Reconstruction of transient vibration and sound radiation of an impacted plate

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ABSTRACT

This paper presents an experimental application of the interpolated time domain equivalent source method based transient near-field acoustic holography for reconstructing the transient pressure field radiated by an impacted plate and the normal acceleration of the plate. In this method, the pressure field radiated by the plate is first modeled by a set of equivalent sources positioned behind the plate, and then the equivalent source strengths at each time step are solved by an iterative solving process and are taken as the input to reconstruct the whole pressure field and the normal acceleration of the plate. An experiment of a clamped rectangular steel plate impacted by a steel ball is presented. The experimental results demonstrate that the proposed method is a powerful tool to visualize the transient vibration and sound radiation of an impacted plate in both the time and space domains.

Keywords: Time domain equivalent source method; Transient near-field acoustic holography; impacted plate

I-INCE Classification of Subjects Number(s): 41.3

1. INTRODUCTION

The transient noise radiated by impacted structures, e.g. riveting and hammering, not only disturbs the living of residents around, but also brings hearing impairment to workers, and therefore it has great engineering significance to study this type of noise. Since the plate is a simple structure encountered in industry, a lot of work has focused on the transient sound radiation from an impacted plate (1-3). Usually, accelerometers attached to the plate are used to measure the normal acceleration of the plate, giving the boundary condition for further analyzing the transient sound radiation. However, the added mass of the accelerometers changes the modes of vibration and sound radiation of the plate to a certain extent, especially for a thin plate. As a non-contact measurement technique, nearfield acoustic holography (NAH) (4, 5) can avoid this change by measuring the acoustic quantities radiated by the plate and reconstructing the whole sound field indirectly. Especially, the time domain NAH is a very useful tool for studying transient vibration and sound radiation of an impacted plate.

Blais et al. (6, 7) introduced numerical Laplace transform to replace Fourier transform in time domain holography (TDH) (8) for investigating the forward and backward projections of the transient sound radiation from an impacted plate by measuring the near-field pressure. They use an experiment with an impacted, free Plexiglas plate to prove that Laplace transform based TDH can improve the precision of recovering transient pressure, acceleration and velocity signals. Other time domain methods also have the capacity to reconstruct the transient sound radiation from an impacted plate, such as real-time nearfield acoustic holography (9), time domain plane wave superposition method (10), transient Helmholtz equation least squares method (11), time domain near-field equivalence source imaging method (12) and time domain boundary element method (13).

Recently, the interpolated time domain equivalent source method (TD-ESM) (14) based transient NAH is presented to reconstruct the transient sound field radiated by an arbitrarily-shaped body. It

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carries out the reconstruction directly in both the time and space domains, and no Fourier and Laplace transforms are needed in this method. However, the method is only examined by a simulation case with the reconstruction of transient pressure field. In this paper, the method is further developed not only to reconstruct the transient pressure field radiated by an impacted plate, but also to reconstruct the normal acceleration of the plate for acquiring an overall understanding of its vibration and sound radiation modes, and also it will be verified experimentally.

2. OUTLINE OF THE PROPOSED METHOD

Figure 1 – (a) Plate in the Cartesian coordinate system \( o(x, y, z) \) and (b) geometric description of the plate plane \( S \), the hologram plane \( H \) and the equivalent source plane \( E \).

As shown in Fig. 1(a), a plate with the length of \( L_x \), the width of \( L_y \) and the thickness of \( h \) is placed in the Cartesian coordinate system \( o(x, y, z) \). A steel ball impacts the plate at the point \( (x_0, y_0) \) radiating a transient sound field.

Figure 1(b) shows the geometry of the plate plane \( S \), the hologram plane \( H \) and the equivalent source plane \( E \). There are \( M \) points distributed on the plane \( S \), \( L \) measurement points distributed on the plane \( H \) and \( K \) equivalent sources placed on the plane \( E \). According to the TD-ESM, the pressure \( p_{hi}(t) \) at the \( h \)th point on the plane \( H \) and the normal acceleration \( a_{Sm}(t) \) at the \( m \)th point on the plane \( S \) at any time \( t \) can be expressed, respectively, as (15)

\[
p_{hi}(t) = \sum_{k=1}^{K} q_k(t) \ast \frac{1}{R_{Hlk}} \delta(t - R_{Hlk} / c) .
\]

\[
a_{Sm}(t) = \frac{\partial v_{Sm}(t)}{\partial t} = \sum_{k=1}^{K} \left\{ \frac{1}{\rho} \frac{\partial R_{Smk}}{\partial n} \left[ \frac{1}{R_{Smk}} q_k(t - R_{Smk} / c) + \frac{1}{c} \frac{\partial q_k(t - R_{Smk} / c)}{\partial t} \right] \right\} ,
\]

where the asterisk denotes the convolution of two time functions, \( c \) is the sound velocity, \( \rho \) is the medium density, \( n \) is the unit normal vector of the plane \( S \), \( \delta(t) \) is the Dirac Delta function, \( H(t) \) is the Heaviside function, \( q_k(t) \) is the \( k \)th equivalent source strength at the time \( t \), \( R_{Hlk} \) is the distance between the \( h \)th measurement point on the plane \( H \) and the \( k \)th equivalent source, and \( R_{Smk} \) is the distance between the \( m \)th point on the plane \( S \) and the \( k \)th equivalent source.

By setting the retarded time \( \tau_{Hlk} = t - R_{Hlk} / c \) and \( \tau_{Smk} = t - R_{Smk} / c \), Eqs. (1) and (2) can be rewritten, respectively, as
\[ p_{hi}(t) = \sum_{k=1}^{K} \left[ \frac{1}{R_{Hik}} q_k(\tau_{Hik}) \right], \]  

(3)

\[ a_{Sm}(t) = \frac{\partial v_{Sm}(t)}{\partial t} = \sum_{k=1}^{K} \left[ \frac{1}{\rho} \frac{\partial R_{Smk}}{\partial n} \frac{1}{R_{Smk}} \left( \frac{1}{R_{Smk}} q_k(\tau_{Smk}) + \frac{1}{c} \frac{\partial q_k(\tau_{Smk})}{\partial \tau_{Smk}} \right) \right]. \]  

(4)

The time \( t \) is discretized by

\[ t_i = t_0 + i\Delta t, \]  

(5)

where \( i = 1, 2, \ldots, I \), \( I \) is the total number of time steps, \( \Delta t \) is the time step, and \( t_0 \) is the initial time. At the time \( t_i \), the retarded time are \( \tau_{Hik}^i = t_i - R_{Hik}/c \) and \( \tau_{Smk}^i = t_i - R_{Smk}/c \), respectively.

Here, a new time coordinate \( \tau \) is defined, in which both \( \tau_{Hik}^i \) and \( \tau_{Smk}^i \) are located. At the retarded time \( \tau \), the \( k \)th equivalent source strength \( q_k(\tau) \) is interpolated as

\[ q_k(\tau) = \sum_{j=1}^{I} \Phi^j(\tau) q_k(\tau^j), \]  

(6)

where \( q_k(\tau^j) \) is the \( k \)th equivalent source strength at the time \( \tau^j (j = 1, 2, \ldots, I) \), and \( \Phi^j(\tau) \) is the Lagrange interpolation function, which is given by

\[ \Phi^j(\tau) = \begin{cases} \frac{1}{\Delta t} (\tau - \tau^{j-1}) , & \tau^{j-1} \leq \tau \leq \tau^j; \\ \frac{1}{\Delta t} (\tau^{j+1} - \tau) , & \tau^j \leq \tau \leq \tau^{j+1}; \\ 0 , & \text{otherwise.} \end{cases} \]  

(7)

The derivatives of the equivalent source strength and the Lagrange interpolation function with respect to the time \( \tau \) can be expressed, respectively, by

\[ \frac{\partial q_k(\tau)}{\partial \tau} = \sum_{j=1}^{I} \frac{\partial \Phi^j(\tau)}{\partial \tau} q_k(\tau^j), \]  

(8)

\[ \frac{\partial \Phi^j(\tau)}{\partial \tau} = \begin{cases} \frac{1}{\Delta t} , & \tau^{j-1} \leq \tau \leq \tau^j; \\ -\frac{1}{\Delta t} , & \tau^j < \tau \leq \tau^{j+1}; \\ 0 , & \text{otherwise.} \end{cases} \]  

(9)

The substitution of Eq. (6) into Eq. (3), yields

\[ p_{hi}(t) = \sum_{k=1}^{K} \sum_{j=1}^{I} \frac{1}{R_{Hik}} \Phi^j(\tau_{Hik}) q_k(\tau^j). \]  

(10)

Equation (10) is the interpolated formulation of the pressure \( p_{hi}(t) \). Similarly, the interpolated formulation of the normal acceleration \( a_{Sm}(t) \) can be expressed by

\[ \]
\[ a_{Sm}(t) = \sum_{k=1}^{K} \sum_{j=1}^{j} g^j(\tau_{Smk})q_k(\tau^j). \]  

(11)

where

\[ g^j(\tau_{Smk}) = \frac{1}{\rho} \frac{\partial R_{Smk}}{\partial n} \left[ \frac{1}{R_{Smk}} \Phi^j(\tau_{Smk}) + \frac{1}{c} \frac{\partial \Phi^j(\tau_{Smk})}{\partial \tau_{Smk}} \right]. \]

(12)

According to Eqs. (10) and (11), the pressures at all \( L \) measurement points on the plane \( H \) and the normal accelerations at all \( M \) reconstruction points on the plane \( S \) at the time \( t_j \) can be expressed, respectively, by

\[ P^j_H = G^j_{Hp} Q^1 + G^j_{2Hp} Q^2 + \cdots + G^j_{iHp} Q^i + \cdots + G^j_{nHp} Q^n, \]

(13)

\[ A^j_S = G^j_{Sa} Q^1 + G^j_{2Sa} Q^2 + \cdots + G^j_{iSa} Q^i + \cdots + G^j_{nSa} Q^n, \]

(14)

where

\[ P^j_H = \begin{bmatrix} p_{H1}(t_j) & p_{H2}(t_j) & \cdots & p_{HL}(t_j) \end{bmatrix}^T, \]

(15)

\[ A^j_S = \begin{bmatrix} a_{S1}(t_j) & a_{S2}(t_j) & \cdots & a_{SM}(t_j) \end{bmatrix}^T, \]

(16)

\[ Q^j = \begin{bmatrix} q_1(\tau^j) & q_2(\tau^j) & \cdots & q_K(\tau^j) \end{bmatrix}^T. \]

(17)

\[ G^j_{Hp} = \begin{bmatrix} \Phi^j(\tau_{H11}) & \Phi^j(\tau_{H12}) & \cdots & \Phi^j(\tau_{H1K}) \\ \Phi^j(\tau_{H21}) & \Phi^j(\tau_{H22}) & \cdots & \Phi^j(\tau_{H2K}) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi^j(\tau_{HL1}) & \Phi^j(\tau_{HL2}) & \cdots & \Phi^j(\tau_{HLK}) \end{bmatrix}, \]

(18)

\[ G^j_{Sa} = \begin{bmatrix} g^j(\tau_{S11}) & g^j(\tau_{S12}) & \cdots & g^j(\tau_{S1K}) \\ g^j(\tau_{S21}) & g^j(\tau_{S22}) & \cdots & g^j(\tau_{S2K}) \\ \vdots & \vdots & \ddots & \vdots \\ g^j(\tau_{SM1}) & g^j(\tau_{SM2}) & \cdots & g^j(\tau_{SMK}) \end{bmatrix}. \]

(19)

By inverting Eq. (13), the equivalent source strengths at the time \( t_j \) can be obtained by

\[ Q^j = \left[ G^j_{Hp} \right]^{-1} \left[ P^j_H - G^j_{2Hp} Q^2 - \cdots - G^j_{(i-1)Hp} Q^{(i-1)} \right]. \]

(20)

Equation (20) provides an iterative solving process for determining the equivalent source strengths at each time step by the pressure measured on the hologram plane \( H \). The standard Tikhonov regularization (16) is applied in the solving process at each time step to obtain the appropriate equivalent source strengths. The solved equivalent source strengths at each time step can then be taken as the input in Eq. (14) for calculating the normal acceleration of the plate. Similarly, the pressure at any field point at each time step can also be calculated by using the solved equivalent source strengths as the input.
3. EXPERIMENT

The experiment was carried out in a semi-anechoic chamber as shown in Fig. 2. A clamped steel plate with the size of 0.45 m×0.45 m was impacted by a steel ball to generate a transient sound field. An accelerometer fixed on the back of the plate was set as the trigger to activate the microphone array recording the pressure information and another accelerometer recording the normal acceleration information.

Figure 3 – Geometric description of the plate plane \( S \), the reconstruction plane \( R \), the measurement plane \( H \), and the equivalent source plane \( E \). Four points \( R_1, R_2, R_3 \) and \( R_4 \) are selected for comparison, marked with the symbol “+”.

Figure 3 shows the position relationships between the plate plane \( S \), the reconstruction plane \( R \), the hologram plane \( H \) and the equivalent source plane \( E \). \( 5 \times 5 \) points were distributed on the planes \( S, R, H \), and \( E \), and the grid space was 0.1 m in both \( x \) and \( y \) directions. The equivalent source plane \( E \) was located at \( z_e = -0.005 \) m. The signal was sampled at a frequency \( f_e = 25.6 \) kHz providing 128 sampling points.

3.1 Reconstruction of the pressure field

In the experiment, the array with \( 5 \times 5 \) microphones was used to acquire the pressures on the hologram plane \( H \) with \( z_h = 0.03 \) m and the reconstruction plane \( R \) with \( z_r = 0.01 \) m.

For assessing the results reconstructed in the time domain, four space points on the plane \( R \) were chosen as shown in Fig. 4 and their positions were \( R_1 (0.3 \) m, \( 0.4 \) m, \( 0.01 \) m), \( R_2 (0.1 \) m, \( 0.3 \) m, \( 0.01 \) m), \( R_3 (0.3 \) m, \( 0.3 \) m, \( 0.01 \) m), and \( R_4 (0.4 \) m, \( 0.2 \) m, \( 0.01 \) m), respectively. Figure 4 shows that the reconstructed pressures at these four points are in good agreement with their measured values. Three time instants with \( t_1 = 1.99 \) ms, \( t_2 = 2.42 \) ms and \( t_3 = 3.52 \) ms were chosen to evaluate the reconstructed results in the space domain. The measured pressure fields at these three time instants are shown in Figs. 5(a), 5(b) and 5(c), respectively, and the reconstructed pressure fields are shown in Figs. 5(d), 5(e) and 5(f),
respectively. It can be seen that the pressure fields vary with the time. It is also obvious that the pressure fields reconstructed by the proposed method are almost the same as the measured ones, which indicates that the proposed method can be used to visualize transient sound fields radiated by an impacted plate effectively at different time instants.

Figure 4 – Time domain waveform comparisons between the measured pressures (solid line) and the reconstructed pressures (dotted line) at four points (a) \( R_1 \) (0.3 m, 0.4 m, 0.01 m), (b) \( R_2 \) (0.1 m, 0.3 m, 0.01 m), (c) \( R_3 \) (0.3 m, 0.3 m, 0.01 m) and (d) \( R_4 \) (0.4 m, 0.2 m, 0.01 m).

Figure 5 – Spatial distributions of the measured pressures at (a) \( t_1 \) = 1.99 ms, (b) \( t_2 \) = 2.42 ms and (c) \( t_3 \) = 3.52 ms versus the reconstructed pressures at (d) \( t_1 \) = 1.99 ms, (e) \( t_2 \) = 2.42 ms and (f) \( t_3 \) = 3.52 ms on the reconstruction plane \( R \).

3.2 Reconstruction of the normal acceleration of the plate

In the experiment, the pressure measured on the hologram plane \( H \) with \( h_z = 0.01 \) m was used to
reconstruct the normal acceleration on the plate plane $S$ with $z_s = 0$ m, and the normal acceleration on the plate plane $S$ was also measured by one accelerometer to serve as the “true” value for comparison.

Figure 6 – Time domain waveform comparisons between the measured normal accelerations (solid line) and the reconstructed normal accelerations (dotted line) at the four points (a) $R_1 (0.3 \text{ m}, 0.4 \text{ m}, 0 \text{ m})$, (b) $R_2 (0.1 \text{ m}, 0.3 \text{ m}, 0 \text{ m})$, (c) $R_3 (0.3 \text{ m}, 0.3 \text{ m}, 0 \text{ m})$ and (d) $R_4 (0.4 \text{ m}, 0.2 \text{ m}, 0 \text{ m})$.

Figure 7 – Spatial distributions of the measured normal accelerations at (a) $t_1 = 1.99 \text{ ms}$, (b) $t_2 = 2.42 \text{ ms}$ and (c) $t_3 = 3.52 \text{ ms}$ versus the reconstructed normal accelerations at (d) $t_1 = 1.99 \text{ ms}$, (e) $t_2 = 2.42 \text{ ms}$ and (f) $t_3 = 3.52 \text{ ms}$ on the plate plane $S$.

Just as the reconstruction of the pressure field, four points $R_1 (0.3 \text{ m}, 0.4 \text{ m}, 0 \text{ m})$, $R_2 (0.1 \text{ m}, 0.3 \text{ m}, 0 \text{ m})$, $R_3 (0.3 \text{ m}, 0.3 \text{ m}, 0 \text{ m})$ and $R_4 (0.4 \text{ m}, 0.2 \text{ m}, 0 \text{ m})$ were chosen to show the comparisons between the time domain acceleration waveforms reconstructed by the proposed method and those measured values,
as shown in Fig. 6. It can be seen that the reconstructed normal accelerations at those four points are close to their measured values. Finally, the comparisons between the normal accelerations reconstructed by the proposed method and those measured at the same time instants with \( t_1 = 1.99 \text{ ms} \), \( t_2 = 2.42 \text{ ms} \) and \( t_3 = 3.52 \text{ ms} \) are shown in Fig. 7, which demonstrates that the proposed method can also visualize the transient normal acceleration distribution on the plate plane effectively.

4. CONCLUSIONS

The interpolated TD-ESM based transient NAH was applied to realize the reconstruction of the transient vibration and sound radiation of an impacted plate. The reconstruction formulas of both the pressure radiated by the plate and the normal acceleration of the plate were deduced by applying the Lagrange interpolation to equivalent source strengths. An experiment with an impacted steel plate as the source was carried out to validate the proposed method. The experimental results demonstrate that the method not only can be used to reconstruct the transient pressure signals and the transient normal acceleration signals directly in the time domain, but also can be used to visualize both the pressure and normal acceleration modes of the plate in the space domain. In particular, the normal acceleration of the plate is obtained by a non-contact measurement in the proposed method, and therefore it is able to avoid the added mass effect associated with the normal acceleration measured directly by the accelerometer.

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