Investigation of Bandgap Structure in Coupled Acoustic-Mechanical System

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ABSTRACT
In this research, we study the applicability of a generic methodology for reducing the interaction and resulting vibration problems in coupled acoustic-mechanical systems based on bandgap structures. Finite element analysis with acoustic-structure interaction (ASI) is used to study the dynamic and acoustic behavior of a representative non-planar plate unit cell and the Bloch–Floquet theory is used to apply the results to an infinite material. Several numerical experiments are provided to gain physical insight into how bandgaps in the acoustic and mechanical domains can be created and located at desired frequency ranges and with specific geometrical dimensions. In addition, the bandgap properties are analyzed in view of the possibility for the control of mechanical waves in the presence of strong ASI. The coupled bandgap phenomenon is investigated based on the result of a vibro-acoustic analysis for an acoustic cavity with a corresponding periodic structure made from the non-planar unit cell. Particular attention is given to the effect on both vibrational/acoustic properties of the finite structure. An experiment investigation of a 3D printed finite structure is explored to study the bandgap phenomenon and vibro-acoustic transmission.

Keywords: Bandgaps, Acoustic-Mechanical interaction, Microstructure

1. INTRODUCTION
Bandgaps, i.e., frequency ranges in which waves cannot propagate, can be found in materials for which there is a certain periodic modulation of the material properties or structure [1-3]. Such a fascinating phenomenon has been exploited effectively in recent years due to their use for filtering and shielding of elastic and acoustic waves. The existence of band gaps for elastic waves may lead to very low vibration levels for structures/devices made from band gap material and exposed to excitation at bandgap frequencies. Several studies have been made on band gaps for acoustic wave. The existence of acoustic bandgaps is of interest for many potential applications in sound insulation, acoustic wave filtering, negative refraction [4]. Also, bandgaps have a potential to be utilized as a novel building block to reduce undesirable vibro-acoustic transmission may generate unwanted vibration and sound radiation. Especially, band gaps for vibro-acoustic transmission potentially could be used in micro-mechanical devices with acoustic functionality such as hearing aids, cell phones.

Although numerous theoretical and experimental works have been made on the existence of bandgaps for elastic and acoustic waves, however, very few attempts have been made at bandgaps in acoustic-structure interaction [5]. Specifically, little attention has been given to the existence of combined acoustic-mechanical bandgaps although of great potential for acoustic-mechanical devices.

In the author’s previous study [6], the bandgap for bending waves in a thin planar square bi-material unit cell was analyzed in view of a possibility for creating bandgaps in the audible frequency. This feature makes periodic plate structure to finding efficient solutions of vibro-acoustic transmission in acoustic-mechanical devices. We investigated the waves and vibration in the frequency range corresponding to the identified bandgap range when the acoustic-mechanical coupling exists. It was found that the footprint of the bandgap frequency range is less identifiable

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than what could be expected, because the response is strongly polluted by the surrounding acoustic medium. It was demonstrated that the coupling to acoustic medium restrains the existence of bandgap for bending waves due to the possibility for the acoustic waves to propagate freely.

The previous result led us to discover that utilization of bandgap structures to control the waves and vibration in the presence of acoustic-mechanical coupling may be potentially useful but requires careful design considerations to make the bandgap phenomenon significant and more pronounced.

Accordingly, this previous study inspires us to investigate a different periodic configuration involving non-planar plate structure which may generate coupled bandgap frequency range where there is a considerable overlap between acoustic and vibrational properties [7]. Hence, we study the existence of coupled bandgaps for a periodic non-planar plate with and without a surrounding acoustic medium. In this respect, the study comprises analysis of the band diagram for the uncoupled acoustic/structure model (section 2 and 3) and the vibro-acoustic analysis of a finite structure (section 4). An experiment investigation of a 3D printed finite structure will be explored to study the total bandgap phenomenon in terms of acoustic and structural waves in the audible frequency range.

2. ANALYSIS OF BAND STRUCTURE

2.1 Dynamic properties of periodic structure

As the first step in the analysis we will investigate the propagation of the waves in the periodic structure combined with an acoustic cavity on top. Specifically, we will examine the possibility for combining vibrational and acoustical bandgap. In order word, we will take a close look at a specified periodicity which is able to create overlapping bandgaps in structure and acoustic domain. For this purpose, we create a periodic vibro-acoustic structure which may generate the coupled bandgaps in acoustic and structural waves (elastic wave) simultaneously. In this respect, the vibro-acoustics unit cell model is applied with a non-planar plate structure coupling to surrounding acoustic medium in the form of one-sided loading. The basic layout is illustrated in Figure 1 where we could exploit the existence of coupled bandgap.

![Figure 1](image_url)

Figure 1 – (a) Unit cell of the periodic structure combined with an acoustic cavity on top. The cell side is quadratic with side length L. (b) Illustration of the irreducible Brillouin zone and wave vector direction.

In order to capture the full behavior in mechanical and acoustic domains, we solve the fully coupled acoustic-structure interaction problem. The both domains are solved by separately and the mutual acoustic-structure coupling is obtained by imposing the explicit boundary conditions. The boundary representation is a multiphysics phenomenon where the acoustic pressure cause a fluid load on the structure domain, and the structural acceleration affects the acoustic domain as a normal acceleration across the acoustic-structure boundary.

In the case of a periodic structure the wave solution to the full dynamic problem can be expanded with the Bloch-wave theorem [8]. The displacement vector $u(x,t)$ should satisfy the form:

$$u(x,t) = \mathbf{c} \cdot \mathbf{e}^{i \mathbf{k} \cdot \mathbf{x}}$$

(1)

In a same manner as for the displacement, for periodic acoustic domain the acoustic pressure should satisfy the form:
where $\omega$ is the wave frequency, $\mathbf{k} = (k_x, k_y)$ is the wave vector, and $\mathbf{u}$ are periodic structure and acoustic modes on the unit cell of the periodic structure, respectively. The Bloch-wave theorem allows for solving the eigenvalue problem for only one unit cell of the periodic structure. The wave vector $\mathbf{k}$ must be varied within the irreducible Brillouin zone shown in Figure 1.

In the analysis we insert the Bloch-wave expansions seen in Eqs. (1) and (2) into the finite element (FE) procedure for coupled acoustic-structure interaction. Thus, FE discretization on the unit cell with periodic boundary conditions using the Bloch-wave expansions results in the full eigenvalue problem formulation:

$$ (\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{v} = 0 $$

where $\mathbf{K}$ and $\mathbf{M}$ are the coupled stiffness and mass matrices consisting of the FE discretization for acoustic and structure domain and coupling component, respectively. The vector $\mathbf{v}$ contains all the discretized nodal values of the displacement and pressure fields, $\mathbf{u}$ and $p$.

We can solve the eigenvalue problem along the irreducible Brillouin zone which is the domain bounded by the line O-A-B-O in Figure 1(b). Generally, solving for the temporal frequency $\omega$ at a specified $\mathbf{k}$, and then we can draw a band diagram to illustrate the dispersion properties, where the eigenfrequencies are plotted versus the wave vectors along with the boundary of the irreducible Brillouin zone. We plot and study the band diagram which reveals the fundamental characteristics of wave propagation through a periodic structure.

We can tune the geometrical and material properties to match the elastic wave bandgap frequency range to the acoustic wave bandgap frequency range. This empirical model is accomplished by parameters in Table 1 and 2. The applied unit cell is presented in Figure 1(a).

<table>
<thead>
<tr>
<th>Table 1 - Geometrical information of the unit cell</th>
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We employ finite element analysis with an acoustic-structure interaction (ASI) model and all analyses are carried out using the commercial software package COMSOL Multi-Physics. In order to identify the presence of coinciding bandgap frequency, in the following we will study the bandgap phenomena in the three different periodic structures based on the unit cell design seen in Figure 1(a).

### 3. NUMERICAL EXPERIMENTS

#### 3.1 CASE 1; Structure part only (uncoupled)

First, we perform eigenvalue problems to plot the band diagram for the pure elastic unit cell model excluding interaction with a surrounding acoustic medium, i.e., uncoupled structural dynamics. In Figure 2, we show the band diagram for the uncoupled periodic structure, which is used for describing a possible modes propagating in the periodic structure. We obtain the elastic wave (vibration) bandgap in the frequency range 11.9 ~ 14.4 kHz as seen in Figure 2. Propagating modes (indicated with solid lines with asterisk marks) do not exist in this range for the particular configuration and set of parameters values seen in Table 1 and 2. This implies that no elastic waves in this range can propagate through the structure regardless of the direction of propagation.
3.2 CASE 2; Acoustic part only (uncoupled)

Secondly, we perform an analysis of wave propagation in acoustic domain of the unit cell as seen in Figure 1. Thanks to the configuration of the solid structure, we naturally obtain the corresponding complex periodic acoustic domain. We also plot the band diagram in the case of the acoustic domain only, i.e., uncoupled acoustics. It is seen from the band diagram that the acoustic wave bandgap is positioned in the frequency range 11.2 ~ 14.1 kHz as seen in Figure 3. Thus, no acoustic waves can propagate through the acoustic domain in this frequency.
For a test case, the bandgap analysis with the T=7mm is repeated to study the effect of the height of the acoustic domain. The other parameters are unchanged but the height is increased to T =7mm. In comparison with the results for T = 5.5mm, it is observed that the bandgap width is smaller. The effect of the smaller bandgap width will be discussed in the subsequent section where a finite structure will be investigated.

We note that the bandgap frequency range is very close to the elastic wave bandgap. In order to identify total bandgap ranges for acoustic waves and vibrational waves, we overlap the band diagrams obtained from the each uncoupled analysis. It is seen that total bandgap is positioned in the frequency range $11.9 \sim 14.1$ kHz as seen in Figure. 4. The band gap size is found as the difference between the lowest point on the second acoustic band ($\omega_2$) and the highest point on the third structure band ($\omega_1$).

### 3.3 CASE 3; Acoustic-Structure Interaction model (fully coupled)

The importance of the coupling on the unit cell models should be investigated. Figure 5 shows the band diagram in the case where coupling is included. The thing to notice from the diagram is that the presence of the acoustic medium is seen to have minimum effect on the vibrational waves. It is noted that when comparing with the uncoupled case, all the computed bands from acoustic-structure interaction analysis are almost identical with the pure acoustics wave and the pure structural waves. Thus, as expected in the total bandgap, the coupled bandgap is kept almost unchanged.

![Figure 5- Band diagrams for the acoustic-structure interaction model. Comparing with the uncoupled structure and acoustic case](image)

### 3.4 Finite structure; ASI model vs Pure Structure/Pure Acoustic model

Investigation of finite structures will allow us to relate the bandgap behavior predicted on unit cell models to the structural behavior of the finite structures build from the periodic structures. This will be done for both the structural behavior as for the acoustical behavior.

![Figure 6- A finite structure with 10 x 10 unit cells subjected to harmonic load.](image)
We create a finite vibro-acoustic structure consisting of 10 x 10 unit cells using the unit cell in Figure 1. Figure 6 displays the finite structure with fixed and rigid wall boundary conditions on the plate and the acoustic domain, respectively. The harmonic point load is applied at the plate. Figure 7 shows the forced vibro-acoustic response of the measurement position. There is a correlation between the identified bandgap range and a low vibration/acoustic response in the measurement point. Since the presence of the acoustic medium do not affect the vibrational waves, we can see that the frequency response curve obtained with the ASI model is very similar to the one obtained with the pure structure model.

It should be noted that some resonance peaks appear within the bandgap frequency range close to the lower and the upper bandgap frequency limit. The reason for that is that the boundary effects become more dominant with fewer unit cells. If the structure is consisting of many unit cells, the bandgap frequency range is more pronounced and a much lower response in most of the bandgap frequency range is achieved. This now raises the question of how many unit cells must be used in order for finite size effects to be neglected from the boundaries. It is challenging to design a periodic structure that has sufficiently small size to fit acoustic-mechanical devices such as hearing aids.

The results also indicate for the ASI model with T=7mm that some resonance peaks and high vibration level are found within the identified bandgap range. As discussed in the band diagram for the model T=7 mm seen in Figure 4, the ASI model with T=7mm allows acoustic waves to propagate for the identified bandgap frequency range. As it turns out the vibrational response within the bandgap frequency range is strongly affected by the acoustic waves. To verify this interpretation, we plot the pressure amplitude above the measurement point. As expected, the corresponding peaks are clearly observed in the bandgap range. Thus it is clear that the presence of acoustic modes in the closed cavity have a great influence on the bandgap footprint observed in a finite structure made from a coupled bandgap structure in acoustics and vibration.

![Figure 7- The vibrational response (upper) and the pressure amplitude (lower) for the finite structure.](image)

4. Experimental investigation

4.1 Experimental setup

The finite periodic structure is fabricated and an experimental investigation is performed to verify the existence of the bandgap effect in acoustic and vibration properties. It should be emphasized that the aim of the experimental study is not to perform a quantitative comparison with numerical results. We therefore would like to focus attention on the existence of the identified bandgap frequency in acoustic and vibration response.
Figure 8- (Left) The fabricated periodic plate made by a 3D printer considering thicker boundaries. (Right) Experimental setup

Figure 8 shows the fabricated periodic plate. In order to clamp the boundaries of the plate, the thicker borders with the same height of acoustic domain, (T=5.5mm) are also fabricated and then assemble with a transparent acrylic plate to make acoustic cavity. The 3D printing is carried with high definition of 32 μm. Geometrical accuracy of the 3D printed periodic plate may vary depending on build parameters, part geometry and size, part orientation, and post-processing, but small discrepancies between the numerical and experimental results are expected to this. However, when it comes to material property, parts created by 3D printing process have variability in the mechanical properties of due to changes in process parameters. Thus, the discrepancies between numerical and experimental results may be attributed the material property. The material for the 3D printer is chosen for density of 1040 kg/m³ and the Young’s modulus of 2168 MPa which are closed to the material parameters listed in Table 2, but large discrepancies between the numerical and experimental results are expected to the practical material property in the 3D printed plate.

The frequency responses of the 3D printed periodic structure are determined by applying a force to the structure and then measuring the vibrational and acoustical response. A shaker is chosen to generate the plate vibration with a random signal to determine the FRF. Plate vibrations are detected with Laser Doffer vibrometer (LDV). The vibrations are generated with a Brüel & Kjær Vibration Exciter 4809. The driving force is measured using a force transducer, allowing the transfer function between the force and the vibrational velocity from LDV to be calculated using the Brüel & Kjær PULSE platform. Radiated sound pressures are detected with the 1/8-inch pressure-field microphone Brüel & Kjær 4138.

4.2 Numerical and experimental comparisons

This section will be devoted to the comparisons between numerical and experimental results.

5. CONCLUSIONS

We studied the use of a bandgap microstructure for reducing the interaction and resulting vibration problems in coupled acoustic-mechanical system. The study comprised analysis of the band diagram for the uncoupled acoustic/structure model and the acoustic-structure interaction model and the vibro-acoustic analysis of a finite structure. From the presented numerical experiments with a non-planar plate unit cell with acoustic domain, it was concluded that a total bandgap in terms of acoustic and structural waves can be achieved. It was however seen that the high vibration peaks appear in the bandgap frequency range due to a clear presence of resonances in the closed cavity configuration.

In further works, an experiment investigation of a 3D printed finite structure will be explored to study the total bandgap phenomenon in terms of acoustic and structural waves in the audible frequency range. Finally, applying advanced optimization techniques, such as topology optimization will be considered. In this connection, special attention will be paid on adequate and cost effective manufacturing of the optimized periodic structures.
ACKNOWLEDGEMENTS

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