A structural design method for five-axis gantry machining center based on stiffness matching

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ABSTRACT

This paper proposes a conceptual design method which is used in the structure design of five-axis gantry machine center. In this method, the main idea is to match the stiffness of the main components of the machining tool. The restrictions are the static stiffness and desired value of the first order natural frequency of the machine tool and the objective function is to get the minimum mass of the machine, respectively. The sub-structure analysis method is employed to analyze the relationship between the basic sizes of the main components and the frequency of the machine tool. According to the relationship and the restriction, the basic sizes of the main components are gained as the basis for the design of the main components of five-axis gantry machining center.

Keywords: conceptual design, stiffness matching, sub-structure analysis method
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1. INTRODUCTION

As the development of the technology, the demand of the performance of the machine tools improves continuously. There are two main research directions of machine tools. One is based on the static performance about which there are many researchers being researching, such as the compensation of the deformations (1). The other one is based on the dynamic performance about which there are a few researchers being researching, such as the vibration reduction (2).

There is different static stiffness in Different structures. The suitable distribution of the stiffness of the parts which is related to the structures can result in high performance (3). So, the static stiffness matching of machine tools becomes a research aspect which belongs to static research.

As the vibration becomes a big question of machine tools, dynamic analysis gets more and more important. For example, the dynamic performance of the feed system of the machine tool has a great influence on machining accuracy (4,5). Dynamic stiffness matching is also a research aspect similar to the static one these years. LIU (6) shows the dynamic stiffness matching design for the feed system of high-speed machine tool. But he just considers about a part of machine tool while other parts are still not considered. In addition, LIU just compares two programs and chooses the better one rather than finds the best one. And the static performance of machine tool is a test item after design rather than in the design program.

In this paper, both the static and dynamic performances are considered while research the stiffness matching and the sub-structure analysis method (7-9) is employed to establish function with which the best design program can be achieved.

The design of a machine tool includes the design of the basic sizes and the details of every component. There are many researches about the details of components (10). So, the

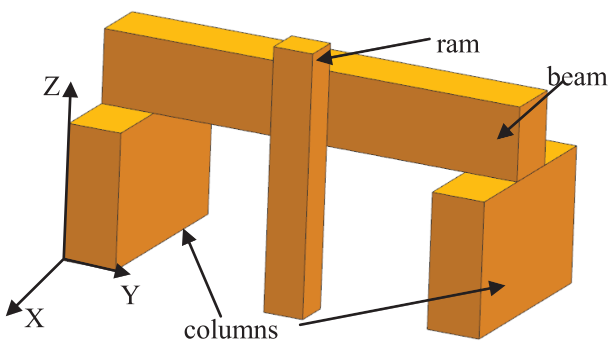

Fig.1 The model of the machining

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basic sizes of the parts of five-axis gantry machining center are as the research objects in this paper. As a conceptual design, the machining center is simplified as Fig.1.

2. THE ANALYSIS OF THE STATIC STIFFNESS OF THE FIVE-AXIS GANTRY MACHINING CENTER

Static stiffness is one of the important indexes of all the machine tools because it affects the machining accuracy and machined surface quality a lot (11). So, designers must take static stiffness into consideration while designing machine tools.

The static stiffness of the conceptual model is calculated according to the mechanics of materials in this section and the relation between the basic sizes of the main components and the static stiffness is achieved.

2.1 The static stiffness in X direction

As is shown in Fig.1, there are four main parts in the conceptual model, including a ram, a beam and two columns. According to the connection relation of the components, corresponding equivalents about the parts are carried out while the static stiffness in X direction is analyzed.

First, the columns subject to torsion, so it is equivalent to an axis model with one end clamped, as is shown in Fig.2 (a). Second, the beam subjects to bending and torsion, so it is equivalent to a beam model with two ends clamped while the bending is taken into consideration and an axis model with two ends clamped while the torsion is taken into consideration, as is shown in Fig.2 (b). Third, the ram subjects to bending, so it is equivalent to a simply supported beam model with extending outer ends, as is shown in Fig.2 (c).

According to the equivalence, the deformation of the machine tool in X direction is achieved while there is a force F_x in X direction acting on the tool of the machine. The deformation r_x can be expressed as:

$$r_x = r_{x1} + r_{x21} + r_{x22} + r_{x3} \tag{1}$$

$$r_{x1} = b_2 \bullet \frac{M_F h_1}{G_1 \beta_1 l_1 b_1^3} \tag{2}$$

$$r_{x21} = \frac{F_x l_2^3}{192 E_2 I_2} \tag{3}$$

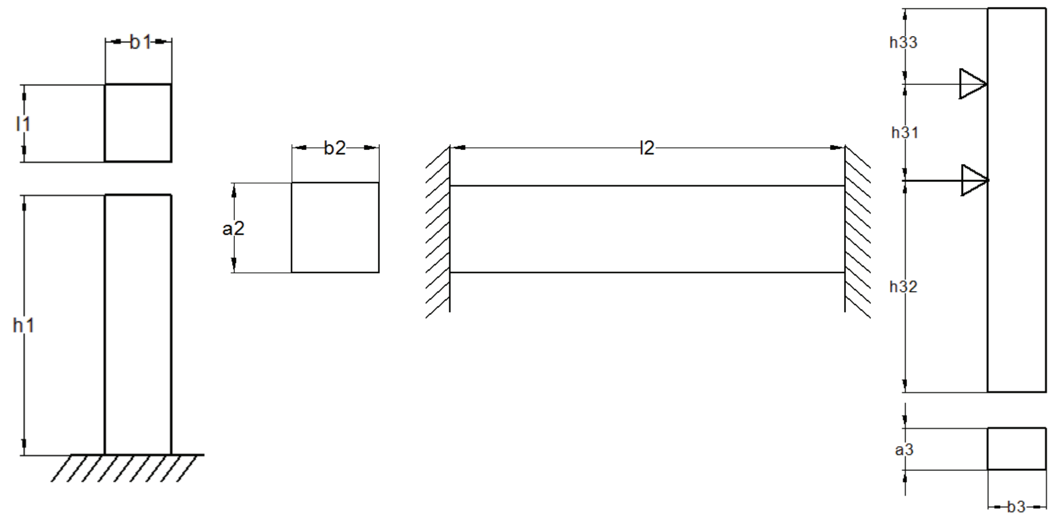
$$r_{x22} = \frac{M_2 l_2}{4 G_2 \beta_2 a_2 b_2^3} \tag{4}$$

$$r_{x3} = \frac{F_x h_{32}^2 (h_{31} + h_{32})}{3 E_3 I_3} \tag{5}$$

$$M_F = F_x l_2 / 8 \tag{6}$$

$$M_2 = \frac{a_2}{2} F_x (1 + 2\frac{h_{32}}{h_{31}}) \tag{7}$$

Where E_1,E_2 and E_3 are the Young's modulus of the column, beam and ram respectively. G_1 and G_2 are the shear modulus of the column and beam respectively. I_1,I_2 and I_3 are the cross-sectional second moment of area of the column, beam and ram respectively. β_1 and β_2 are the torsional factors of the rectangular cross section of the column and beam respectively. Other parameters are shown in Fig.2.



(a) (b) (c)

Fig.2 The equivalent models of the static stiffness in X direction.

2.2 The static stiffness in Y direction

The analysis of the static stiffness in Y direction is similar to which in X direction. Corresponding equivalents are also carried out.

First, the columns subject to bending and tension, so it is equivalent to a beam model with one end clamped while the bending is taken into consideration and a bar model with one end clamped while the tension is taken into consideration. Second, the beam subjects to bending and tension, so it is equivalent to a beam model with two ends clamped while the bending is taken into consideration and a bar model with two ends clamped while the tension is taken into consideration. Third, the ram subjects to bending, so it is equivalent to a simply supported beam model with extending outer ends.

Further details of the equivalent models and the expression of the static stiffness in Y direction are not given for reasons of space. The analysis progress of the details can refer to 2.1.

3. THE ANALYSIS OF THE FIRST ORDER NATURAL FREQUENCY OF THE FIVE-AXIS GANTRY MACHINING CENTER

As is known to all, the first few natural frequencies are one of the most important factors which influence the dynamic stiffness of the machine tool. The first order natural frequency is analyzed by sub-structure analysis method in this paper. And the method proposed in this paper can also be used to analyze other natural frequencies.

According to orthogonality, the modes in X direction and in Y direction are analyzed individually. The natural frequency of first order mode may be in X direction or in Y direction due to the different structures.

3.1 The analysis of the first order natural frequency in X direction

There are many different methods which have been proposed to analyze the modes of different structures, such as Lumped Mass Method and Finite Element Method (FEM). Lumped Mass Method is easy but only used in simple structures while FEM is complex though can be used in complex structures. The sub-structure analysis method and Lumped Mass Method are employed synthetically to analyze the modes of the machine tool in this paper.

The main idea of this method is that Lumped Mass Method is used to analyze the components which have been equivalent to simple structures while sub-structure analysis method is used to connect these components. Then, the natural frequencies of the machine tool can be achieved.

One of the important steps of this method is to simplify the components. The analysis of the first mode in X direction is shown below.

As are parts of the machine tool, the general shapes of the columns, beam and ram have been determined. It is easy to find that the first modes of them are all bending modes. So, these main parts are equivalent to beams when considering the first mode of the machine tool in X direction. Other equivalent models should be considered when these components are analyzed because the geometrical compatibility conditions of the interface of the sub-structures are needed when the sub-structure analysis method is used.

So, the ram is divided into three parts by two fulcrums, and each part is equivalent to a beam model when the first mode of the machine tool in X direction is analyzed. The beam is divided into two parts by the midpoint, and each part is equivalent to a beam model, too. The parts of the beam should also be equivalent to axis models because the twist angle of the midpoint is needed when the geometrical compatibility condition is considered. The columns are also equivalent to beam models and axis models because of the geometrical compatibility condition. These seven parts are the sub-structures which are needed to be analyzed.

There are two equivalent models in the analysis in X direction, including beam model and axis model. The stiffness and mass matrices of the beam elements are (12)

$$K=\frac{EI}{l^3}\begin{bmatrix}12 & 6l & -12 & 6l\\ 6l & 4l^2 & -6l & 2l^2\\ -12 & -6l & 12 & -6l\\ 6l & 2l^2 & -6l & 4l^2\end{bmatrix}, M=\frac{\rho Al}{420}\begin{bmatrix}156 & 22l & 54 & -13l\\ 22l & 4l^2 & 13l & -3l^2\\ 54 & 13l & 156 & -22l\\ -13l & -3l^2 & -22l & 4l^2\end{bmatrix} \tag{8}$$

Where E is Young's modulus, ρ is mass density, I is cross-sectional second moment of area, A is area, l is length of the element. The stiffness and mass matrices of the axis elements are

$$K=\frac{GI_t}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}, M=\frac{\rho Al}{24}(a^2+b^2)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{9}$$

Where G is shear modulus, ρ is mass density, I_t is polar moment of inertia of area, A is area, l is length of the element, a and b are the length and width of the cross section respectively.

It is easy to find that the first mode of the machine tool in X direction is identified by the first modes of the seven sub-structures. So, only the first modes of the sub-structures are analyzed in this paper.

Generalized coordinates of the first mode of each sub-structure contain the main constrained mode with fixed interface and fixed-interface mode. Generalized coordinates are easy to be achieved based on Eq. (8) and Eq. (9). First, the generalized coordinates of three sub-structures of the ram can be expressed as:

$$q_{32x}=\left[p_1\, f_1\;\; \theta_1\right]^T, q_{31x}=\left[f_2\;\; \theta_2\, p_2\, f_3\;\; \theta_3\right]^T, q_{33x}=\left[p_3\, f_4\;\; \theta_4\right]^T \tag{10}$$

Second, the generalized coordinates of two sub-structures of the beam can be expressed as

$$\begin{aligned} q_{21x}&=\left[f_5\;\; \theta_5\, p_4\, f_6\;\; \theta_6\right]^T, q_{22x}=\left[f_7\;\; \theta_7\, p_5\, f_8\;\; \theta_8\right]^T\\ q_{23x}&=\left[\theta_9\, p_6\;\; \theta_{10}\right]^T, q_{24x}=\left[\theta_{11}\, p_7\;\; \theta_{12}\right]^T \end{aligned} \tag{11}$$

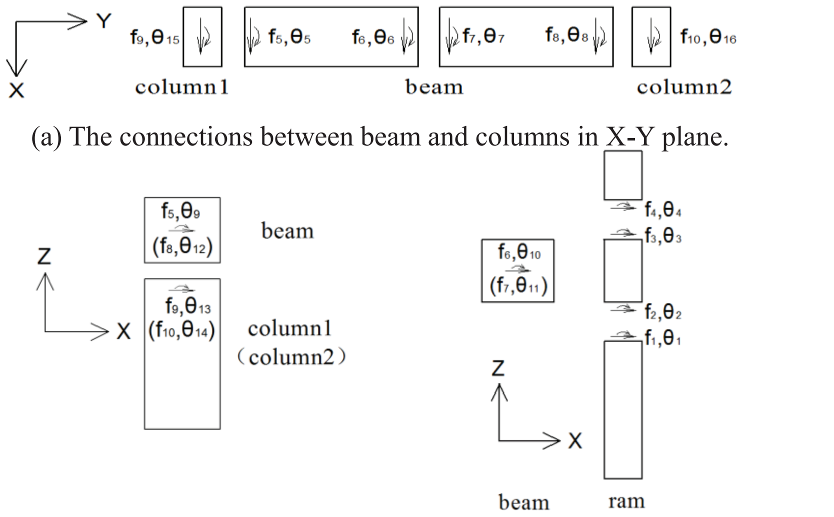
Third, the generalized coordinates of two columns can be expressed as

$$\begin{aligned} q_{11x}&=\left[p_8\, f_9\;\; \theta_{13}\right]^T, q_{12x}=\left[p_9\, f_{10}\;\; \theta_{14}\right]^T\\ q_{13x}&=\left[p_{10}\;\; \theta_{15}\right]^T, q_{14x}=\left[p_{11}\;\; \theta_{16}\right]^T \end{aligned} \tag{12}$$

Where $p_1 \ldots p_{11}$ are the first main constrained modes with fixed interface of seven sub-structures while $f_1 \ldots f_{10}$ and $\theta_1 \ldots \theta_{16}$ are the first fixed-interface modes.

Another important step of the method proposed in this paper is building connections between sub-structures. Some connections are hard to be set up and need to be simplified. According to Fig.3, the connections are expressed as:

$$\begin{aligned} & f_1 = f_2, f_3 = f_4, f_6 = f_7 = \frac{f_1 + f_3}{2} \\ & \theta_1 = \theta_2, \theta_3 = -\theta_4, \theta_{10} = \theta_{11} = \frac{\theta_1 + \theta_3}{2} \\ & f_5 = f_9, f_8 = f_{10}, \theta_9 = \theta_{13}, \theta_{12} = \theta_{14} \\ & \theta_6 = \theta_7, \theta_5 = \theta_{15}, \theta_8 = \theta_{16} \end{aligned} \tag{13}$$



(a) The connections between beam and columns in X-Y plane.

(b) The connections between beam and columns in X-Z plane.

(c) The connections between beam and ram in X-Z plane.

Fig.3 The connections between components of the machine tool.

3.2 The analysis of the first order natural frequency in Y direction

The analysis of the first order natural frequency in Y direction is similar to the analysis in X direction. The main equivalent models of the columns, beam and ram are all beam model. In addition, the parts of the beam and the columns should be equivalent to bar model because of the geometrical compatibility condition.

The stiffness and mass matrices of the bar elements are

$$K = \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, M = \frac{\rho Al}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{14}$$

Where E is Young's modulus, ρ is mass density, A is area, l is length of the element.

The generalized coordinates of the sub-structures of the components can also be achieved according to Eq. (8) and Eq. (14), such as $q_{32y}, q_{31y}, q_{33y}, q_{21y}, q_{22y}, q_{23y}, q_{24y}, q_{11y}, q_{12y}, q_{13y}$ and q_{14y}. As is similar to the analysis in X direction, the expressions and the connections are not given in this paper.

3.3 The solution of the first order natural frequency of the five-axis gantry machining center

As different basic sizes, the first order mode of the machine tool may be in X direction or in Y direction. It depends on the values of the first frequencies of the mode in X direction and in Y direction.

The first order natural frequency in X direction can be calculated according to 3.1. The modal stiffness matrices $K_{1x},\ldots K_{7x}$ and modal mass matrices $M_{1x},\ldots M_{7x}$ of the components of the machine tool can be achieved based on Eq. (8), Eq. (9) and the practical constraints. So, the modal stiffness and modal mass matrices of the machine tool without connections between the components are expressed respectively as:

$$\overline{K} = diag(K_{1x}, \quad \cdots, \quad K_{7x}) \tag{15}$$

$$\overline{M} = diag(M_{1x}, \quad \cdots, \quad M_{7x}) \tag{16}$$

The independent transformation matrix S_x is introduced to connect the components. S_x satisfies the following equation.

$$q_x = [S_x]\overline{q_x} \tag{17}$$

Where

$$\begin{aligned} q_x &= \begin{bmatrix} q_{32x}{}^T & q_{31x}{}^T & \cdots & q_{14x}{}^T \end{bmatrix}^T, \\ \overline{q_x} &= \begin{bmatrix} p_{1x} & \cdots & p_{11x} & f_1 & \theta_1 & f_3 & \theta_3 & f_5 & \theta_5 & \theta_6 & f_8 & \theta_8 & \theta_9 & \theta_{12} \end{bmatrix}^T \end{aligned} \tag{18}$$

So, according to the connections between sub-structures in Eq. (13), S_x satisfies that:

$$\begin{aligned} & S_{1,1} = S_{2,12} = S_{3,13} = S_{4,12} = S_{5,13} = S_{6,2} = S_{7,14} = S_{8,15} = S_{9,3} = S_{10,14} = S_{12,16} = S_{13,17} \\ & = S_{14,4} = S_{16,18} = S_{18,18} = S_{19,5} = S_{20,19} = S_{21,20} = S_{22,21} = S_{23,6} = S_{26,7} = S_{27,22} = S_{28,8} \\ & = S_{29,16} = S_{30,21} = S_{31,9} = S_{32,19} = S_{33,22} = S_{34,10} = S_{35,17} = S_{36,11} = S_{37,20} = 1 \\ & S_{11,15} = -1 \\ & S_{15,12} = S_{15,14} = S_{17,12} = S_{17,14} = S_{24,13} = S_{24,15} = S_{25,13} = S_{25,15} = \frac{1}{2} \end{aligned} \tag{19}$$

Then, the modal stiffness and modal mass matrices of the machine tool can be achieved respectively as:

$$K = \left[S_x\right]^T \overline{K} \left[S_x\right] \tag{20}$$

$$M = \left[S_x\right]^T \overline{M} \left[S_x\right] \tag{21}$$

The first order natural frequency w_{x1} in X direction can be calculated based on Eq. (20) and Eq. (21).The first order natural frequency w_{y1} in Y direction can also be calculated according to 3.2. So, the first order natural frequency w_1 of the machine tool is the smaller value between w_{x1} and w_{y1}.

4. THE STIFFNESS MATCHING OF THE FIVE-AXIS GANTRY MACHINING CENTER

Suitable structure design is important to the performance and economic efficiency of the machine tool. Unreasonable structure design may result in inconformity of the demands of the static and dynamic stiffness. Moreover, some designs let the machine tool cumbersome though the machine tool satisfies the demands.

An understanding to stiffness matching is proposed in this paper. The static and dynamic stiffness of the machine tool which just need to satisfy the demand do not need to be too high as the purpose to achieve the high economic efficiency. So, the static and dynamic stiffness are set as the restrictions. The lighter the machine tool is, the more economic efficiency the machine tool owns. So, the minimum mass of the machine is set to be the objective function. Then, the design function based on the stiffness matching can be expressed as:

$$\begin{aligned} &\min f\left(x_1, x_2, ..., x_n\right) \\ &s.t. \quad w_1 \geq w_{demand} \\ &\qquad r_x \leq r_{xdemand} \\ &\qquad r_y \leq r_{ydemand} \end{aligned} \tag{22}$$

Where function $f(x_1, x_2,\ldots, x_n)$ is the mass function of the machine tool while $x_1, x_2,\ldots, x_n$ are the variables of the basic sizes of the components which are need to be designed, w_{demand}, $r_{xdemand}$ and $r_{ydemand}$ are the demands of the first order natural frequency, static stiffness in X and Y directions of the machine tool respectively. There are also restrictions in basic sizes of the components sometimes according to the actual situation.

5. DESIGN EXAMPLE

Consider a model like Fig.1 which is consisted of four components. Three main sizes including the thickness b_1 of the columns, the length a_3 and width b_3 of the cross section of the ram are chosen to be the variables while others are set to be constants. The constants in this model are shown in Table 1.

Table 1-The constants in the model

Column		beam		ram	
ρ_1/(kg/m^3)	7900	ρ_2/(kg/m^3)	7900	ρ_3/(kg/m^3)	7900
b_1/(m)	4	a_2/(m)	0.82	h_{32}/(m)	1.78
h_1/(m)	2.172	b_2/(m)	0.65	h_{31}/(m)	0.82
E_1/(Pa)	1.2e11	l_2/(m)	4.8	h_{33}/(m)	0.305
μ_1	0.3	E_2/(Pa)	2.1e11	E_3/(Pa)	1.2e11
		μ_2	0.3		

According to the method proposed in this paper, the suitable structure design of the machine tool can be achieved while the restrictions are given. The restrictions in this example are given as:

$$w_1 \geq 359 rad/s\,, r_x \leq 1.2e-6m\,, r_y \leq 1.2e-6m \tag{23}$$

Then, the variables including b_1, a_3 and b_3 can be determined according to Eq. (22) and are shown as:

$$b_1 = 0.52m, a_3 = 0.40m, b_3 = 0.41m \tag{24}$$

6. CONCLUSION

In this paper a design method based on stiffness matching is proposed to achieve a suitable structure design of the five-axis gantry machining center. The static and dynamic stiffness of the machine tool are both concerned as restrictions. Moreover, the sub-structure analysis method is employed in the paper while the dynamic stiffness is analyzed. The minimum mass of the machine tool is set as the objection of the structure design. Then, the design function based on the stiffness matching is established as Eq. (22). A design example in this paper describes the analysis process clearly. It should be noted that the method proposed in this paper is a conceptual design method of the machine tool. There is guidance in this structure design and the model of the design needs further refinement before practical application.

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