Vibro-acoustic response of panels excited by a diffuse acoustic field: experimental estimation of sensitivity functions using a reciprocity principle

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ABSTRACT
The experimental characterization of the vibroacoustic response of panels excited by a Diffuse Acoustic Field (DAF) is of great interest, especially for building acoustics and industrial applications. Theoretically, a DAF is usually defined as an infinite number of uncorrelated plane waves with uniformly distributed incidence angles. However, completely diffuse conditions can hardly be reached in practice, and important variabilities are also seen between sound insulation tests performed in different laboratories. To circumvent these issues, a methodology is developed for experimentally characterizing a panel response to a DAF excitation, defined by its theoretical model. Indeed, the formulation of the problem in the wavenumber domain allows estimating the system response at a given point from the DAF cross-spectral density function and so-called “sensitivity functions”. These functions characterize the panel vibroacoustic response to parietal plane waves and can be experimentally determined using a reciprocity principle, which states that they are equal to the panel velocity field expressed in the wavenumber domain when the system is excited by a point source at a point. For validation purposes, the proposed technique is applied numerically and experimentally on a simply supported rectangular plate. The interests and possible applications of this approach are finally discussed.

Keywords: diffuse acoustic field, vibroacoustic response, panel

1. INTRODUCTION
The experimental vibroacoustic characterization of panels submitted to Diffuse Acoustic Field (DAF) is of great interest for the industry. Theoretically, a DAF is defined as an infinite set of uncorrelated plane waves with uniformly distributed incidence angles. For practical applications, the DAF is reproduced using a reverberant room and it rarely corresponds to its theoretical definition, particularly below the Schroeder frequency. Moreover, the absence of grazing incidence waves and the niche effect close to the transmission window tend to affect the acoustic field. It has also been shown in the literature that there is a large variability between measurements in different laboratories.

In this context, the aim of this study is to set up a low-cost and robust test mean to experimentally characterize a panel’s response to DAF excitation by only using its theoretical model to overcome representativity issues of the DAF reproduced in reverberant room. Indeed, the mathematical formulation of the problem in the wavenumber domain allows us estimating the system’s response at point (belonging to the structure or the acoustic media) from wall-pressure Cross Spectrum Density (CSD) functions (characterizing the DAF) and from so-called “sensitivity functions”. The latter are defined as the panel’s response to wall-pressure plane waves expressed in the wavenumber domain and therefore characterize the panel’s vibroacoustic behavior. It is therefore only needed to experimentally determine those sensitivity functions in the acoustic wavenumber domain to characterize the experimental behavior of a panel submitted to a DAF.
In this paper, we propose a method for estimating the sensitivity functions using a reciprocity principle. The latter states that they are equivalent to the panel’s displacement expressed in the wavenumber domain when excited by a unitary source at point \( M \). For validation purposes, this technique is applied on a simply supported plate. The sensitivity functions obtained experimentally are compared to theoretical sensitivity functions. Moreover, we confront the panel’s vibration response to a DAF obtained using the proposed method to the one measured in a reverberant room.

2. VIBRATORY RESPONSE OF PANELS UNDER DAF EXCITATION

2.1 DAF model

A DAF is characterized by its wall-pressure CSD function in the space-frequency domain:

\[
S_{pp}(r, \omega) = S_{pp}(\omega) \frac{\sin(k_0 r)}{k_0 r},
\]

where \( \omega \) is the angular frequency, \( r \) the separation between two points, \( k_0 \) the acoustic wavenumber and \( S_{pp}(\omega) \) the wall-pressure Auto Spectrum Density (ASD) function. Applying a 2D-Fourier Transform (FT) gives us the wall-pressure CSD function in the wavenumber-frequency spectrum:

\[
S_{pp}(k, \omega) = \frac{\pi \frac{S_{pp}(\omega)}{k_0}}{\sqrt{k_0^2 - |k|^2}} \text{ if } |k| < k_0,
\]

\[
0 \text{ if } |k| \geq k_0,
\]

where \( k = k_x e_x + k_y e_y \) is the wavenumber defined in the plane \((e_x, e_y)\).

2.2 Vibratory response and sensitivity function

The theory of random processes allows describing physical quantities using statistical tools such as ASD and CSD functions. The ASD function of the velocity at point \( x = x e_x + y e_y \) of a panel under DAF excitation can be expressed as follows:

\[
S_{vv}(x, \omega) = \frac{1}{4 \pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |H_v(x, k, \omega)|^2 S_{pp}(k, \omega) \, dk,
\]

\( H_v(x, k, \omega) \) is the sensitivity function and characterizes the dynamical behavior of the panel. It can be expressed in the wavenumber domain by:

\[
H_v(x, k, \omega) = \int_S H_v(x, \bar{x}, \omega) e^{i k \bar{x}} \, d\bar{x},
\]

where \( \bar{x} = \tilde{x} e_x + \tilde{y} e_y \) and \( H_v(x, \bar{x}, \omega) \) is the panel velocity response at point \( x \) excited by a normal force applied at point \( \bar{x} \). Two interpretation can be given to the sensitivity function, one that can be directly read from Eq. (4) and the other based on the reciprocity principle.

2.2.1 Direct interpretation

As shown in Fig. 1(a), Eq. (4) can be interpreted as the response of the panel at point \( x \) excited by a wall-pressure plane wave of wave vector \( k \) (i.e. excited by the wall pressure field: \( e^{i k \bar{x}} \)).

2.2.2 Interpretation based on the reciprocity principle

The second interpretation is based on the reciprocity principle which indicates that the ratio of the normal velocity of the panel at point \( x \) over the applied normal force at point \( \bar{x} \) is equal to the ratio of the normal velocity of the panel at point \( \bar{x} \) over the normal force applied at point \( x \):

\[
\left[ \frac{V(x)}{F(x)} \right] = \left[ \frac{m}{s \cdot k g \times m} \right] = \left[ \frac{V(\bar{x})}{F(\bar{x})} \right] = \left[ \frac{m}{s \cdot k g \times m} \right].
\]
Following the previous notation, we obtain
\[ H_v(x, \bar{x}, \omega) = H_v(\bar{x}, x, \omega), \] (6)

Introducing Eq. (6) in Eq. (4) and expressing the sensitivity function in \(-k\) gives us
\[ H_v(x, -k, \omega) = \int_S H_v(\bar{x}, x, \omega) e^{-i k \bar{x}} d\bar{x}. \] (7)

The right hand side of Eq. (7) is interpreted as the 2D-FT of \(H_v(\bar{x}, x, \omega)\). To sum up, the sensitivity function \(H_v(x, -k, \omega)\) is obtained by exciting the panel with a normal force at point \(x\) and by calculating the 2D-FT of the vibratory response of the panel, as illustrated in Fig. 1(b).

\[ H_v(x, k) = v_x \]
\[ H_v(x, -k) = DFT(v_x) \]

Figure 1 – (a) direct interpretation; (b) interpretation based on the reciprocity principle.

2.2.3 Cutoff wavenumber criterion
To estimate the panel’s velocity response from Eq. (3), it is necessary to calculate the generalized integral over the wavenumber space. In practice, the integral is approximated by truncation and discretization in the wavenumber space
\[ S_{vp}(x, \omega) = \frac{1}{4\pi^2} \sum_{k \in \Omega_k} |H_v(x, k, \omega)|^2 S_{pwp}(k, \omega) \delta k, \] (3)

where \( \Omega_k \) and \( \delta k \) are respectively the truncated wavenumber space and the wavenumber resolution. Eq. (3) shows that the sensitivity function filters out the excitation \(S_{pwp}\), which helps determining the limits of the summation. For a DAF excitation, the wall-pressure CSD function is null for wavenumbers greater than the acoustic wavenumber (see Eq. (2)). It is therefore only necessary to sum between \(-k_0\) and \(k_0\) and, consequently, the sensitivity function should only be determined in the acoustic domain.

3. VALIDATION OF THE METHOD
In order to numerically and experimentally validate the method, a test case is considered which consists in a simply supported plane plate submitted to a DAF excitation.

3.1 Description of the study

Figure 2 – Description of the simply supported plate and coordinate system.
The considered panel (see Fig. 2) is a rectangular plate made of aluminum whose geometrical and mechanical properties are detailed in Table 1. The plate is simply supported on its four edges. This type of boundary conditions has been chosen due to its simplicity in modeling. In addition, the experimental setup proposed by Robin⁷ for reproducing these boundary conditions has been validated and proven to be accurate. This will help us for the comparison of numerical and experimental results.

The average structural loss factor has been experimentally estimated using the -3 dB bandwidth method on the first few resonances of the plate, with a mean value of 0.5%.

### Table 1 – Properties of the simply supported aluminum plate.

<table>
<thead>
<tr>
<th>Parameter (Symbol), Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (E), GPa</td>
<td>70</td>
</tr>
<tr>
<td>Poisson’s ratio (ν)</td>
<td>0.3</td>
</tr>
<tr>
<td>Mass density (ρ), kg/m³</td>
<td>2740</td>
</tr>
<tr>
<td>Length (Lₓ), mm</td>
<td>480</td>
</tr>
<tr>
<td>Width (Lᵧ), mm</td>
<td>420</td>
</tr>
<tr>
<td>Thickness (h), mm</td>
<td>3.17</td>
</tr>
</tbody>
</table>

In this study, we will calculate the velocity response of the plate to a DAF at point M of coordinates (x = 0.06 m, y = 0.3 m), as illustrated in Fig. 2. According to the reciprocity principle described in §2.2.2, the normal force is applied at point M. A uniform mesh of (Nₓ, Nᵧ) = (37,27) points in both directions eₓ and eᵧ respectively is defined and a gap of 1 cm to the edges of the plate is left, leading to a spatial resolution of δₓ = 12.8 mm and δᵧ = 15.4 mm. The plate’s velocity is measured on the entire grid in a frequency range of [170,2000 Hz] with a frequency resolution of 0.625 Hz. At 2000 Hz \( k₀ = 37 \text{ m}^{-1} \), which will be the highest wavenumber of interest for evaluation of the sensitivity functions.

A 2D-FT of the vibration field needs to be performed for estimating the sensitivity function. According to the theory of the discrete FT, the highest wavenumber \( k_{x,y}^{\text{max}} \) and the wavenumber resolution \( δk_{x,y} \) in both directions eₓ and eᵧ can be calculated as follows

\[
k_{x,y}^{\text{max}} = \frac{\pi}{\delta x} \approx 245 \text{ m}^{-1}; \quad k_{y}^{\text{max}} = \frac{\pi}{\delta y} \approx 204 \text{ m}^{-1},
\]

\[
δk_{x} = \frac{2\pi}{L_{x}} \approx 13 \text{ m}^{-1}; \quad δk_{y} = \frac{2\pi}{L_{y}} \approx 15 \text{ m}^{-1}.
\]

To improve the wavenumber resolution the “zero-padding” method is used in order to obtain a wavenumber resolution of 1 m⁻¹ along kₓ and kᵧ.

### 3.2 Numerical application

In this section, a numerical validation of the method based on the reciprocity principle is presented. To this end, both interpretation of the sensitivity function are simulated and compared, which will validate the spatial discretization and serve as reference for the experimental validation. Sensitivity functions obtained from the direct interpretation (§2.2.1) are noted \( H_{v}^{\text{direct}} \) and those determined using the reciprocity principle \( H_{v}^{\text{recip}} \).

Fig. 3 shows sensitivity functions obtained at three different frequencies, the lowest corresponding to the frequency of the (2,1) mode and the two others are off-resonance cases. The circles of radius \( k₀ \) and \( k_{y} \), corresponding to the acoustic and flexural natural wavenumbers respectively, are also indicated in this figure. Results obtained simulating the two methods, direct and reciprocal, are in close agreement. This validates the considered spatial mesh and the use of zero-padding for improving the wavenumber resolution without affecting the results.
Figure 3 – Sensitivity function:
Direct interpretation (on top) and based on reciprocity principle (at the bottom)

(a) $f = 178$ Hz; (b) $f = 600$ Hz; (c) $f = 1710$ Hz

$k_0$  $k_f$

Fig. 4 shows the frequency response of the plate vibration $S_{pp}$ excited by a DAF, with unitary wall-pressure ASD function ($S_{pp}(\omega) = 1$ Pa$^2$), obtained by simulating the direct and reciprocal interpretations to determine the sensitivity functions. As the two curves match perfectly, proving that sensitivity functions are properly determined on the entire frequency range using the reciprocity principle.

Figure 4 – Velocity ASD functions: direct interpretation vs. reciprocity principle.
3.3 Experimental application

To validate experimentally the proposed method, an experiment has been performed at INSA Lyon (France). A simply supported plate has been excited with a shaker at point $M$ (see Fig. 5). The vibratory response of the panel has been measured on a mesh of $N_x \times N_y$ points with a laser vibrometer. The sensitivity functions are then deduced by a 2D-FT.

The sensitivity functions obtained experimentally ($H_{y}^{exp}$) and estimated numerically ($H_{y}^{th}$) are compared in Fig. 6. The experimental results agree well with numerical results, particularly within the acoustic wavenumber circle (represented by a continuous line).

Figure 6 – Experimental setup

1) shaker; 2) simply supported plate.

Figure 6 – Sensitivity function:
Theoretical simulations (on top) and experimental (at the bottom)

(a) $f = 178$ Hz; (b) $f = 600$ Hz; (c) $f = 1710$ Hz

$k_0$ –– $k_f$. 

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The velocity ASD functions obtained with experimental and numerical sensitivity functions 
$S_{pp}(\omega) = 1 \text{Pa}^2$ are presented in Fig. 7 to verify that the latter are well estimated on the whole frequency range. One observes that the velocity ASD function estimated with the experimental sensitivity functions is in good agreement with the numerical results. This validates experimentally the proposed method.

![Figure 7 – ASD velocity functions: theory vs. experimental.](image)

4. COMPARISON WITH REVERBERANT CHAMBER MEASUREMENTS

Finally, the previous results are compared with measurements in a reverberant room. Those measurements were performed at the “Groupe d’Acoustique de l’Université de Sherbrooke” (Canada) using a plate similar to the one used in the previous section (i.e. same dimensions, same material, same mounting). The plate was mounted in an existing niche between coupled anechoic-reverberant rooms (the panel being flush mounted on the reverberant room side). A double-walled structure was also mounted, with mechanical decoupling, to prevent acoustic leaks and flanking paths. A loudspeaker fed via a white noise signal excited the reverberant chamber. A $9 \times 9$ microphone array centered on the plate and positioned 2 mm away from it allowed measuring the wall-pressure ASD function (presented in Fig. 8). A laser vibrometer was finally used to measure the plate’s velocity ASD function.

![Figure 8 – Wall-pressure ASD function measured in reverberant room.](image)

The plate’s velocity ASD function measured with the reverberant room is compared to the one obtained with the proposed method in Fig. 9. The reciprocity method accurately reproduces the plate’s response to DAF excitation up to 800 Hz. Above this frequency, slight differences are noticeable. Resonance peaks appear on one curve and not on the other, which can be explained with the poor representativity of the pressure field in reverberant room (inhomogeneity, lack of grazing incidence.
waves, etc.) and more likely, with the positioning of the measuring point in reverberant chamber (and the point of the applied force for the reciprocity method) whose influence increases with the frequency considering the mode shapes get more complex and local.

Figure 9 – Velocity ASD functions: measurement in reverberant room vs. reciprocity method.

5. CONCLUSIONS

In this paper, a method for experimentally characterizing the panel’s response to a DAF was proposed. This approach is based on the mathematical formulation of the random excitation problem in the wavenumber domain. This one indicates that a panel’s response to a DAF depends on the wall-pressure CSD function of the excitation and on a so-called “sensitivity function”, which can be interpreted as the response of the panel to a wall-pressure plane wave at a given wavenumber.

However, the reciprocity principle allows giving a second interpretation to the sensitivity function and shows that the sensitivity function at point \( P \) (on the panel or in the acoustic media) can be determined by exciting the panel with a unitary source at the same point \( P \) and by expressing the vibration field in the wavenumber domain.

This method has been validated numerically and experimentally on a test case which consists in a simply supported plate submitted to DAF excitation. In the near future, one proposes to apply the reciprocity method when the point \( P \) belongs to the acoustic environment in order to deal with transmission loss problems in the case of panels excited by a DAF.

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