ABSTRACT

An existing 2.5D FEM - BEM model is extended to predict sound pressure levels inside buildings. Previous studies have showed that the 2D model is insufficient. A comparison of 2.5D and 3D computations has been done in the case of a simple two-storey building parallel to the tracks. The 2.5D model can be improved to provide reliable results. The mixing of 2D $\frac{1}{2}$ and 3D models has been named 2D$\frac{3}{4}$. Two different ways to compute sound radiation have been programmed: an SEA-like radiation or using a 3D modal approach. The model is first validated against 3D FEM without ground and the validity of this simplified approach is discussed.

Keywords: BEM, FEM, 2.5D

1. INTRODUCTION

Annoyance created by the passage of trains is a subject that has become particularly relevant in large cities where both underground and surface rail transportations are constantly implemented. In France, for instance, tramways have been installed or re-installed in the largest cities (Grenoble, Lyon, Nantes to name a few). Unused train tracks are also sometimes been put back into service. Also, with the high value of land, buildings are now being constructed very close to railways despite the extra-cost incumbent to the need for protection against vibrations and re-radiated noise. Last, underground projects are also in full expansion with for instance in France, the “Grand Paris” project where many tunnels will be constructed. As a consequence, developers and city planners can’t avoid the ‘vibration-noise’ problem and there is a need for numerical tools to consider such issues well ahead of new projects, but also in rehabilitation processes.

Several aspects must be considered. First, is the problem mostly 3D or can 2.5D approaches be used? Then, can straightforward FEM codes be employed or should one prefer coupled FEM/BEM approaches? What approach should be employed to predict noise levels inside buildings?

The problem of train induced noise levels inside buildings is not new but the means employed for its study have evolved with time thanks to the increase of computing resources. In the 1990s, SEA based approaches have been employed, despite the low frequencies of the problem (between 10 and 300 Hz) (1). Later, the coupling between a 2.5D BEM model for the ground-foundations with a full 3D FEM model for a building by means of a mobility approach inspired from soil-machinery coupling has been proposed (2-3).

In this work, FEM/BEM code named Mefissto (4-6) has been employed and further extended to include noise radiation. Several past studies have showed (by comparing 2D, 2.5D and 3D computations) that, first, 2D computations miss important physical phenomena leading to erroneous results while 2.5D approaches are particularly well suited to the ground/structure problems mainly because of large dissipation in soils (5).

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The problem of underground tunnel excitation is presented in two companion papers at ICSV2016 (7,8) where the precision of Mefissto results against measurements in existing tunnels with train traffic is shown to be very satisfactory, later allowing train/track models to be precisely tuned in order to forecast vibration transfer from tunnel up to buildings in future projects.

In the present paper, the question of noise prediction inside buildings is tackled with an original hybrid model in the case of surface excitation. A combination of 2.5D results for the vibration of walls is combined with 3D computations in rooms either with an SEA-like approximation or with a full modal approach. The modal-based approach has been called 2D 3/4, and has been assessed by comparing results against fully coupled 3D FEM computations. Next, the influence of the “missing” walls in the 2.5D computations is evaluated against reference 3D FEM computations.

2. AN HYBRID 2D 3/4 APPROACH

The Mefissto software is a 2D, 2.5D and 3D FEM/BEM model (3-8) which allows the user to divide an existing problem into any number of sub-domains and to apply either the FEM or the BEM approach in each-subdomain. FEM approaches are usually preferred for structures (tracks, tunnels, buildings, barriers,..) while the BEM offers the advantage of economically dealing with not fully bounded sub-domains, typically soils but also massive structures where the BEM can become faster than FEM (4). Part of the ground with inhomogeneous properties can be modelled with FEM. Using BEM for buildings has been found slow to converge. Soil interlayers, can in principle either be modelled as BEM boundaries (actually as boundaries of varying length according to frequency) or introduced into a complex Green function. The latter has only been tested in 2D with the conclusion that it is preferable to actually mesh the free surface and interlayers both for cost reasons and also because meshing these boundary avoids the restriction to canonical situations. Note that, in 3D, computation times argue in favour of the use of complex Green functions. It should be noted that Mefissto is a self-contained software where meshings are done automatically at each new frequency.

The 2D reference plane is the xy plane and the third infinite direction is chosen as z. The 2.5D computation is based on a series of computations in the xy plane for different kz wave numbers (9,3). In each case what appears as a point force in the xy plane is a line force which varies as \( e^{-ikz} \) along z. Each 2D node has 3 components for displacement and/or tractions at each node. The 2.5D solution is obtained by a Fourier-like integration.

Multi-tractions problems which occur at geometry corners or when more than two media meet (i.e. more unknowns than equations due to the separation of tractions) are automatically taken dealt with using the “fictitious node” approach proposed in 2D in (10) and successfully extended in 3D in (6). After the kz integration is done one has access to the displacement (or velocity) vector at any point in the 3D domain keeping in mind that although the solution is 3D for the excitation (point forces or any combination of point forces) the geometry is actually a 3D extrusion (therefore infinite along z) of the original 2D geometry.

Consequently, the 2.5D geometry does not contain any transverse xy partition. Each 2D “volume” is in fact, in 2.5D, an infinite duct (Figure 1). In order to compute sound pressure levels in a finite volume, we define fictitious partitions say between \( z_1 \) and \( z_2 \) and compute the normal velocity \( V \) on the four zx and yz finite surfaces, thus defined, on regular grids. These velocity fields are then used to compute the sound pressure at any point in the volume. Absorption in each volume is entered as a reverberation time \( Tr \). Two different computation models have been modelled.

2.1 Approach 1: SEA-like energy approach

With this approach, the sound radiated by each of the four surfaces is simply computed using an SEA-like approach by means of a sound radiation coefficient \( \sigma \) leading to the sound pressure \( P \) computed as:

\[
W_i = \rho_c S_i \bar{V}_i^2 \quad \frac{p^2 A}{4\rho c} = \sum_{i=1}^{4} W_i \quad \quad A = S \cdot \alpha = \frac{60 \cdot \lambda}{1.08 \cdot 6\cdot c \cdot \alpha \cdot Tr} \tag{1}
\]

where \( A \) is total absorption area, \( \lambda \) the total volume, \( \alpha \) the absorption coefficient; \( \sigma \) the radiation factor is computed by means of analytic expressions such as Maidanick’s (11) or the Ver (12) formulas or can be input from separate computations.
2.2 Approach 2: modal volume Green function (2D3/4)

The sound pressure radiated by each surface is approximated as (13):

\[ P(M) = -j \rho \omega \int_{S} V(Q). G_{V}(M, Q). dS(Q) \]  \hspace{1cm} (2)

Where \( G_{V} \) is the response (Green function) of the volume for non-vibrating boundaries but including absorption deduced from the value of \( Tr \). \( G_{V} \) is computed using the standard modal approach with cosine modes (13). This approach assumes the absence of feedback from the volume onto its boundary velocity which is a reasonable approximation for large volumes such as rooms. This is verified in the next section.

Figure 1 – Infinite “duct volume” and definition of a fictitious 3D volume \( \Omega \) between \( z_1 \) and \( z_2 \).

The mean sound pressure level can be computed in any sub-volume of \( \Omega \) by selecting a random number of positions. Usually 10 receivers are sufficient.

3. VALIDATION & NUMERICAL RESULTS

Several aspects of the 2D3/4 approach must be looked at: (i) the one way coupling between wall vibrations and volume acoustics (ii) the validity of approaching a building of finite length (Lz) by a 2.5D geometry of infinite extend (iii) the influence of the missing transverse xy walls, as well as the reduced number of structural junctions compared to a full 3D situations where each volume is delimited by 6 surfaces. We also want to assess the influence of the number of applied forces along the z axis.

3.1 Validation of the decoupled approach

In order to validate the decoupled approach we first consider the case of a single 4x4x4 m\(^3\) 6-walled volume excited by a normal point force in the middle of one wall. The 8 corners are blocked. Each concrete wall is 20 cm thick. Two 3D-FEM computations have been done, either with or without acoustic elements (\( \eta=0.01 \)). In the second case the normal velocity on each surface is used to compute the sound pressure level by means of the modal Green function (eq. 2) (\( Tr=1 \) sec and 10 receivers). The comparison showed in Figure 2 is very satisfactory. Differences at certain picks can probably be attributed to the different models for room absorption. One should note that convergence proved to be faster using the hybrid FEM+modal approach than the full FEM model for walls+volumes.
Using a third octave representation, Figure 3 shows the convergence of rms pressure level in the volume with the number of random receiver positions considered (10, 20 or 30). Ten receivers is sufficient.

We call 2.5D a building which is infinite along z.

We call duct-building, a 3D geometry without the transverse xy walls, having a finite length of Nz*4m (z=-2 Nz to +2.Nz).

The finite 3D building is obtained by adding transverse walls thus delimiting real 3D volumes. In the 2.5D and 3D-duct geometries, the volumes have the same z positions as for the corresponding 3D cases. Figure 4 illustrates these points. Figure 4a,b,c show respectively the generic 2D geometry, the 3D-duct case with Nz=3, and the corresponding 3D geometry with 18 cm transverse walls. Figure 3d represents the 3D case when Nz=11. The building rests on a 20 cm thick slab with a loss factor $\eta_d$. In order to have a 3D (FEM) reference solution of moderate cost with replace the ground by using a value $\eta_d=0.50$. In the rest of structure $\eta=0.05$. Unless otherwise stated, walls have a thickness of 18 cm and the floors are 20 cm-thick. The reference 3D FEM code employed is Nastran (structural and acoustic 3D elements). A vertical Force is placed at the median z position 1 m away from the building, at $y=0.2$, $z=0$. 

![Figure 3 – Convergence with the number of random receiver positions (1/3 octave)](image-url)
Figure 4 a,b,c – 2D, 3D-duct and 3D Geometries (Nz=3)

Figure 4d- 3D FEM geometry (Nz=11)
3.2 Effect of Lz - comparison between MEFISSTO and 3D FEM

Mefissto provides a 2.5D solution which corresponds to infinitely long structures (Lz=∞). In order to evaluate the influence of the building’s length Lz, we have carried a set of 3D FEM computations of varying lengths (Nz*4m, N=3,5,7,9,11). Figure 5 shows the pressure levels in the 4 median volumes (same z position as the force). We observe that even with Nz=3 the comparison of sound pressure levels obtained in 2.5D (Lz=∞) and in 3D (Lz=12 m) is very satisfactory. As could be expected, the agreement improves as Nz increases.

![Figure 5 - Comparison of 2.5D (Lz=∞) and 3D (Lz finite) results.]

3.3 Sound Pressure levels : influence of the missing transverse xy walls

The idea is to compare 2.5D computations with the reference situation (3D-FEM). Different xy partitions are successively considered: (A) no xy partitions (duct situation), (B) 18 cm-concrete, (C) 18 cm-plaster, (D) 10 cm-plaster. Pressure levels are plotted in third-octave bands. The 2.5D results (Figure 6), compare well with the full 3D FEM results particularly for plaster partitions. Although not exact it shows that the present 2D 3/4 model provides a reasonable agreement with real 3D solutions assuming that two walls out of four are lightweight.
3.4 Influence of the number of averaging Comparison with the SEA-like approach

We want to compare the two proposed radiation models (energy or modal). We do so using the 3D FEM velocities. For the energy computations we use the radiation loss factor \( \sigma \) proposed by Maidanik. The pressure levels (Figure 7) obtained using the energy approach are significantly higher than the values obtained using the modal Green functions. One should keep in mind that the energy approach does not take into account phase relations between the 4 radiating surfaces. One might argue that with a more precise \( \sigma \) (obtained from separated pre-computations for instance) a better agreement might be obtained.

3.5 Influence of the excitation

In order to approximate a train excitation, here approximated as an incoherent finite line source, we replace the single vertical force placed at \((-1, 0.2, 0)\) by several forces along the \( z \) axis with a spacing of 4 m i.e. at \((-1, 0.2, i\times4)\). We have considered two finite buildings: \( Nz=7 \) (\( Lz=28 \) m, NF=7 forces) and \( Nz=11 \) (\( Lz=44 \) m, NF=11 forces). The corresponding 2D3/4 computations are done with the same number of uncorrelated forces. The sound pressure level is plotted in central volumes \((1,0,1)\) and \((2,0,2)\). So, we get 4 results in each volume (Figure 8). The four results are very close which tells that the 2D3/4 approach also provides reliable results for the present excitation closer to the train excitation. The differences between 7 and 11 forces are small; this is to be attributed to the high attenuations in the ground so that distant force positions...
contribute little to the overall sound pressure levels.

3.6 Computations with a semi-infinite Ground

We now consider the presence of a ground with the 2D\(\frac{3}{4}\) approach. The only difference with the previous situation is that ground losses rather than being introduced by means of an equivalent loss factor, are now properly taken care of through coupling with a semi-infinite soil. The soil properties are: \(E=269\, MPa, \rho=1550\, kg/m^3, \eta=0.10, \nu=0.257\). We remind that the previous computations without soil were done in order to a simple and fast reference situation for validation purposes.

The first question considered is the validity of the previous case where the soil was replaced by an increased loss factor in the supporting slab. Figure 9 reports the third-octave pressure levels obtained with both approaches (a slab with \(\eta=0.3, 0.5, 2\)) and the semi-infinite soil (the slab is now \(\eta=0.05\) as in the rest of structure). A fairly good agreement with the full computation is observed using \(\eta=2\) for the bottom slab. This indicates that when the problem considered does not involve propagation within the ground (therefore excluding transfer from tunnels to buildings for instance), the presence of the ground might be replaced by an equivalent loss factor.
3.7 Comparison with the RIVAS engineering method

The Rivas project (14) aimed dedicated to the reduction of railway induced noise levels and has proposed simple formula to estimate noise pressure levels in buildings by adding two correction spectra to surface vibration levels (LVa): a first correction to compute foundation velocity levels (LVb), a second one to estimate floor velocity levels (LVc) and simple correction terms to compute sound pressure levels LP. The correction spectra are provided for surface excitation, for 3 classes of soil (average, soft and hard), 3 types of buildings (houses, low and high buildings). The prediction of LVc and LP are provided for concrete and wooden floors and volumes are supposed to be the size encountered in lodgings. Note that these correction terms have been obtained as mean terms obtained from a large set of measured levels in many real situations.

We have applied this simplified approach to a slightly modified building (with two vertical foundations) both for a surface excitation (vertical force 5 m away from the building) and for an underground excitation (vertical force at a depth of 10 m, below the building). Figures 10a and 10b report, for both excitations, the comparison of the SPL computed with the sea-like, the 2D$^{3/4}$ approach and the RIVAS approach. The LP results obtained from Mefissto have been averaged over the four volumes. The energy approach either uses the Maidanick or the Ver radiation factor. The use of the Ver’s formula for $\sigma$ leads to pressure levels closer to the 2D$^{3/4}$ results. It seems that the simplified approaches (energy of RIVAS) must be used with caution or as an upper bound to actual pressure levels. The Rivas results appear to be closer to the simplified energy approach therefore in excess compared to the more precise 2D$^{3/4}$ computations. This goes in the direction of safety as far as engineering predictions are concerned. Finally, note that the results obtained with the RIVAS corrections overestimate the SPL at the highest frequencies. Computations without the two vertical foundations showed little differences. However it seems that, for the surface excitation, if
The proposed 2D 3/4 model provides velocity levels in the structure as well as sound pressure levels at chosen positions in the fictitious volumes. We may therefore visualize both vertical velocity and pressure at given frequencies (15). Figures 11 shows, at 200 Hz such a representation (displayed without the roof) in the previous case (with a surface excitation).

Velocity levels are showed on the outer surfaces and the pressure levels are represented in two planes in each volume. Fictitious walls are symbolized with a thin grid. No information is given on the inner side of the walls (green surfaces).

Figure 12 shows the vertical velocity at 100 Hz in a more complex situation with two tunnels and a point force applied in the right tunnel.

3.8 3D display

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Figure 11. 200 Hz excitation by a surface point force. LP and LV.

Figure 12. 100 Hz. Two tunnels and 2 point forces in right tunnel. LP and LV.
4. CONCLUSION

The 2.5D FEM/BEM approach has been coupled to a 3D modal calculation of Green functions in order to predict sound pressure levels in fictitious volumes. The validity of the proposed model has been assessed against full 3D solutions where the error due to the missing transverse xy partitions has been found to be acceptable for lightweight missing walls. Using a more simple energy radiation is much less satisfactory and a more precise radiation loss factor than the formula proposed by Maidanick or Ver should be employed. The RIVAS engineering approach also provides upper bound pressure levels. However above the plate critical frequencies the energy simplification is worth using. First results also indicate that in certain cases, where transfer through the ground does not come into play, the ground might be replaced by a properly calibrated global loss factor applied to the lower portion of the structure.

REFERENCES

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