets. Typical example of applying wavelets is biomedical engineering, especially processing of electrocardiogram.

Different types of wavelets are used in different applications, and different wavelets' parameters are implemented to provide the best results. The two most common wavelets used for de-noising are Haar and Daubechies. Haar's function is the same function as the first Daubechies function (14,15). Some other wavelets that have provided good results are Coiflets, Symlet, Biorthogonal, Reverse biorthogonal and Mayer. Wavelet functions for some of the wavelets are presented in Figure 2.

3. METHOD OF ANALYSIS OF APPLYING WAVELETS IN DE-NOSING OF RIRS

For the analysis of wavelets application in RIR de-noising, two types of synthesized RIRs are used. The first one is generated by the image source model (ISM) (see, e.g. Fig. 1) and contains certain irregularities such as non-exponential decay and irregular initial decay. The second type of synthesized RIRs represents the responses with truly exponential decay as in diffuse sound fields. The RIR of this type and its IIDC is presented in Fig. 3. This IIDC is rather similar to that of the synthesized RIR generated by the ISM, except the irregularities that are present, but hardly visible in the shown curve. The synthesized RIRs are sampled at 44100 Hz and polluted with the same Gaussian random noise to compare the results of de-noising process. The noise level is determined to be a certain amount of dB below the level of RIR. Numerical value of noise level is the same as in the equation used for generation of the RIR with exponential decay, as done in Ref. 12.

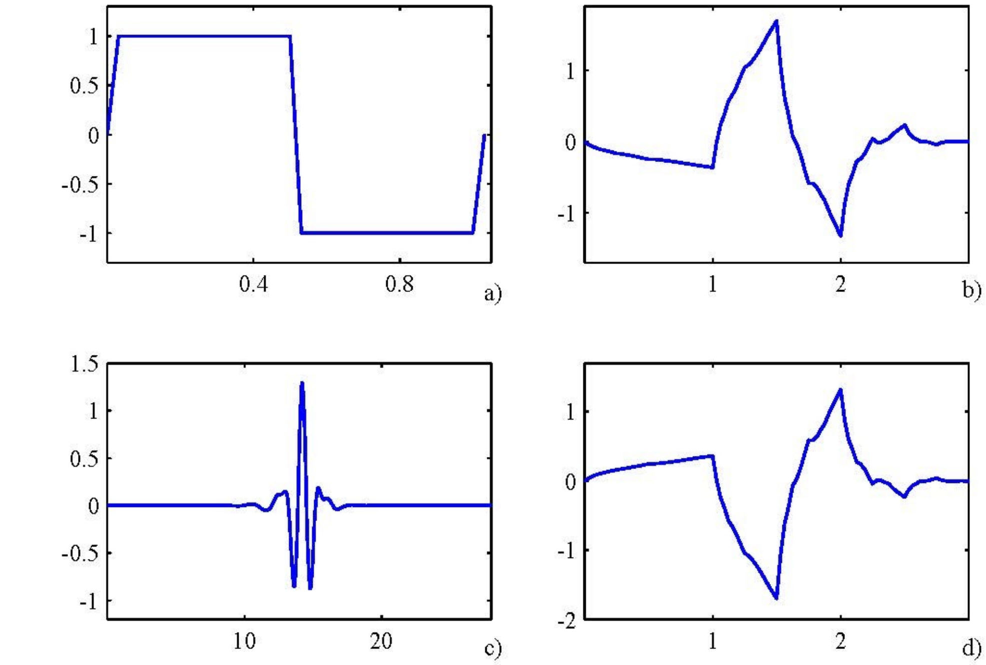

Figure 2 – Wavelet functions: a) Haar wavelet function, b) Deubechies wavelet function, c) Coiflet wavelet function and d) Symlet wavelet function

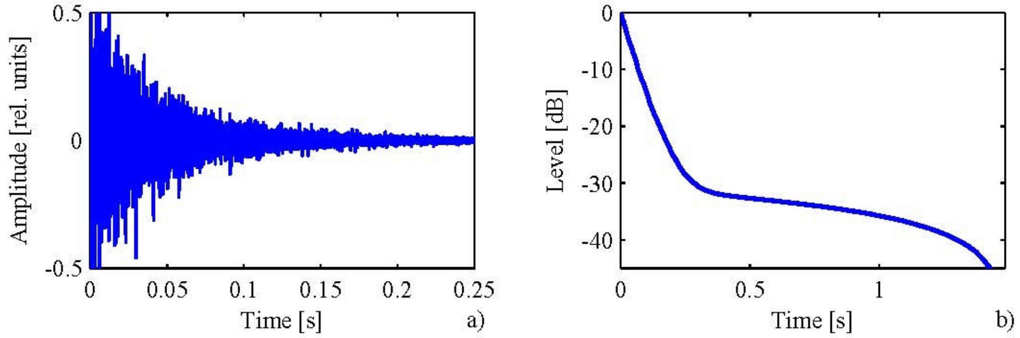

Figure 3 – Synthesized RIR with exponential decay and its backward integrated decay curve

The whole procedure of RIR measurement is simulated in the following way. The synthesized RIR is convolved with the excitation signal (exponential swept sine) in the first place. Then synthetic noise is added to the obtained room response to the excitation. The room response with noise is deconvolved with the inverse filter and RIR with noise is extracted. The idea was to apply different wavelets with different parameters to both the room response with noise and to the extracted RIR with noise and analyze the effects on the extracted RIRs and IIDCs. In both analyses, the effects of wavelets application are observed in the time and frequency domains, but also observing the IIDCs. Special attention is paid to the IIDCs and their dynamic range. The IIDCs of noiseless RIRs are used as the reference ones, and they are compared with the ones of the RIRs with noise before and after the wavelets application.

Different wavelets are applied to both room responses to the excitation signal and extracted RIRs in order to reduce the impact of noise. Implementation of wavelets is done in the Matlab software package using the function *wden* (16). Parameters of the wavelets are set directly in the function *wden*, These parameters include *minimax/sqtwolog/heursure* thresholding, *hard/soft* thresholding, rescaling, and wavelet decomposition level.

The initial results are obtained with initial set up of the wavelet parameters. It includes *sqtwolog* for universal thresholding, *soft* thresholding and different decomposition levels of the wavelets. These initial results were satisfying, but not so impressive. The analysis is continued by changing the wavelet parameters and analyzing the results in order to find the optimal combinations of the parameters leading to the largest increase in the dynamic range of the IIDC and causing only minor change of the decay of the IIDC that is below the pre-defined threshold. The pre-defined threshold for determination of dynamic range improvement has a value of 0.2 dB here. For other values of this threshold, the results will be somewhat different, but general trends will be similar to those presented in this paper.

It is also worth noting that different levels of Gaussian random noise are used. In this way, different signal-to-noise ratios (SNR) of the room responses and RIRs are generated.

4. RESULTS OF APPLYING WAVELETS IN DE-NOSING OF RIRS

The application of the wavelets on the room responses to the excitation signal leads to rather different results depending on the wavelets parameters. Some of the parameters do not give the decay curves of acceptable quality. Two illustrative examples are presented in Figure 4. In the first example presented in Figure 4 a), the IIDC obtained after applying the wavelet has only slightly increased dynamic range, although it seems that this IIDC follows the noiseless IIDC in considerably longer range than the noisy IIDC. In another example presented in Figure 4 b), the dynamic range of the IIDC obtained after the wavelet application is increased a bit more, but the decay rate is also changed (increased) in a significant part of the decay curve, see also the solid line IIDC following the dotted noiseless IIDC to a certain point, and then going below that reference curve.. Dynamic range of both IIDCs with noise (before and after the wavelet) is very small in this example. These curves coincide with the reference curve only in a small time segment, or in other words, the deviation between these curves and the reference one becomes greater than the pre-defined threshold of 0.2 dB in rather small decay (dynamic) range. If the level of wavelet is larger (the example from Figure 4 b), where the level is 8), the IIDC obtained with the wavelet very quickly goes beneath the reference IIDC.

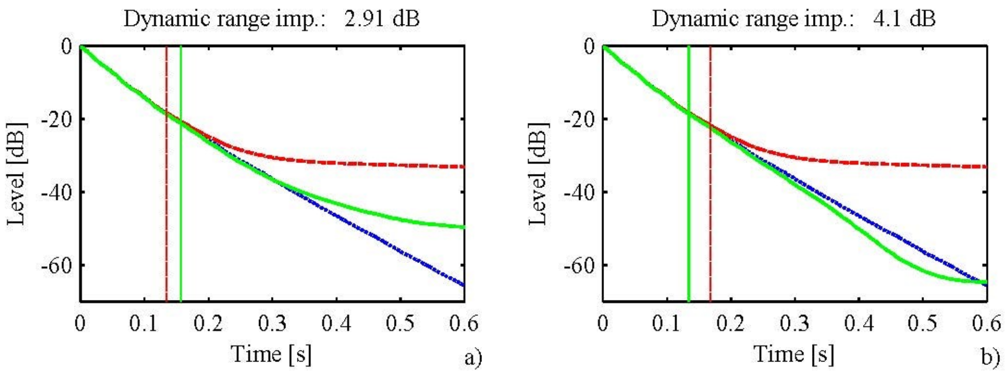

Figure 4 – IIDCs of noiseless RIR (reference curve ···), noisy RIR before wavelet (---) and after wavelet (–), where the a) Daubechies 2 wavelet at level 3, and b) Daubechies 32 wavelet at level 8 are applied to the room response to the excitation (synthesized RIR generated by the ISM is used)

The effects of wavelets applied directly on the extracted RIRs with noise are presented here in more details. These effects are first illustrated by the examples in the time domain given Figure 5, where the RIR synthesized by the ISM is used and the Daubechies 2 wavelet at level 3 is applied on this RIR. The RIRs before and after the wavelet application are similar to each other, especially in the first part where the reverberant energy is dominant. The difference between them becomes more visible after a certain point along the RIR, where noise contribution becomes larger, as presented in Figure 5 a). However, the difference between the RIRs is almost the same along the responses, and the amplitude of the difference is close to the amplitude of added noise.

The noisy RIRs before and after the wavelet application are transformed to the frequency domain and their spectra are presented in Figure 6. The presented spectra are also rather similar to each other. There is only a slight difference of order of a few tenths of dB, where the levels of the RIR before the wavelet are somewhat greater than the levels of the RIR after the wavelet, see Figure 6 b). This is an expected result since the noise is somewhat reduced after the wavelet. The figure shows that the wavelet reduces dominantly noise with frequency content above 1 or 2 kHz.

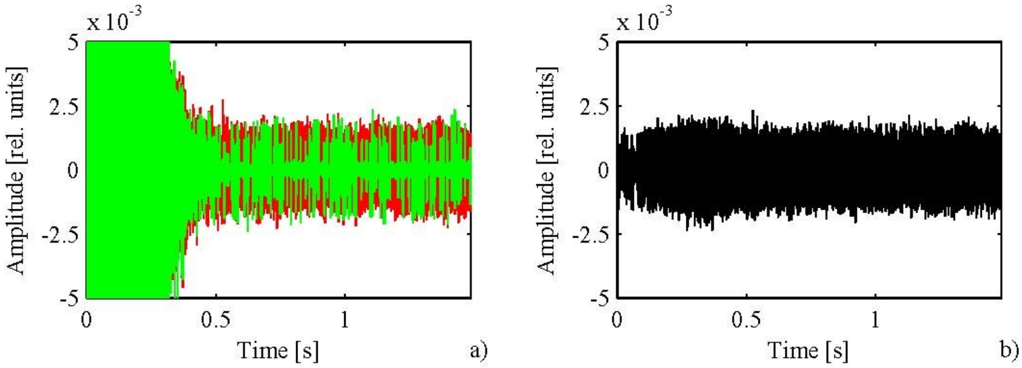

Figure 5 – RIR synthesized by the ISM with added -50 dB noise in the time domain a) before wavelet (without wavelet application) (---) and after wavelet (with Daubechies 2 wavelet at level 3) (–), as well as b) the difference of these RIRs

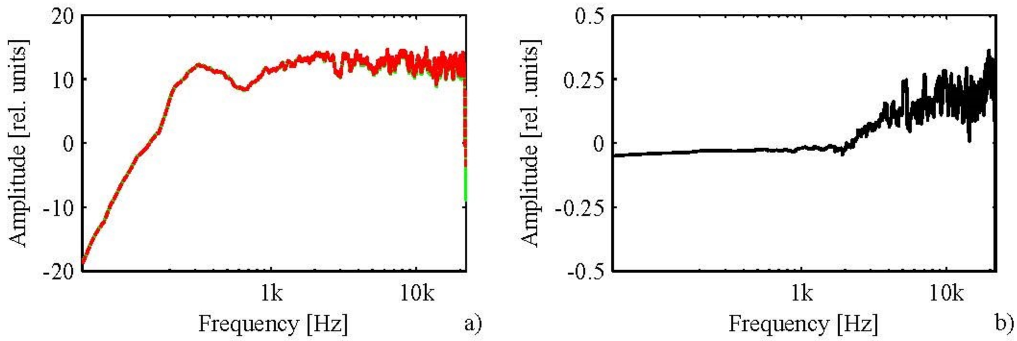

Figure 6 – RIR synthesized by the ISM with added -50 dB noise in the frequency domain a) before wavelet (without wavelet application) (---) and after wavelet (with Daubechies 2 wavelet at level 3) (–), as well as b) the difference of these RIRs

As a consequence of the discussed changes made by applying the wavelets, there is a change of the decay curve obtained by the Schroeder backward integration (IIDC). To observe this change, the decay curves generated from the RIR without noise (reference curve) and with noise before applying the wavelets (without wavelets) are plotted together with the IIDC generated from the RIR obtained after applying the wavelets (with wavelets). In this paper, representative results for some wavelets are shown in the figures, while the results for other wavelets are summarized in the tables below.

As it is shown in Figure 7, there is a significant improvement in dynamic range of the IIDC after applying the wavelet. In this case, the Daubechies 2 wavelet is used at level 3, while the other wavelet parameters are the same as above (used for Figures 5 and 6). In this particular case, the dynamic range improvement of 20.52 dB is obtained. Figure 8 shows the same decay curves, but for the Haar wavelet function, and level is now 4. This wavelet also gives significant increase in the dynamic range of the IIDC - in this case it is 22.20 dB.

Change of the noise level leads to somewhat different results, where the improvement of dynamic range is reduced with an increase in noise level, but general trends are similar to those obtained with smaller noise levels. The results for noise level of -40 dB are presented in Figure 9 for the Daubechies 2 wavelet, and in Figure 10 for the Haar wavelet. The dynamic range improvement is now smaller than in the case with noise level of -50 dB, however the improvement is still significant. In order to achieve such an improvement, it is necessary to use the wavelet decomposition level 6, which is greater level than for the noise level -50 dB.

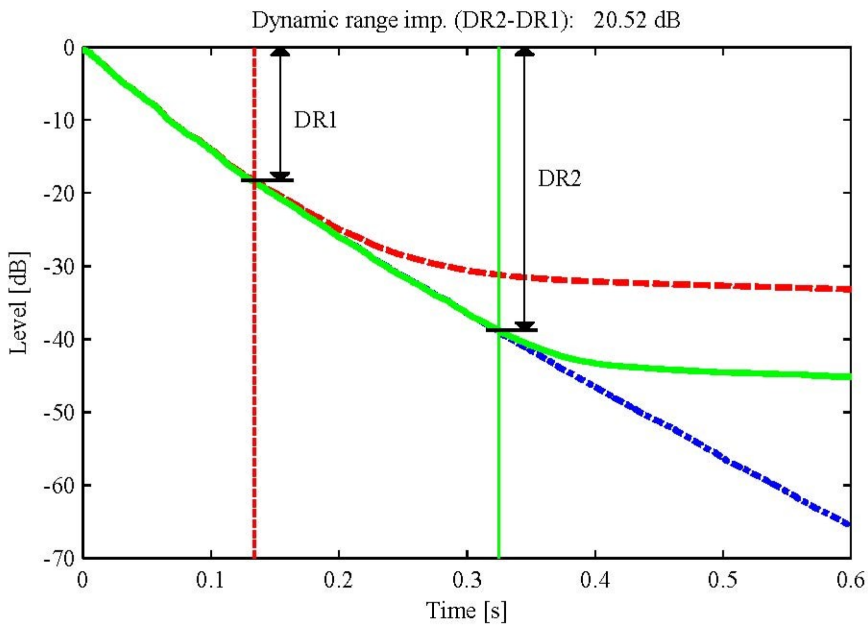

Figure 7 – IIDCs of the RIR synthesized by the ISM: without noise - reference curve (···), with noise of level of -50 dB before the wavelet (---) and after applying the Daubechies 2 wavelet at level 3 (–)

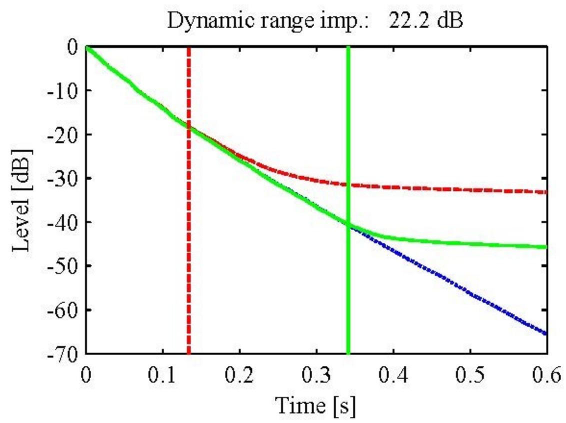

Figure 8 – IIDCs of the RIR synthesized by the ISM: without the noise (···), with noise of level of -50 dB before the wavelet (---) and after applying the Haar wavelet at level 4 (–)

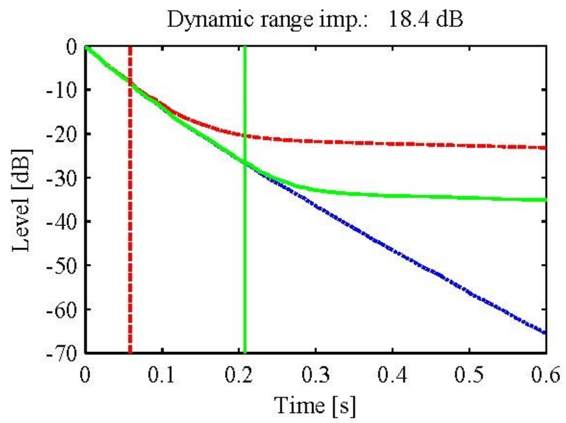

Figure 9 – IIDCs of the RIR synthesized by the ISM: without the noise (···), with noise of level of -40 dB before the wavelet (---) and after applying the Daubechies 2 wavelet at level 6(–)

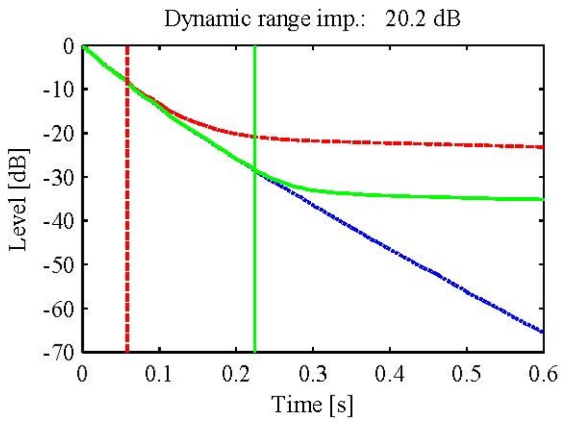

Figure 10 – IIDCs of the RIR synthesized by the ISM: without the noise (···), with noise of level of -40 dB before the wavelet (---) and after applying the Haar wavelet at level 6(–)

The effects of changing the wavelet (and its parameters) applied in the RIR de-noising on change of the dynamic range of the IIDC are summarized in Table 1 for the RIR synthesized by the ISM and noise level of -50 dB, while the results for the noise level of -40 dB are given in Table 2. The results for the RIRs with exponential decay are very similar to the results presented in Table 1 and 2.

Table 1 – Improvement of the dynamic range of the IIDC in dB after applying various wavelets on the RIR synthesized by the ISM for noise level of -50 dB

Wavelet / Level	Haar	Daubechies 2	Daubechies 8	Daubechies 32	Coiflets 3	Coiflets 5
3	18.97	20.52	21.44	21.89	20.75	20.88
4	22.20	15.56	14.80	14.79	18.38	18.30
5	14.80	14.80	14.66	14.04	15.55	15.37
Wavelet / Level	**Mayer**	**Symlet 2**	**Biorthogonal 2.2**	**Biorthogonal 2.8**	**Reverse biorthogonal 2.8**	**Reverse biorthogonal 6.8**
3	20.87	20.52	16.65	19.65	18.71	20.53
4	15.56	15.56	20.42	22.25	20.93	18.35
5	14.92	14.80	21.68	22.71	22.52	15.55

Table 2 – Improvement of the dynamic range of the IIDC in dB after applying various wavelets on the RIR synthesized by the ISM for noise level of -40 dB

Wavelet / Level	Haar	Daubechies 2	Daubechies 8	Daubechies 32	Coiflets 3	Coiflets 5
4	18.35	18	18.39	19.02	18.17	18.21
5	19.33	18.35	18.98	20.16	18.40	18.49
6	20.20	18.40	19.05	20.18	18.62	18.74
Wavelet / Level	**Mayer**	**Symlet 2**	**Biorthogonal 2.2**	**Biorthogonal 2.8**	**Reverse biorthogonal 2.8**	**Reverse biorthogonal 6.8**
4	18.46	18	12.50	12.65	12.51	17.48
5	19.06	18.35	12.77	12.77	12.67	18.13
6	19.21	18.40	14.39	16.58	12.72	18.2

As we can see from the tables, there is a certain difference among the results for different wavelets and levels. The dynamic range improvement is in the range from about 14 dB to about 22 dB for noise level of -50 dB, and from 12 dB to about 20 dB for the noise level of -40 dB. For the noise level of -50 dB, any of the used wavelets can give the dynamic range improvement above 20 dB with an adequate wavelet decomposition level. This is not completely the case for the noise level of -40 dB, where there are some wavelets that lead to dynamic range improvement close to 20 dB, but there are some wavelets (Biorthogonal and Reverse biorthogonal wavelets) that give the improvement of up to about 12 dB, 14 dB or eventually 16 dB. These two wavelets yield smaller dynamic range improvement compering with other wavelets. It should be noticed that the decomposition level is chosen to yield the greatest dynamic range improvement. It can be concluded that several wavelets can be applied in the de-noising of the RIRs leading to the similar improvement of the decay curve dynamic range. However, attention should be paid to choose adequate wavelet parameters, especially level. If the level of some wavelets is increased above the level given in Table 1 and 2, the dynamic range will remain very similar to the range for the level 5 in Table 1, that is, for the level 6 in Table 2. The dynamic range variation (for the levels above the presented ones) is typically around 0.4 dB.

It is also important to mention what will happen if other parameters are changed, not only level. In Figure 11, the impact of the Daubechies 2 wavelet at level 3 on the RIR synthesized by the ISM and noise level of -50 dB is presented. Difference in comparison to previous figures is that now other parameters are used: *sqtwolog* for universal thresholding and *soft* thresholding. As can be seen from this figure, the same wavelet decomposition level gives in this case twice as bad result as in the previous cases. The IIDC obtained after the wavelet application goes after some point beneath the reference IIDC. This phenomenon is also valid for the other wavelets with these parameters.

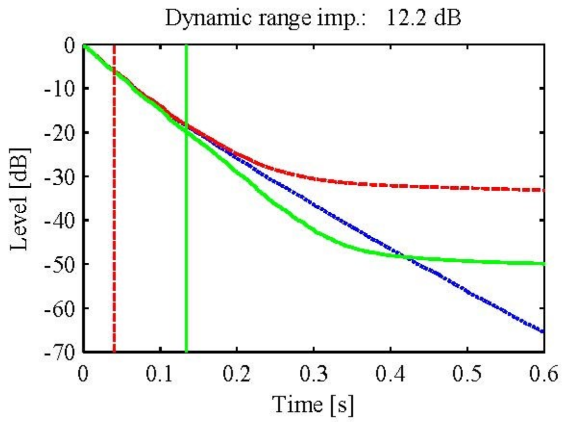

Figure 11 - IIDCs of the RIR synthesized by the ISM: without the noise (···), with noise of level of -50 dB before the wavelet (---) and after applying the Daubechies 2 wavelet with *sqtwolog* for universal thresholding, *soft* thresholding and with level 3 (–)

5. CONCLUSIONS

Wavelets have played an important role in the field of signal processing, especially in de-noising of speech, medicine signals and images. Usage of wavelets is widespread, but in the field of acoustic measurements and processing of RIRs, application of wavelets cannot be considered to be common.

Potentials of the wavelets in de-noising of the RIRs are investigated here. In order to provide insights into different aspects of wavelet application, the effects of changing the wavelet parameters are analyzed. The results show that the wavelet type and parameters have a significant influence on the de-noised RIR. Thus, even some negative effects can appear such as disturbance or loss of a part of RIR decay. Regarding the dynamic range of backward integrated decay curve, it can remain similar as before applying wavelets, but also it can be improved significantly, even more than 20 dB. With optimally chosen wavelet parameters, the dynamic range improvement is typically about (or even above) 20 dB for the RIRs with noise of level of -40 dB or lower. Increase of noise level leads to reduction of the dynamic range improvement, although the improvement can still be of a large value.

Choosing an adequate wavelet and optimal wavelet parameters seems not to be an easy task. Some conclusions what to apply can be drawn from the results presented in this paper. However, the authors will continue this research in order to study the effects of wavelet application in de-noising of RIRs using the RIRs measured in different rooms and using other combinations of wavelet parameters.

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