Fatigue life analysis of the plate structure with random vibro-acoustic loading

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ABSTRACT
Vibro-acoustic loading is one of the random loadings, which acts on the structure surface. The vibro-acoustic load will excite the random vibration response of the structure, which will lead to fatigue failure. In this investigation, the random response and fatigue failure of the typical plate structure with random vibration loading is derived. The transformation relation between acoustic loading and random vibration loading is presented. The acoustic loading is simulated with random vibration environment. Then the power spectrum density of the acceleration is treated as the loading condition. The random vibration response of typical plate structure is calculated with numerical method. Based on the calculated results of the random stress and fatigue analysis method, the evaluation of fatigue life for the structure is carried out and compared with the criterion life to demonstrate the reliability of the current method.

Keywords: 4 vibro-acoustic loading, fatigue life, finite element analysis  I-INCE Classification of Subjects Number(s): 03.1

1. INTRODUCTION
Vibro-acoustic loading is important environment that weapon equipment always experiences when transported and used, so Vibro-acoustic loading environmental suitability is significant capability for weapon equipment. In order to acquaint the structure property of the equipment in the design stage, methods of numerical analysis, experiment and so on can be used to assess it fatigue life under vibration loads, which can provide support to optimize design.

Traditionally, there are two methods to analyze structure’s fatigue life under vibration loading (1,2). One is time domain method based on data statistics, which needs to count the numbers of stress cycles on dangerous point of structure firstly, then sum the amount of damage with cumulative fatigue damage method and assess its fatigue life. However, it is not efficient enough because of its large amount of data, and the exact time domain stress on dangerous point is also difficult to get. Another method is frequency domain method based on power spectrum density, in this method the PSD (power spectrum density) of the stress responds of dangerous point is calculated through statistics, and then assess its fatigue life. Frequency domain method is low cost and convenient, so it is more used in the engineering.

In this paper, the acoustic loading is simulated with random vibration environment. Then the power spectrum density of the acceleration is treated as the loading condition. The random vibration response of typical plate structure is calculated with numerical method. Based on the calculated results of the random stress and fatigue analysis method, the evaluation of fatigue life for the structure is carried out and compared with the criterion life to demonstrate the reliability of the current method.

2. DEDUCTION OF FATIGUE LIFE
The stress amplitude $S_a$ and the numbers of loading cycles have relation bellow, when the structure is under sine alternating loads (3, 4).

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In equation (1), \( k \) and \( c \) are both the material constants.

According to Miner Liner Method, the accumulation of fatigue damage of the material under sine alternating stress whose amplitude is \( S_a \) is as equation (2).

\[
D_m = \sum_i D_i = \sum_i n_i / N_i
\]

In equation (2), \( n_i \) is the cycles number of the stress \( S_{ai} \), and \( N_i \) is the cycles number of stress \( S_a \) which make the material destroyed. \( D_i \) is the damage degree of \( S_{ai} \), the material will be destroyed, when \( D_m \) reached 1.

In the process of vibration, the frequency the stress whose amplitude is between \( S_a \) and \( S_a + dS_a \) emerges in time of \( T \) is \( n(\frac{S_a}{T}) \).

\[
r(\frac{S_a}{T}) = \nu T p(S_a) dS_a
\]

In equation (3), the power spectrum density of the stress on the dangerous point is \( G(f) \), the spectrum moment is \( m_a = \int_0^\infty f^2 G(f) df \). In narrow band random process, \( \nu \) is the numbers of slop than pass through average value, \( \nu^* = (m_2/m_0)^{0.5} \). In wide random process, \( \nu \) is the expected value of stresspeak, \( \nu_p = (m_4/m_2)^{0.5} \), \( p(S_a) \) is the probability density of the stress amplitude.

The probability damage of the material under random stress \( S_a \) is as bellow:

\[
\frac{n(S_a)}{N(S_a)} = \nu T \frac{n(S_a)}{N(S_a)} dS_a
\]

According to Miner Liner Method, the total damage of material under random stress is \( D_M \):

\[
D_M = \nu T \int_0^\infty \frac{p(S_a)}{N(S_a)} dS_a
\]

When \( D_M = 1 \), the material will be destroyed, and its fatigue life \( T \) can be calculated.

\[
T = \frac{1}{\nu T \int_0^\infty \frac{p(S_a)}{N(S_a)} dS_a} = \frac{c}{\nu \int_0^\infty S_a^k p(S_a) dS_a}
\]

In the stationary Gauss process \( x(t) \) with zero average value, its probability peak \( p(t) \) is as bellow:

\[
p(t) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2}\frac{t^2}{\sigma^2}\right)
\]

In narrow band random process, it assumed that, the stresspeak is accord with Rayleigh distribution, and the stress amplitude and stresspeak have the same probability function.

\[
p(S_a) = \frac{S_a}{\sigma^2} e^{-\frac{S_a^2}{2\sigma^2}}
\]

Integrate the equation (8) from zero to infinity, it transform to equation (9):

\[
\int_0^\infty \frac{p(S_a)}{N(S_a)} dS_a = 2^{\frac{k}{2}} \cdot \sigma^k \cdot \Gamma\left(1 + \frac{k}{2}\right) / c
\]

In equation (9), \( \sigma \) is root mean square value of stress, \( \Gamma(x) \) is gamma function, \( \nu = \nu^* \), merge equation (9) and equation (6), the fatigue life can be calculated:
\[ T = \frac{c}{2^k \cdot \nu^* \cdot \sigma^k \cdot \Gamma(1 + \frac{k}{2})} \tag{10} \]

For wide random process, the material’s damage \( D_{WB} \) can be obtained from amending \( D_{NB} \) in narrow process.

\[ D_{WB} = \lambda D_{NB} \tag{11} \]

In equation (11), \( \lambda \) has the bellow relation with coefficient \( \varepsilon \) and slop \( k \) of \( S-N \) curve.

\[ \lambda = a + (1-a)(1-\varepsilon)^b \tag{12} \]

Where \( a=0.926-0.033k \), \( b=1.587k-2.323 \), \( \varepsilon = \sqrt{1-\alpha^2} \), \( \alpha \) is irregular coefficient

So, the fatigue life \( T \) of material under wide random loading can be acquired.

\[ T = \frac{c}{\lambda 2^k \cdot \nu^* \cdot \sigma^k \cdot \Gamma(1 + \frac{k}{2})} \tag{13} \]

### 3. FATIGUE LIFE ANALYSIS OF PLATE STRUCTURE

Circular plate (5) is typical structure in weapon equipment that may be destroyed under random vibration loading or vibro-acoustic loading. Numerical simulation of the structure’s responds under random vibration loading is carried out. Firstly, the dangerous point is found in the numerical modal. Then the relation between power spectrum density of stress in dangerous point is calculated. Firstly, the root mean square \( \sigma \), the spectrum moment \( m_n \) and expectation of stresspeak \( v \) are calculated through integration. Take these parameters into equation (10), the fatigue life of circular plate will be gotten.

The finite element model is set up in the software as shown in Figure 1, where the modulus of elasticity is 2e11, and Poisson’s ratio is 0.3. The displacement of four points around the border is restricted.

![Figure 1 – Finite element modal](image)

#### 3.1 Model Analysis

Modal analysis is the premise of spectrum analysis, the main modals of the structure can be acquired through some software. Table 1 shows the former five modals of the plate, always, the former two modals is what is most focused on (6. 7).
Table 1 – The former five natural frequency of the plate

<table>
<thead>
<tr>
<th>modal</th>
<th>Natural frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>231.33</td>
</tr>
<tr>
<td>2</td>
<td>292.77</td>
</tr>
<tr>
<td>3</td>
<td>307.5</td>
</tr>
<tr>
<td>4</td>
<td>308.1</td>
</tr>
<tr>
<td>5</td>
<td>379.52</td>
</tr>
</tbody>
</table>

**Figure 2 – PSD of random loading**

### 3.2 Spectrum Analysis

In the process of spectrum analysis, the curve of power spectrum density shown in Figure 2 is loaded on the restricted points of the plate. The displacement and stress can be acquired through the Post-processing. Fig. 3 shows the stress nephogram, with stress probability of 1σ which means that under the given loading, the probability the maximum stress or displacement less than the calculated result is 68%. From Fig. 3, the position with maximum stress is near the restricted point, with $\sigma_x=6.0e6$, $\sigma_y=6.9e6$ and $\sigma_y=1.3e7$. Aimed at this position, the changing relation between stress and frequency is calculated as shown in the curve in Figure 4. Then, spectrum moment and the numbers of slope than pass through average value can be calculated through equation (14) and equation (15). If the random responds under the given loading is assumed narrow random process, fatigue life of the plate could be obtained by taking the $m_0$ and $v^*$ from equation (14) and equation (15) into equation (10). Through the process of calculation, the fatigue life of plate is about 140 hours.

$$m_0 = \int_0^\infty f^2 G(f) df$$

\*(14)\*

$$v^* = (m_2/m_0)^{0.5}$$

\*(15)\*
Compared with the standardized life, the result calculated though the method above is slightly smaller. The most probable reason is that there some difference between numerical modal and real structure. In the real structure, the displacement of the connection position is not zero, which is assumed zero in the numerical modal, so the stress calculated become larger leading to the fatigue life smaller. Also, there are some other reasons such as algorithm in the software and so on. All the same, the calculated result by means above has reference value for engineering.

4. CONCLUSIONS

Based on Miner Liner Method, and relation between stress amplitude $S_a$ and the action times $N$ by the load, the prediction modal of structure’s fatigue life under random vibration is deducted, referencing document. Through software, the stress distribution of plate structure and the power spectrum density of stress in dangerous point are calculated. Taking the result from numerical analysis into the prediction, its fatigue life is calculated and compared with standardized life. The means through numerical analysis and frequency domain method to predict structure’s fatigue life is doable.

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