Comprehension of Frequency Response Function from the Viewpoint of Wave Energy Propagation

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ABSTRACT

Frequency Response Function (FRF) is known as primary indicators for vibration design. However, vibrational design based on profound comprehension of the time evolution of FRF has not come into wide use, while steady state FRF are popular in vibration analysis. Then we focused our efforts on the comprehension of the time evolution of FRF as the superposition of waves. Formation process of FRF may be designed by the view point of wave energy propagation, after the comprehension of the mode excitation by the wave energy flow. By three major noise and vibration analysis techniques, such as wave analysis, modal analysis and energy analysis, we can obtain a cross-sectional method to devise structural modification to realize intended shaping of FRF. This paper adopts the ray-trace method which is developed for one-dimensional structures and is used to comprehend the FRF as the superposition of waves. Then, the ray-trace method is combined with throw-off element, and accordingly wave energy flow is tied with time evolution of FRF. Finally, it is shown that intended shaping of FRF is realized by controlling wave energy path by structural modifications.

Keywords: Transmission, Propagation I-INCE Classification of Subjects Number: 43.2.1

1. INTRODUCTION

Frequency Response Function (FRF) is a major indicator for the evaluation of countermeasure for noise and vibration issues. FRF is steady-state response for a unit force excitation, and hence is expressed by superposition of many waves which spread on the structures and are reflected by and transferred through boundaries and discontinuity of the structures. Therefore, our previous paper (1, 2) showed that it is useful to decompose the vibration into the contribution of two types of progressive waves: direct waves from excitation points to evaluation points, and reflection waves from boundaries to evaluation points, to come up with an idea of structural modification to obtain desired FRF. Employing numerical method such as Finite Element method is very powerful to obtain exact FRFs but simple and physical insight cannot be obtained to have better understandings of the coupling between some elements.

One approach to analyze the effect of the coupling is to estimate power transmissions. Hugin (3) showed that a method for calculating the response and transmitted power of bending waves in structures consisting of beams coupled extendedly. This method is useful to obtain a better physical insight for the effect of the coupling. Many papers have also dealt with the power transmission and maximization of the power transmission through junctions (4). However, a more generic derivation is needed to have better understandings of the coupling for complex structures.

This paper is concerned with the comprehension of the time evolution of FRF as the superposition of waves to obtain simple and physical insight of the coupling. As for the tool of wave analysis, the ray-trace method which is developed for wave propagation analysis of one-dimensional structures is adopted (5). The application of the ray trace method allows us to calculate the formation process of

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FRF from the view point of detailed dispersion and scattering characteristics of waves on a structure. Then the comprehension of FRF from the viewpoint of wave energy propagation is also addressed. One of the authors proposed that the baseline of FRF is utilized to understand the time evolution of FRF (6). The baseline of FRF is calculated by throw off element to the each end. Therefore the baseline of FRF is translated to wave propagation amount as the wave propagation between some elements. As wave amplitude in various elements of a structure relates to wave energy propagation, wave energy flow is tied with the time evolution of FRF.

2. WAVE ANALYSIS

This chapter briefly introduces ray tracing approach (5, 7), which is one of matrix approach for wave propagation analysis. It is convenient for wave analysis to use the reflection and transmission coefficients to describe the characteristics of the discontinuities and the end of waveguides. The ray tracing approach can be used to analyze the dynamic responses, that is, wave amplitude of waveguides by using the reflection and transmission coefficients.

It is also described that both FRF and energy transmission ratio are available by the ray tracing approach. Wave amplitude ratio between an excitation point and a response point directly associated with FRF. Moreover, wave amplitude ratio between two elements is directly associated with energy transmission ratio.

2.1 Ray-trace Approach for One-Dimensional Structure

A coupled beam in plane shown in Figure 1 is considered in this section. Longitudinal and bending displacements can be described as the sum of waves travelling in the positive and negative x-directions:

$$u(x,t)=(a_L e^{-jk_L x}+b_L e^{jk_L x})e^{j\omega t}, \tag{1a}$$

$$w(x,t)=(a_P e^{-jk_B x}+b_P e^{jk_B x}+a_N e^{-k_B x}+b_N e^{k_B x})e^{j\omega t}. \tag{1b}$$

In these expressions, a_L and b_L are the complex amplitudes of longitudinal waves propagating in positive and negative x-directions respectively, a_P and b_P are the complex amplitudes of bending propagating waves in positive and negative x-directions respectively and a_N and b_N are the complex amplitudes of bending near field waves in positive and negative x-directions respectively. Here j is the imaginary unit, ω is a circular frequency, k_L is a longitudinal wavenumber such that:

$$k_L=\sqrt{\frac{\rho\omega^2}{E}}, \tag{2}$$

and k_B is a bending wavenumber such that:

$$k_B=\sqrt[4]{\frac{\rho A\omega^2}{EI}}, \tag{3}$$

where A is cross-sectional area, I is second moment of area and E is Young's modulus.

Ray-trace model of the coupled beam shown in Figure 1 is given by

$$\begin{Bmatrix} \boldsymbol{a}'_1 \\ \boldsymbol{b}'_1 \\ \boldsymbol{a}'_2 \\ \boldsymbol{b}'_2 \\ \boldsymbol{a}'_3 \\ \boldsymbol{b}'_3 \end{Bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{r}_A\boldsymbol{G}_{b1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{r}_{12}\boldsymbol{G}_{a1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{t}_{21}\boldsymbol{G}_{b2} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{t}_{12}\boldsymbol{G}_{a1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{r}_{21}\boldsymbol{G}_{b2} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{r}_{23}\boldsymbol{G}_{a2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{t}_{32}\boldsymbol{G}_{b3} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{t}_{23}\boldsymbol{G}_{a2} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{r}_{32}\boldsymbol{G}_{b3} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{r}_D\boldsymbol{G}_{a3} & \boldsymbol{0} \end{bmatrix} \cdot \begin{Bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{b}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{b}_2 \\ \boldsymbol{a}_3 \\ \boldsymbol{b}_3 \end{Bmatrix}, \tag{4}$$

where, $\boldsymbol{r}_{12}$, $\boldsymbol{t}_{12}$, $\boldsymbol{r}_{21}$, $\boldsymbol{t}_{21}$, $\boldsymbol{r}_{23}$, $\boldsymbol{t}_{23}$, $\boldsymbol{r}_{32}$, $\boldsymbol{t}_{32}$ are the reflection and transmission coefficient matrices at each discontinuity. The first subscript indicates element number of the incident side, the second represents the element number of the reflected or transmitted side. Here, $\boldsymbol{G}_{a1}$, $\boldsymbol{G}_{a2}$, $\boldsymbol{G}_{a3}$, $\boldsymbol{G}_{b1}$, $\boldsymbol{G}_{b2}$, $\boldsymbol{G}_{b3}$ are the dispersion matrices, The first subscript indicates positive going or negative going waves, the second represents the element number. Wave amplitude vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ represent a set of wave amplitudes consisting of longitudinal waves, bending propagating and near field waves: $\boldsymbol{a}=\{a_L, a_P, a_N\}$ and $\boldsymbol{b}=\{b_L, b_P, b_N\}$. The subscript of the wave amplitude vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ indicates the element number. This model shows that the new wave amplitude vector $\boldsymbol{\alpha}' = \{\boldsymbol{a}_1', \boldsymbol{b}_1', \boldsymbol{a}_2', \boldsymbol{b}_2', \boldsymbol{a}_3', \boldsymbol{b}_3'\}^T$ can be given as

$$\boldsymbol{\alpha}' = \boldsymbol{T}(\omega)\boldsymbol{\alpha}, \tag{5}$$

after transmission has taken place.

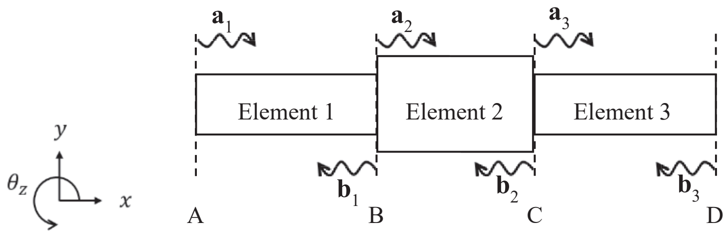

Figure 1 – A coupled beam with two discontinuities

2.2 Calculation of Frequency Response Function by Ray-Tracing

FRF is described by the ray trace model. After repeated propagation, waves are superposed to form FRF. The change of wave amplitude after one ray trace is calculated by Equation (5). Then the wave amplitudes after the next ray trace is also calculated by Equation (5). After an infinite number of ray traces, final set of wave amplitudes vector $\boldsymbol{\alpha}$ is given by

$$\boldsymbol{\alpha} = (\boldsymbol{I} - \boldsymbol{T})^{-1}\boldsymbol{\alpha}_0, \tag{6}$$

where, $\boldsymbol{\alpha}_0$ is the initial wave amplitude vector on excitation point. For example, the initial longitudinal wave amplitude vector $\boldsymbol{\alpha}_{0\mathrm{L}}$ for the case that the first element shown in Figure 1 is excited by a harmonic force $f = Fe^{j\omega t}$ is represented by

$$\boldsymbol{\alpha}_{0L} = \frac{F}{E \cdot A \cdot k_{\mathrm{L}}} \cdot \begin{bmatrix} -j & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \tag{7}$$

and the initial bending wave amplitude vector $\boldsymbol{\alpha}_{0\mathrm{B}}$ is represented in the same manner by

$$\boldsymbol{\alpha}_{0B} = \frac{F}{2 \cdot E \cdot I \cdot k_{\mathrm{B}}} \cdot \begin{bmatrix} -(1-j) & -(1-j) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}. \tag{8}$$

In the calculation of FRF based on ray-tracing approach, the displacement with respect to any evaluation point is calculated by the sum the wave amplitude with respect to that point, represented by $\boldsymbol{\alpha}$ in Equation (6).

2.3 Wave Propagation and Energy Propagation

In this section, the calculation method of wave amplitude ratio between two points on different elements, hereinafter referred to as "wave propagation amount", is introduced by using ray-trace model under the semi-infinite boundary condition shown in Figure 2. Then, the calculation method of energy transmission coefficient is also introduced in the same manner.

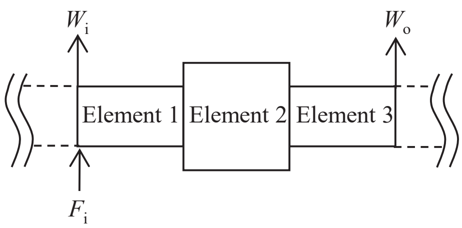

Figure 2 – Semi-infinite boundary condition

Considering a semi-infinite boundary conditions $\boldsymbol{r}_{\mathrm{A}} = \boldsymbol{0}$, $\boldsymbol{r}_{\mathrm{D}} = \boldsymbol{0}$, wave amplitude vectors $\boldsymbol{a}_1$ and $\boldsymbol{b}_3$ for steady state response no longer exist in Equation (4). Then the unknown wave amplitude vectors $\boldsymbol{a}_2$, $\boldsymbol{a}_3$, $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ can be determined by

$$\begin{Bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{b}_2 \\ \boldsymbol{a}_3 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{r}_{12}\boldsymbol{G}_{a1} \\ \boldsymbol{t}_{12}\boldsymbol{G}_{a1} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{Bmatrix} \boldsymbol{a}_1 + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{t}_{21}\boldsymbol{G}_{b2} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{r}_{21}\boldsymbol{G}_{b2} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{r}_{23}\boldsymbol{G}_{a2} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{t}_{23}\boldsymbol{G}_{a2} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \cdot \begin{Bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{b}_2 \\ \boldsymbol{a}_3 \end{Bmatrix}. \tag{9}$$

This equation is utilized to calculate wave propagation amount. Consider a situation that a positive going bending wave only is incident on the cross section B in Figure 1. Amplitude of reflected and transmitted waves for each element can be computed by

$$\begin{Bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{b}_2 \\ \boldsymbol{a}_3 \end{Bmatrix} = (\boldsymbol{I} - \boldsymbol{T}')^{-1} \boldsymbol{t}_{ab1} a_{b1} \,. \tag{10}$$

The ratio of the amplitude of transmitted bending propagation wave a_{b3} and the incident bending propagation wave a_{b1} can be obtained by this calculation such that:

$$V \equiv \frac{a_{b3}(\omega)}{a_{b1}(\omega)} \,. \tag{11}$$

This amount V relates to the wave amplitude ratio between two points on different elements. The amount is, therefore, referred to as wave propagation amount for unit wave amplitude a_{L1}=1 in this paper.

Then, the relationship between wave propagation amount and Coupling Loss Factor (CLF) of Analytical Statistical Energy Analysis (ASEA) is clarified. Energy transmission coefficient at discontinuity in the steady state of semi-infinite rod is given by Cremer et al. (8). An energy transmission coefficient of a semi-infinite rod is given by wave amplitude as follows

$$T_{inf} = \frac{\rho_2 A_2 (a_{b3}^2)}{\rho_1 A_1 (a_{b1}^2)} \,. \tag{12}$$

This equation shows that energy transmission coefficient is given by the cross sectional information and the square of a wave propagation amount. Then a CLF used in ASEA is given by

$$\eta_{13} = \frac{c_1}{2\omega L_1} \frac{\rho_2 A_2 (a_{b3}^2)}{\rho_1 A_1 (a_{b1}^2)} \,. \tag{13}$$

This equation indicates a CLF is also calculated by using a wave propagation amount. It means that there is a relationship between a wave propagation amount and a CLF of ASEA

3. FORMATION PROCESS OF FREQUENCY REAPONCE FUNCTION

Formation of FRF is addressed by the viewpoint of wave propagation and energy flow. As a first step of FRF formation, baseline of an FRF is formed by the waves, which reached on an evaluation point from an excitation point, under semi-infinite boundary conditions. The baseline is, therefore, equivalent to wave amplitude on the evaluation point per unit force on the excitation point. Then, for certain kind of boundary condition, the superposition of transmitted wave and the reflected wave is started. Eventually, resonance peaks of FRF are formed in a steady state. It is clarified that baseline of FRF is related to wave propagation amount, and accordingly baseline of FRF is also related to energy propagation.

To comprehend the relationship between baseline of FRF and energy propagation, it is interesting to note that CLF in ASEA have the energy transmission coefficient in the equation. Since CLF is a value that indicates the energy propagation between the coupling systems, the relationship between CLF and wave propagation amount mentioned above indicates that the baseline of FRF also relates to energy propagation. This relationship can be schematically shown in Figure 3. At the first step of the comprehension of FRF formation, a ray-trace model is utilized to calculate “wave amplitude amount” under semi-infinite conditions. The “wave amplitude amount” is related to “energy propagation”. At the second step, “baseline of FRF” is calculated by using both the “wave amplitude amounts” and “a driving point FRF”. The “baseline of FRF” relates to “CLF” of ASEA.

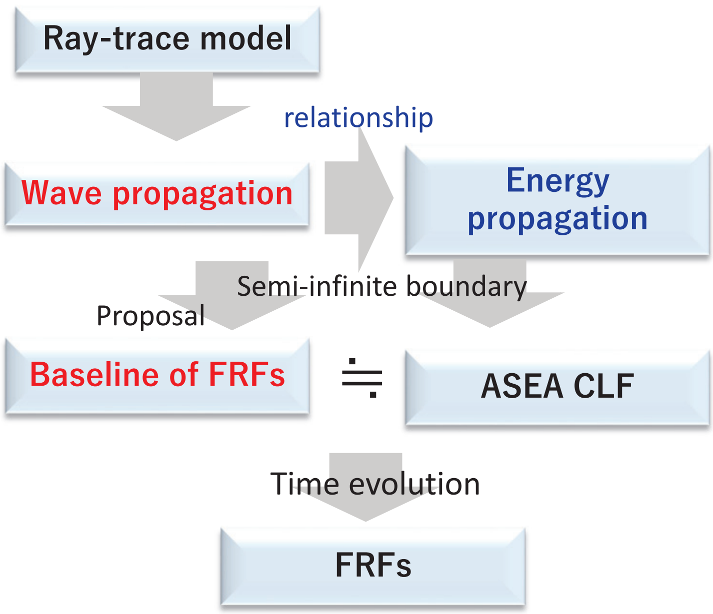

Figure 3 – Relationship between FRF and Energy flow

3.1 Calculation of Baseline of Frequency Response Function

As a case study, FRFs are calculated for two boundary conditions: under semi-infinite boundary conditions and under free boundary conditions. Analysis object in this section is shown in Figure 4. The FRF under semi-infinite boundary corresponds to the baseline of FRF.

Figure 5 shows the relationship between a conventional FRF and a baseline of FRF. Red solid line indicates the magnitude of the baseline of FRF under semi-infinite boundary conditions and blue solid line indicates the magnitude of the FRF under free boundary conditions. The baseline of FRF matches nearly exactly with skirt portions of the FRF. The FRF represents the response in case that there are reflected waves from the boundary. By the wave superposition, the FRF becomes generally larger than the baseline of FRF, and forms resonance peaks in some cases. For the case of the phase of the sum of reflection waves, the evaluation point is in anti-phase to the baseline of FRF and the amplitude of the sum of reflection waves is smaller than the magnitude of the baseline of FRF, there may be cases where the magnitude of FRF is smaller than the magnitude of the baseline of FRF. Thus, formation process of FRF is much influenced by wave propagation amount from the excitation points to the evaluation points. This wave propagation amount also relates to energy propagation, and accordingly formation process of FRF is also much influenced by energy flow.

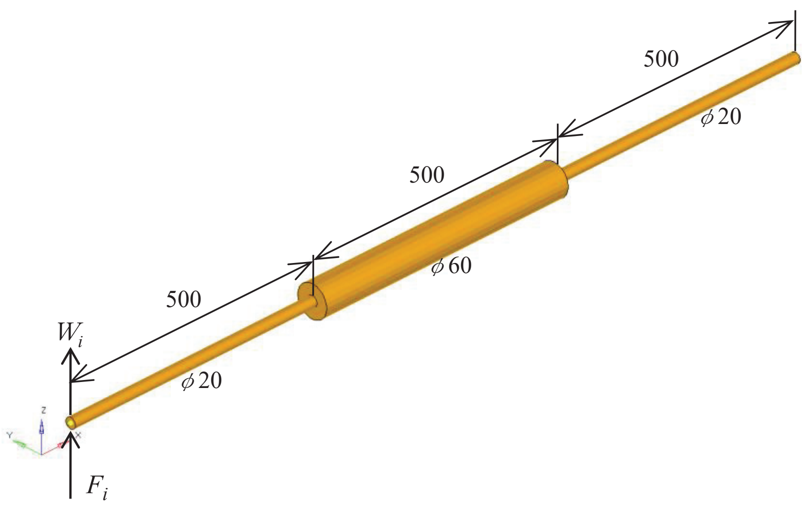

Figure 4 – Analysis object

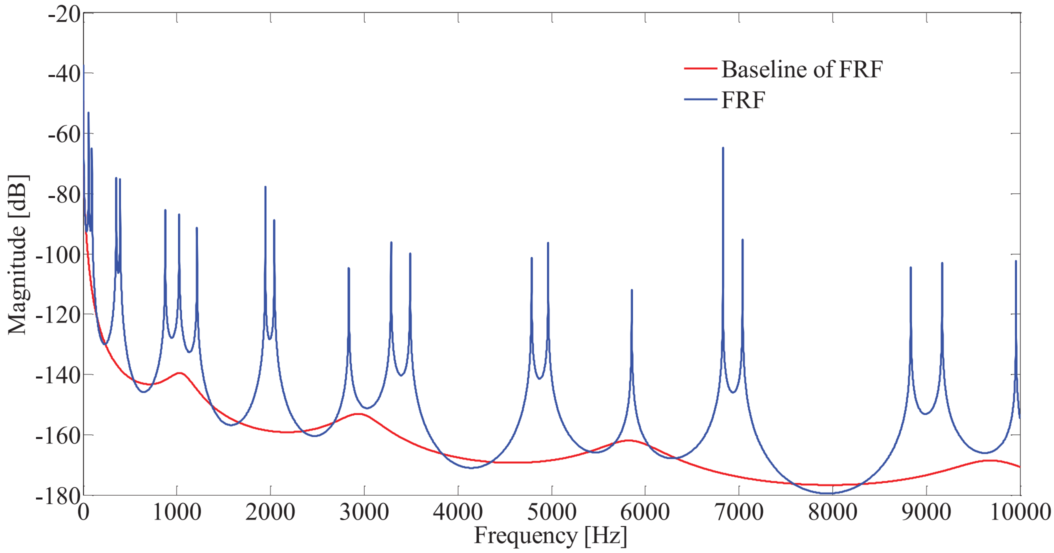

Figure 5 – Relationship between a FRF and a baseline of FRF

3.2 Relationship between Wave Propagation and Energy Propagation

This section describes both the physical meanings of wave propagation amount and the relationship between wave propagation amount and energy transmission coefficient. Wave propagation amount is obtained with the semi-infinite boundary conditions, and represented by the ratio of the wave amplitude between an evaluation point and an excitation point, as shown in Equation (11). A Baseline of FRF is, therefore, calculated by multiplying the wave propagation amount a_3/a_1 and the driving point FRF a_1/F which is limited to positive going waves as follow

$$H_{13}(\omega)=\frac{a_1(\omega)}{F_1(\omega)}\cdot\frac{a_3(\omega)}{a_1(\omega)}, \tag{14}$$

Here, above-mentioned driving point FRF for bending direction is shown in Figure 6 for the analysis object shown in Figure 4. Then the above-mentioned wave propagation amount for bending direction is shown in Figure 7 by red solid line. Further, energy transmission coefficient shown in Equation (12) can be calculated by the wave propagation amount, which is picked up from ray-tracing. In Figure 7, energy transmission coefficient is also shown by blue solid line. As shown in Figure 7, the wave propagation amount is reduced at a particular frequency band. In addition, the same is true for energy transmission coefficient.

Baseline of FRF is calculated by multiplying above-mentioned driving point FRF shown in Figure 6 and wave propagation amount shown in red solid line in Figure 7. Resultant baseline of FRF is compared with the driving point FRF (Figure 8). Baseline of FRF is reduced about 10 dB from the driving point FRF. This is because the maximum diminution of the wave propagation amount in Figure 7 is approximately 10 dB. As the red solid line in Figure 5 is strictly the same as the red solid line in Figure 8, both formation process of FRF and resultant FRF (black solid line in Figure 5) is influenced by the wave propagation amount and energy transmission coefficient.

On the other hand, as described in Section 2, CLF in ASEA is represented by the energy transmission coefficient and the characteristics of the structure. As CLF is generally a parameter that indicates the energy flow, it is calculated from wave propagation amount. From the viewpoint of energy propagation, baseline of FRF is also indicate the energy flow between some elements.

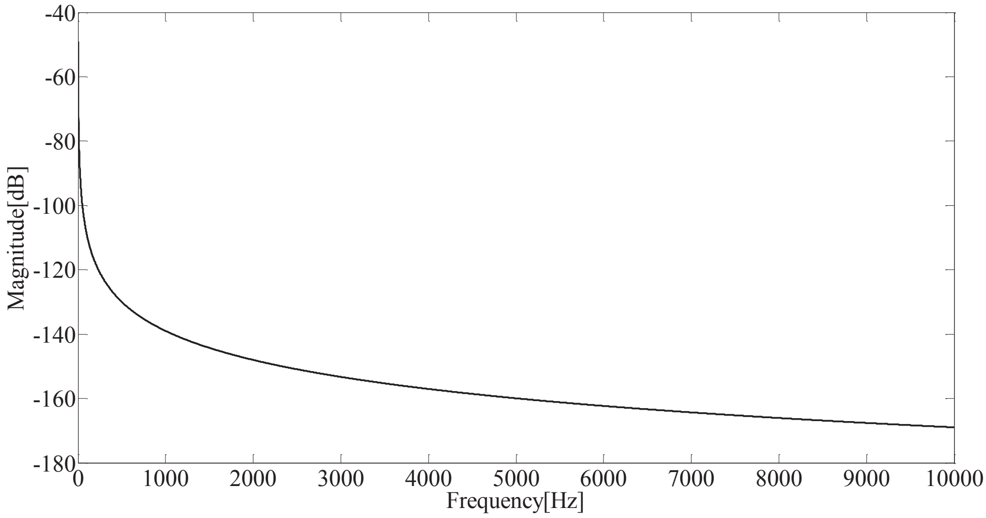

Figure 6 – Driving point FRF limited to positive going bending wave

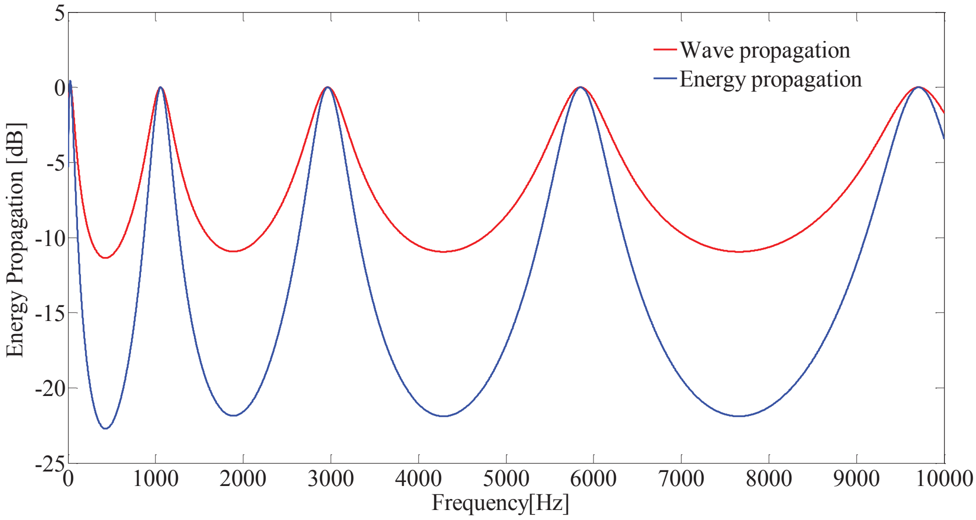

Figure 7 –Wave propagation amount and energy propagation for bending direction

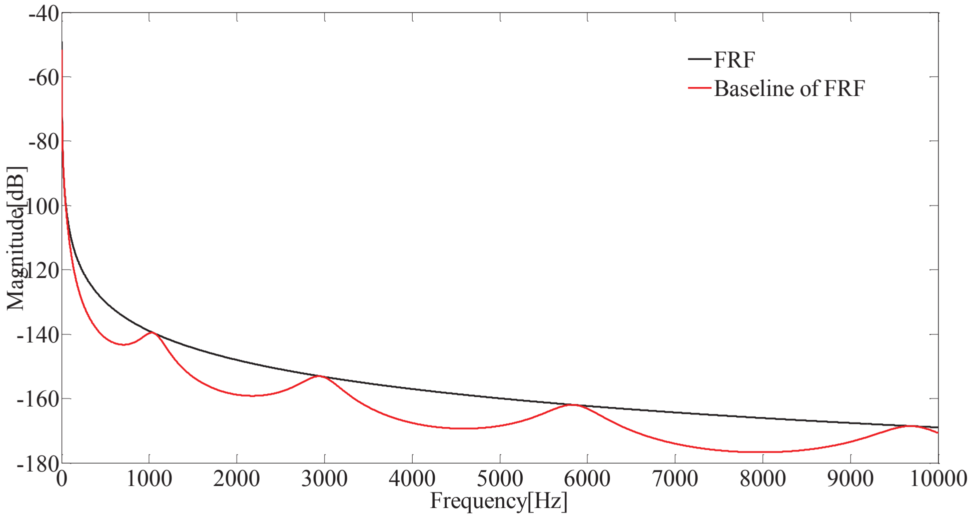

Figure 8 – Comparison between a baseline of FRF and the driving point FRF shown in Figure 6

4. CONCLUSION

This paper concerns with the formation process and time evolution of FRF through wave superposition. To obtain a more generic derivation to have better understandings of the coupling for complex structures, ray-tracing approach was adopted. The ray tracing approach allows us to calculate the formation process of FRF from the view point of detailed dispersion and scattering characteristics of waves on a structure. Then the comprehension of FRF formation, from the viewpoint of wave energy propagation, was also addressed. Baseline of FRF was utilized to unite wave analysis, modal analysis and energy analysis. As wave amplitude in various elements of a structure relates to wave energy propagation, wave energy flow was tied with the time evolution of FRF.

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