A Study on Structural Optimization of SEA Subsystems
using Finite Element Model
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ABSTRACT
This paper describes a structural optimization method for subsystems in statistical energy analysis (SEA). The objective function is power flow (PF) shows the power flow between the subsystems. Under the constraint function is the total mass, the design variable is taken as the thickness of each FE element in each subsystem in the case of proposed structural optimization method. The proposed method is based on a combination of SEA and FEM calculation, calculating repeatedly until satisfying the value of objective functions under arbitrary constraints. As a result of applying the proposed method to a simple structure consisting of two flat plates connected in an L-shaped configuration. The validity of the optimal thickness distribution by proposed method were verified by comparison of subsystem energy, input power, natural frequency and coupling loss factor before and after optimization. In spite of the difference of optimization problem, the results of thickness distribution are very similar.

Keywords: Statistical energy analysis, Shape design, Forced vibration

1. INTRODUCTION
Automotive industry requires for improving the fuel consumption is lighting the weight of car. Accordingly, vibro-acoustic analysis to the high frequency is desired by using high stiffness thin plate which is thinner than conventional one. When implementing the structural optimization considered the energy flow or power flow between structural subsystems for attempting to reduce structure-borne sound radiated from machinery, it is difficult to examine how the energy flow changes structural subsystems with conventional structural optimization methods reported by Furuya (1), in which the objective function is the natural frequency or the frequency response function (FRF). The conventional method using the peak value of FRF is not easy to set the objective function because of considering the peak of the magnitude in the discrete frequency for the case of existing the plural natural frequencies in the target frequency. Conversely, statistical energy analysis (SEA) is a method for vibro-acoustic analysis which regards the system as composed of high modal density and focuses on the power equilibrium between the subsystems (2). In SEA, the coupling loss factor (CLF) denotes the energy flow between the subsystems, and power flow (PF) denotes the power flow between the subsystems during machine operation. Therefore, it is considered that setting the CLF or PF to the objective function is easy to realize the structural optimization which considers the energy flow or power flow between the subsystems. In addition, the subsystem is averaged over space and frequency, so it is possible to become the uniformly thickness distribution of subsystem structure and decrease the number of objective function compared with the conventional method.

The authors developed a structural design process on the basis of experimental SEA for reducing structure-borne sound, whose efficiency was subsequently verified by applying it to various machines (3). The process identifies the internal loss factors (ILFs) or CLFs which should be changed in order to reduce the noise radiated from the machinery. Thus far, the subsystem structures examined for the purpose of adjusting the CLF appropriately have been studied with the analytical formula for the CLFs used in analytical SEA (4). However, detailed examination of the subsystem structures have been impossible to perform with analytical SEA because of not creating the specific shape of the subsystem.
For this reason, in pursuit of a structural optimization method based on finite element method (FEM), which is capable of accounting for the details of the structure, the authors proposed the FEM-SEA method (5), where the evaluation of the CLF was performed with respect to the target subsystem instead of the entire system. It is possible to examine the detail of the subsystem structures (for example, structure with irregularity surface or curvature, and discontinuity structure with open hole) by using the FE model.

Accordingly, the authors developed a formulation of a structural optimization method for SEA subsystems for which the realization of the desired value of the loss factors is necessary. This method is based on the FEM-SEA, targeting only subsystems which include the subsystem determined in the implementation of the structural design process for reducing structure-borne sound.

Regarding the previous research in which SEA has been employed in optimization algorithms, there are case studies concerning the minimization of sound pressure level in a car interior using analytical SEA by Aran and Dhanesh (6), where the thickness of the subsystem and the ILFs as design variables. Bartosch and Egger (7) take the loss factors including the ILFs and CLFs as design variables. However, analytical SEA has been used in these cases, it has been impossible to examine the structure of the SEA subsystems in detail.

So, in this paper, the aim is expanding the formulation of a structural optimization method for SEA subsystems to conduct them in which the objective function is the power flow. As a test structure, an L-shaped plate consisting of two finite, elastic plates coupled at a right angle is considered, where the mass is taken as a constraint function, and the one and multiple one-third octave bands PF is taken as a objective function.

2. BASIC THEORY OF SEA AND STRUCTURAL OPTIMIZATION METHOD

2.1 SEA Power Balance Equation

In SEA, a system is regarded as an assembly of subsystems. If the system has $r$ subsystems, consideration of the power balance between them leads to a basic set of SEA equations (2):

$$ P = LE $$(1)

$$ L = \omega 
\begin{bmatrix}
\eta_{r,1} & -\eta_{r,1} & \cdots & \eta_{r,1} \\
-\eta_{1,1} & \eta_{1,1} + \sum_{i=1}^{r} \eta_{i,1} & \cdots & \eta_{1,1} \\
-\eta_{2,1} & \eta_{2,1} + \sum_{i=1}^{r} \eta_{2,i} & \cdots & \eta_{2,1} \\
\vdots & \vdots & \ddots & \vdots \\
-\eta_{r,1} & \eta_{r,1} + \sum_{i=1}^{r} \eta_{r,i} & \cdots & \eta_{r,1}
\end{bmatrix} $$

(2)

Here, $\omega$ is the center angular frequency of the band, $E$ is a vector containing the subsystem energies, and $P$ is the external input power vector. The loss factor matrix, $L$, comprises Internal Loss Factors (ILFs), $\eta_{i,j}$, and Coupling Loss Factors (CLFs), $\eta_{i,j}$. Estimation of the ILFs and CLFs is referred to as the construction of the SEA model.

2.2 Structural Optimization Procedure

The structural optimization method is implemented in accordance with the structural design process for reducing structure-borne sound. First, the SEA parameters which can effectively reduce the acoustical or vibrational energy of the subsystem are determined by the structural design process. Next, the partial model simulating the boundaries of the entire system is constructed. Finally, the structure with the desired values of the loss factors or the power flows under arbitrary constraints is obtained by applying a combination of SEA calculation and the optimization procedure.

2.3 Structural Optimization Method

The flowchart of the developed structural optimization method is shown in Figure 1. First, calculating the subsystem energies and input power of subsystem by applying rain-on-the-roof-excitiation (8) on the basis of initial value of the design variables. The design variables are the density, Young's modulus, the damping values associated with the material properties,
the thickness of the plate elements, the shape, and the coupling between the subsystems related to the structures, and so on. Second, calculating the SEA parameters on the basis of the power injection method (9) of the objective functions using the calculated subsystem energies and input powers. Finally, calculating the constraints functions by performing static analysis. The optimization algorithm defines new value for the design variables, and a new set of SEA parameter and constraints functions calculation are performed until satisfying the value of objective functions.

For the case of calculating the subsystem energies and input powers, the way of excitations to the structure is needed to examine. According to the SEA theory (2), all modes in the target frequency range are excited, inducing individual excitations at multiple points. The constraints functions are the total mass of structure, the stiffness (displacement), the stress, the buckling load, and the natural frequency, and so on.

2.4 Formulation of the Structural Optimization Problem by SEA

The formulation of the optimization problem by taking into account the subsystem structure is considered together with past structural optimization problems. The objective function is power flow $PF_{i,j}$ between subsystems $i$ and $j$ is predicted by

$$PF_{1,2} = \omega (\eta_{i,1}E_1 - \eta_{2,2}E_2)$$

The structure for which the objective function is maximized (minimized) or satisfies the target value is generated using a numerical method such as FEM. For example, the objective function is assumed to be PF at an arbitrary frequency band and is used to formulate the minimization of the objective function.

In the case of the minimization of the objective function $PF_i (\{x_j\})$ at multiple frequency bands ($i=1,..., n$) on the basis of the constraint function $g (\{x_j\})$ in a feasible design region $D$, the following
equations can be written by,

\[
\begin{align*}
\text{Minimize} \quad & \sum_i PF_i(\{x_i\}) \\
\text{Subject to} \quad & g(\{x_j\}) - g_{\text{max}} \leq 0 \\
& \{x_j\}^L \leq \{x_j\} \leq \{x_j\}^U \quad (j = 1, \ldots, n)
\end{align*}
\]

(4a) (4b) (4c)

Here, \(g_{\text{max}}\) is the upper limit of the constraint function \(g(\{x_j\})\), and \(\{x_j\}^L \leq \{x_j\} \leq \{x_j\}^U\) is for lower limit (upper limit) on design variables \(\{x_j\}\).

3. APPLICATION OF THE STRUCTURAL OPTIMIZATION METHOD

3.1 Test Structure and Problem Settings

As shown in Figure 2, the target structure consists of two rectangular steel plates coupled in an L-shaped configuration. This is corresponding to the partial model described in Section 2.2. The length of the plates 1 and 2 are \(L_1 = L_2 = 0.3\) m. Both plates have a width of \(L_3 = 0.3\) m. The thickness of the plates is 1 mm. The white circle in Figure 2 is used by the explanation of the optimization results mentioned later.

The design variable is taken as the thickness of the FEM element, which is a commonly manipulated design variable in optimization problems regarding plate and shell elements. In the case of considering the actual structure, it is considered that the maximum stress and maximum displacement are taken as the constraint functions. In this paper, the total mass and the thickness of the plate corresponding to the design variable are taken as a constraint function. The upper and the lower limit for the design variables with consideration of the product available plate thickness are 2.0 mm and 0.6 mm, respectively. The target optimization is the minimization of \(PF_{1,2}\) at the construction of the SEA model when the power is injected to subsystem 1, and the CLF is set between zero and unity. The total mass is taken as a constraint function. The upper limit for the design variable is the original value (1.42 kg).

3.2 Structural Optimization Method

ANSYS Ver. 11.0 is used for constructing the partial model, the SEA parameters are calculated using MATLAB, and the optimization results are obtained using OPTIMUS 10.9, which is software for automation, integration, and optimization.

3.2.1 Constructing the Partial FE Model

The material density is \(\rho = 7860\) kg/m\(^3\), and Poisson’s ratio is \(\nu = 0.3\). An elastic shell element (shell 63) is used that consists of 4 nodes, with three translational and three rotational degrees of freedom per node. The size of each element in the mesh is about 0.03 m \(\times\) 0.03 m, which is sufficient to contain five nodes per bending wavelength up to 1k Hz. The total numbers of nodes and elements are 231 and 200, respectively. All edges of the plate are pin supported.

3.2.2 Setting the Initial Values for Some Parameters

Various functions, such as objective functions, design variables, and constraint functions are set in accordance with the structural optimization method.

The design variables is taken as the thickness of the FEM element, which is a commonly manipulated design variable in optimization problems regarding shell elements. Plate 1 and 2 are selected as a structural element, thus, there are 200 design variables. The target frequency band, the value of the objective functions, and the constraint functions are set on the basis of the analytical results of initial model.

In the evaluation of the normalized energy for the subsystem, it is desirable to use rain-on-the-roof excitation as the excitation method described in Section 2.3. The number of excitation is to set to be 27 excitations, the magnitude of the excitation force is to set to be a unit force, and the response points for the subsystem energy are selected the 81 nodes per each subsystems excluding the junction (11 nodes) and all edges of the plate (60 nodes). All modes (44 modes) within the frequency range (0-1k Hz) are used. The loss factors are assumed to be 0.05 for all modes. The displacement of the response is calculated for the
range between 5 Hz and 900 Hz at 5 Hz steps. Regarding the SEA parameters, the one-third octave band frequencies from 20 Hz to 800 Hz is calculated by using the following equation:

\[
\begin{align*}
\begin{pmatrix}
\eta_{1,1} \\
\eta_{1,2} \\
\eta_{2,1} \\
\eta_{2,2}
\end{pmatrix} &= \begin{pmatrix}
E_1^1 & E_1^i & -E_2^i & 0 \\
0 & -E_2^i & E_1^i & E_2^1 \\
E_1^2 & E_2^2 & -E_2^i & 0 \\
0 & -E_1^i & E_2^2 & E_2^2
\end{pmatrix}^{-1}
\begin{pmatrix}
P_1 \\
P_2
\end{pmatrix} \\
\eta_{1,2} &= \frac{1}{\omega}
\begin{pmatrix}
E_1^2 & E_2^2 & -E_2^i & 0 \\
0 & -E_1^i & E_2^2 & E_2^2
\end{pmatrix}
\begin{pmatrix}
P_1 \\
P_2
\end{pmatrix}
\end{align*}
\]

Figure 3 – Initial values for the power flow from subsystem 1 to 2

Figure 4 – Iteration history for the objective function in the 125 Hz band

Figure 5 – Iteration history for the parameters related to the objective function

Regarding the SEA parameters, the one-third octave band frequencies from 20 Hz to 800 Hz is calculated by using the following equation:
where $E_{i\, j}$ is the energy of subsystem $i$ when subsystem $j$ is excited by input power $P_j$.

The total mass is calculated in static analysis of ANSYS.

In the case of applying this optimization method to the actual structure, it will be distinguished whether SEA is applicable or not by means of considering these threshold values for the number of modes in bands and a value for the modal overlap factors.

Figure 3 shows the $PF_{1,2}$ of initial model in one-third octave band frequencies from 20 Hz to 800 Hz. When the value of the $PF_{1,2}$ at 50 Hz and 250 Hz are negative, points are not plotted in Figure 3. The CLF take a negative value, besides subsystem energy 1 is larger than the one of subsystem 2. From these results, the target objective function is comparatively large value ($1.19 \times 10^0$) in $PF_{1,2}$ at 125 Hz in one-third octave band. The target value of the objective function is determined by focusing on specifying the PF which should be changed in order to reduce the acoustical or the vibrational energy of the subsystem.

After the setup of the above mentioned, the optimization algorithms are set in the OPTIMUS software. In this paper, in case of the single objective optimization problem in Section 3.3, the non-linear programming by quadratic lagrangian method (NLPQL), which is a kind of local optimization method applicable to non-linear programming problems, is chosen here. Since the time required for obtaining the optimization results is rather long, the number of iterations is set to 10 times.

In case of the multi objective optimization problem in Section 3.4, the normal-boundary intersection method (NBI) (10), which is completely independent of the relative scales of the functions and is quite successful in producing an evenly distributed set of points in the Pareto set given an evenly distributed set of weights, a property which the popular method of linear combinations lacks, is chosen here. Since the time required for obtaining the optimization results is rather long, the number of iterations is set to 45 times.
3.3 Optimization Results by Single Objective Optimization Problem

The iteration history of the objective function \( PF_{1,2} \) and the parameters related to the objective function are shown in Figure 4 and 5. Figure 6 shows the comparative values of the power flow between the initial value and the optimization results at the 10th iteration, which reaches the minimum value from 10th iterations. From Figure 4, the value of the \( PF_{1,2} \) at first iteration decreased by about 60% as compared with the initial value. According to the repetition, all the values except for the subsystem energy \( E_{1,1} \) are decreasing in Figure 5. From Figure 6, the value of the \( PF_{1,2} \) at target frequency band decreased by about 94% and become \( 7.25 \times 10^{-2} \) as compared with the initial value of \( 1.19 \times 10^{0} \). The mass of subsystem 1 and 2 are 0.701 kg and 0.714 kg. The optimization results in this case indicate that the all the values of \( PF_{1,2} \) include the target frequency band are smaller compared with the initial value.

3.3.1 Verification of the Optimization Results by Single Objective Optimization Photographs

Figure 7 shows contour map of the thickness distribution for the structure in the optimization results. The minimum thickness (0.6 mm) corresponds to the maximum brightness (white color). The white circle in Figure 7 corresponds to the location in Figure 2.

Table 1 shows the values for some SEA parameters related to the objective function between the initial and the optimization results in the target frequency band.

The thickness distribution is linear symmetry in the Y-axis in Figure 7, and the increasing of subsystem energy \( E_{1,1} \) is related to the increasing of input power on the excitation side in Table 1.

Table 1 – Comparison between the initial and optimum values in the 125 Hz band

<table>
<thead>
<tr>
<th></th>
<th>Initial value</th>
<th>Optimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{1,2} )</td>
<td>( 9.55 \times 10^{-2} )</td>
<td>( 3.95 \times 10^{-3} )</td>
</tr>
<tr>
<td>( \eta_{2,1} )</td>
<td>( 9.55 \times 10^{-2} )</td>
<td>( 8.47 \times 10^{-3} )</td>
</tr>
<tr>
<td>( E_{1,1} (J) )</td>
<td>( 2.27 \times 10^{-2} )</td>
<td>( 2.81 \times 10^{-2} )</td>
</tr>
<tr>
<td>( E_{2,1} (J) )</td>
<td>( 6.76 \times 10^{-3} )</td>
<td>( 2.19 \times 10^{-3} )</td>
</tr>
<tr>
<td>( P_{1}(W) )</td>
<td>( 7.69 \times 10^{-3} )</td>
<td>( 1.08 \times 10^{-2} )</td>
</tr>
<tr>
<td>( E_{1,2} (J) )</td>
<td>( 6.76 \times 10^{-3} )</td>
<td>( 4.39 \times 10^{-3} )</td>
</tr>
<tr>
<td>( E_{2,2} (J) )</td>
<td>( 2.27 \times 10^{-2} )</td>
<td>( 2.80 \times 10^{-2} )</td>
</tr>
<tr>
<td>( P_{2}(W) )</td>
<td>( 7.69 \times 10^{-3} )</td>
<td>( 7.87 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

Table 2 – Comparison between the initial and optimum values of the natural frequencies (unit:Hz)

<table>
<thead>
<tr>
<th>Order</th>
<th>Initial value</th>
<th>Optimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54.6</td>
<td>53.0</td>
</tr>
<tr>
<td>2</td>
<td>65.3</td>
<td>62.3</td>
</tr>
<tr>
<td>3</td>
<td>136.1</td>
<td>132.4</td>
</tr>
<tr>
<td>4</td>
<td>136.1</td>
<td>134.8</td>
</tr>
<tr>
<td>5</td>
<td>142.4</td>
<td>138.9</td>
</tr>
<tr>
<td>6</td>
<td>161.6</td>
<td>165.4</td>
</tr>
<tr>
<td>7</td>
<td>216.6</td>
<td>220.2</td>
</tr>
<tr>
<td>8</td>
<td>235.7</td>
<td>232.9</td>
</tr>
<tr>
<td>9</td>
<td>271.9</td>
<td>264.0</td>
</tr>
<tr>
<td>10</td>
<td>271.9</td>
<td>272.5</td>
</tr>
</tbody>
</table>

Table 2 shows a comparison between the initial and optimum values of the first tenth natural
The third and the forth natural frequencies influence on the target frequency 125 Hz band (from 112 Hz to 141 Hz) in the initial condition.

In the optimum condition in Table 2, the fifth natural frequencies influence on the target frequency.

The modal shapes in the third, forth and fifth in the initial condition are changed to ones of the fifth, third and forth, respectively.

3.4 Optimization Results by Multi Objective Optimization Problem

The target objective function are 125, 160, 200 Hz in one-third octave band frequencies, avoiding the negative value. The iteration history of the objective function $PF_{1,2}$ is shown in Figure 8. Figure 9 shows the comparative values of the power flow between the initial value and the optimization results at the 36th
iteration, which reaches the minimum value from 45th iterations. From Figure 8, the value of the \( PF_{1,2} \) in 125 Hz and 200 Hz at first 10th iteration are decreased by about 8% and 20% as compared with the initial value, and then they increase with the value of the \( PF_{1,2} \) in 160 Hz decreases. From Figure 9, the values of the \( PF_{1,2} \) at target frequency band decreased by about 97%, 78% and 97%, and become \( 3.56 \times 10^2 \), \( 1.75 \times 10^1 \) and \( 1.44 \times 10^2 \) as compared with the initial value of \( 1.19 \times 10^0 \), \( 7.99 \times 10^1 \) and \( 4.61 \times 10^1 \). The mass of subsystem 1 and 2 are 0.613 kg and 0.756 kg. The optimization results in this case also indicate that the all the values of \( PF_{1,2} \) include the target frequency band are smaller compared with the initial value.

3.4.1 Verification of the Optimization Results by Multi Objective Optimization Problem

Figure 10 shows contour map of the thickness distribution for the structure in the optimization results. The thickness distribution is also linear symmetry in the Y-axis in Figure 10, and the thickness distribution is also similar to the results of single optimization from Figure 7 in spite of the difference of optimization problem.

Table 3 – Comparison between the initial and optimum values of the natural frequencies (unit:Hz)

<table>
<thead>
<tr>
<th>Order</th>
<th>Initial value</th>
<th>Optimum value</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>54.6</td>
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<tr>
<td>2</td>
<td>65.3</td>
<td>61.9</td>
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<tr>
<td>3</td>
<td>136.1</td>
<td>118.3</td>
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<tr>
<td>4</td>
<td>136.1</td>
<td>119.9</td>
</tr>
<tr>
<td>5</td>
<td>142.4</td>
<td>136.7</td>
</tr>
<tr>
<td>6</td>
<td>161.6</td>
<td>165.1</td>
</tr>
<tr>
<td>7</td>
<td>216.6</td>
<td>194.1</td>
</tr>
<tr>
<td>8</td>
<td>235.7</td>
<td>222.5</td>
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<td>9</td>
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<td>236.8</td>
</tr>
<tr>
<td>10</td>
<td>271.9</td>
<td>264.7</td>
</tr>
</tbody>
</table>

Table 3 shows a comparison between the initial and optimum values of the first tenth natural frequencies. The third and the forth natural frequencies influence on 125 Hz band (from 112 Hz to 141 Hz), the fifth and sixth influence on the 160 Hz band (from 141 Hz to 178 Hz), and the seventh influence on the 200 Hz band (from 178 Hz to 224 Hz) in the initial condition. In the optimum condition in Table 3, the fifth and the eighth are shifted to the lower frequency band. The modal shapes in the third, forth, eighth, ninth and tenth in the initial condition are changed to ones of the forth, third, tenth, ninth and eighth, respectively.

4. CONCLUSIONS

In this study, a structural optimization method for SEA subsystem was applying to realize the desired value of power flow between subsystems for the one and multiple one-third octave bands frequency. As a result of applying the developed method to a simple structure consisting of two flat plates connected in an L-shaped configuration, a subsystem structure with the desired value of power flow for the target frequency band was constructed. The validity of the optimal thickness distribution were verified by comparison of subsystem energy, input power, natural frequency and coupling loss factor before and after optimization.

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