Do wavelet filters provide more accurate estimates of reverberation times at low frequencies.

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ABSTRACT

It has been amply demonstrated in the literature that it is not possible to measure acoustic decays without significant errors for low BT values (narrow filters and or low reverberation times). Recently, it has been shown how the main source of distortion in the time envelope of the acoustic decay is the frequency dependent group delay of the common implementations of the 1/3 and 1/1 octave filters. Some authors report good results using wavelet filter banks as an alternative to the usual filters. In this paper, a critical review of the performance of wavelet filter banks is undertaken. A filter bank using the continuous wavelet transform (CTW) has been implemented using a Morlet mother function. Although in general, the wavelet filter bank performs better than the usual filters, the influence of decaying modes outside the filter bandwidth on the measurements has been detected, leading to a biased estimation of the reverberation time in the frequency band of interest.

Keywords: Reverberation Time, Acoustic decay, Wavelets, Continuous Wavelet Transform.

1. INTRODUCTION

In the last few years, the need of measurements of acoustic decays at low frequencies is increasing. Some applications require measurement of reverberation time to obtain a measurand value which is a function of the decay time: this is the case of acoustic insulation measurements [1] or structural loss factors. Other applications need the measurement of acoustic decays directly, also at low frequencies. As an example, the paper published by Torres-Guijarro [2] in 2008: it can be seen how the reverberation time in control and listening rooms is measured below 100 Hz. The recommendations given in some documents [3, 4] consider very short reverberation times for such rooms: acoustic decay times in the order of 200 ms are reported. As Jacobsen [5] reported, measurements of acoustic decays for low BT products cannot be carried out without large errors.

The standard ISO 3382, parts 1 and 2, includes several equations for calculating the standard deviation of measurement of the reverberation time in rooms, based on papers published by Davy in 1979 and 1980 [6, 7]. Later, in 1988, Davy proposed some empirical corrections to these expressions for low frequency measurements [8]. The corrections were needed because of discrepancies observed in real measurements compared with the analytical expressions obtained during his previous research. Davy assumed a smooth magnitude response of the filters and thus his expressions were based on the behaviour of decays of sound in reverberant spaces without taking any influence of the filters into account. One of the reasons for the differences between Davy’s estimation and real measurements could be due to the large errors that narrow filters may introduce in the evaluation of acoustic decays. The filters performance do not affect only to the estimation of the slope of the acoustic decay, but also to the statistical distribution of the measurement of the reverberation time. Sobreira, Cabo and Jacobsen [9] shows that the probability density function of the measurement of reverberation...
time at low frequencies is not normal. It is clear that the filtering process affects the acoustic decay slope mainly at low frequencies. Different authors have tried different approaches to overcome the limit $BT=4$ to be able to make accurate measurements of reverberation time at low frequencies. Lee [10] has described the use of the Continuous Wavelet Transform (CWT) to measure acoustic decays, but he presents a limited analysis of the performance of the wavelets. He uses an ideal decay, containing a single decaying sinusoid at $f=125$ Hz, to check the performance of the wavelet 1/3 octave filter bank he proposes. Lee concludes [11] that a wavelet filter banks performs better at low frequencies (between 63 and 150 Hz) than the usual 1/3 octave band filters.

In this paper we are mainly interested in analysing the possible influence of decaying modes whose resonance frequency lies in some of the adjacent frequency band. First the wavelet filter bank performance at low frequencies is evaluated at the 63 Hz 1/3 octave frequency band, using synthesised acoustic decays. Then, some measurements using impulsive signals (balloons explosions) and a limited band signal have been performed at the listening room of the Acoustic Technology, Dpt. of Electrical Engineering, Technical University of Denmark-DTU, Denmark and at the Heminanechoic Chamber at Vigo University—figure 1.

2. THE CONTINUOUS WAVELET TRANSFORM – CTW.

The word wavelet, which means little wave, comes from the French word "Ondalette". The first mention to this signals was on the Haar’s thesis appendix in 1909 [12]. The Wavelets did not become very important for the researchers until thirty years later, when they realised that these waves were better than the Fourier transform for applications where good time and frequency resolution is needed. For many decades scientists have wanted more appropriate functions than the sines and the cosines, which are basis of Fourier analysis. By their definition, these functions are nonlocal (and stretch out to infinity). They therefore do a very poor job in approximating sharp transients. The fundamental idea behind wavelets is the use of a mother function which is scaled and shifted to follow the changes of the signal under study. Then, wavelet algorithms process the data at different scales or resolutions. If a signal is observed through a large window, only gross time domain features can be noticed, although frequency domain resolution if high. The time domain details, as transients, can be studied with a short window, but there will be a poor frequency resolution. Wavelet analysis tries to have both possibilities: good resolution in time domain and good resolution in frequency domain. This makes wavelets interesting and useful. The wavelet analysis procedure takes a prototype function, called an analysing wavelet or mother wavelet. Temporal analysis is performed with a contracted, high-frequency version of the prototype wavelet, while frequency analysis is performed with a dilated, low-frequency version of the same wavelet. Because the original signal or function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data operations can be performed using just the corresponding wavelet coefficients [13]. Some applied fields are using wavelets include astronomy, acoustics, nuclear engineering, subband coding, signal and image processing, etc.

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(a) Hemiaechoic Chamber (University of Vigo)  (b) Listening Room at DTU

Figure 1: Rooms used to perform the measurements.
The continuous wavelet transform, CWT, of a signal $x(t)$ is defined as [14]:

$$C(a, b, x(t), \Psi(t)) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{a}} \Psi^* \left( \frac{t - b}{a} \right) dt,$$  \hspace{1cm} (1)

where:

- $\Psi$ is the mother function or mother wavelet, and $^*$ denotes the complex conjugate.
- $a, b \in \mathbb{R}$, with $a > 0$ a dilation factor which scales the mother function in frequency domain while $b$ shifts the function in time domain.

In order to discuss the effect of the evaluation of acoustic decays using the CWT, the same mother function as in references [10, 11] has been used. The Morlet wavelet is defined by equation (2) as:

$$\Psi(t) = \frac{1}{B} e^{j\omega_o t - (t^2/B)},$$  \hspace{1cm} (2)

where $\omega_o$ is the center frequency and $B$ is the bandwidth, defined as the variances of the Fourier transform of the mother function:

$$B = \int_{-4}^{+4} f^2 \Psi^*(f) df.$$  \hspace{1cm} (3)

The figure 2 shows the Morlet mother function. The wavelet transform, allows to represent the signal in terms of the scale factors $a$ and the corresponding shifts in time domain $t$. There is a relationship between scale and frequency. A shrunken wavelet, with a small scale factor, is used to follow the rapidly changing details. A dilated wavelet is used to follow the slowly changing parts of the signal (low frequency components). But this is a general approach. When talking about CTW, scale factors are used in stead of frequency.

In the implementation in discrete time of the CTW, there is a relation between the central frequency of the wavelet and the frequency:

$$f = \frac{f_o}{a \cdot T_s},$$  \hspace{1cm} (4)

where the term $\frac{1}{a}$ has dimensions of frequency [14].

3. ANALYSIS OF THE INFLUENCE OF MODES OUTSIDE THE FILTER BAND

In this section, the model defined by Kob and Vörlander and [9, 15] is used to study the performance of the wavelet filter bank when there are modes with long reverberation time whose resonance frequency is outside the band of interest. The equation (5) models the joint acoustic decay of several modes:

$$h_{\text{model}}(t) = \sum_{i=1}^{N} A_i \cos(2\pi f_i t + \phi_i) \exp(-3 \cdot \ln 10 \frac{T_i}{T_s} t).$$  \hspace{1cm} (5)

were:
Figure 3: Case 1: Acoustic decay at the input (top) and the output (bottom) of the filters with central frequencies 63, 80 and 100 Hz. Two decaying modes with resonances $f_1 = 63$ Hz, $f_2 = 80$ Hz and $T_{60}(f_1) = T_{60}(f_2) = 0.4$ s.

- $\phi$ models the phase of the modes. Averaging over $\phi$ has a similar effect as averaging several decays measured at different points of the system under test. During the measurement process of an acoustic decay the squared pressure is observed; therefore the phase of a given normal mode can get any possible value in the interval $[0, \pi]$ with uniform distribution.

- $A_i$ models the amplitude of the modes at the measurement point at $t = 0$.

- The term $\exp\left(-\frac{3\ln(10)}{T_i}t\right)$ models an exponential decay with an attenuation of 60 dB when $t = T_i$ (therefore $T_i$ is the reverberation time associated with a given normal mode with resonance frequency $f_i$). $T_i$ has been assumed constant and it is used to establish the true value of the measurand.

- $f_i$ are the resonant frequencies of the different modes. As we do not have any “a priori” information on the system under test, and as no standard related to the evaluation of acoustic decays requires estimating the resonant frequencies, they are selected randomly, taking random values in the frequency band of interest following a uniform distribution.

- $N$ is the number of modes used to synthesise the acoustic decay.

Two cases have been defined. In both cases, the simulation includes two decaying modes with resonance frequencies at $f_1 = 63$ Hz and $f_2 = 80$ Hz. In the case 1, both modes have the same reverberation time, $T_{60} = 0.4$ s. In the second case, the reverberation times are $T_{60}(f_1) = 0.4$ s and $T_{60}(f_2) = 1.2$ s. The possibility to have of two modes with such a different reverberation time is described by Rindel [16], where he points out that “At low frequencies the reverberation time can vary strongly, being rather long in axial modes and much shorter in tangential and oblique modes. This is further exaggerated if the sound absorption is unevenly distributed on the surfaces, which is normally the case. Note that the frequency 80Hz is clearly outside the 63 Hz 1/3 octave filter band. If the filter magnitude response is ideal, there should be no influence of the decay of the mode at 80 Hz at the output of the 63 Hz 1/3 octave band.

3.1 Case 1:

The figure 3 shows the original signal, obtained applying the model function (5) with $f_1 = 63$ Hz, $f_2 = 80$ Hz and $T_{60} = 0.4$ s for both models. The outputs of the filters with central frequencies 63 Hz, 80 Hz and 100 Hz are also shown. The input shows clearly the beat between the two sinusoidal components of the signal while the output of the filters shows perfect sinusoidal decays. It should be noticed how even though there is not any tonal component at f=100 Hz, the output of the 100 Hz 1/3 octave band filter shows a sinusoidal decay of small amplitude. This reveals the existence of a reverberation tail at 100 Hz even though...
there is not a decaying mode with its resonance within this band.

In this case, as we have pure exponential decays at the output of the filters, the integrated response is linear, as shown in figure 4. The table 1 shows the results of the evaluation of the reverberation time using different dynamic ranges: the EDT, $T_{10}$, $T_{20}$ and $T_{30}$. As it can be seen, all the values tends to be very close to the expected value ($T=0.4$ s). Then, when there are modes outside the frequency band with similar damping (i.e. similar reverberation time), the influence on the evaluation of the reverberation of those modes on the decays of the frequency band of interest cannot be observed and a good estimation of the reverberation time is obtained (as expected).

Table 1: Estimation of reverberation time from the acoustic decays (case 1).

<table>
<thead>
<tr>
<th>Freq. Band</th>
<th>63 Hz</th>
<th>80 Hz</th>
<th>100 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDT</td>
<td>0.3972</td>
<td>0.3993</td>
<td>0.4054</td>
</tr>
<tr>
<td>$T_{10}$</td>
<td>0.4035</td>
<td>0.4005</td>
<td>0.4010</td>
</tr>
<tr>
<td>$T_{20}$</td>
<td>0.3992</td>
<td>0.3992</td>
<td>0.4007</td>
</tr>
<tr>
<td>$T_{30}$</td>
<td>0.3999</td>
<td>0.3999</td>
<td>0.4003</td>
</tr>
</tbody>
</table>

3.2 Case 2:

In this case, an acoustic decay with two modes with resonances at $f_1 = 63$ Hz, $f_2 = 80$ Hz and different reverberation times is synthesised: $T_{60}(f_1)=1.2$ s ;$T_{60}(f_2)=1.2$ s. Figure 5 shows the input and the outputs of the filters. As in case 1, a small amplitude pure acoustic sinusoid is observed at the output of the 100 Hz 1/3 octave filter band. At the output of the 80 Hz filter the acoustic decay of dominant mode of the decay, with longer reverberation time, is observed. It should be noticed that it appears as a pure sinusoid with exponential attenuation as expected. The characteristic beat of the addition of two sinusoids can be noticed at the output of the 63 Hz 1/3 octave band filter, see figure 6, showing that in this case the dominant mode with resonance at 80 Hz is affecting the decay of the mode whose resonance is at 63 Hz. Figure 7 shows in detail the Shroeder integral of the acoustic decay at the output of the 63 Hz 1/3 octave band. Its double slope reveals the influence of the mode with resonance frequency at 80 Hz, although this one is beyond the cutoff frequency of the 63 Hz 1/3 octave band. The table 2 shows the result of the estimation of the reverberation time in this case. It clearly shows how in the 63 Hz band, there is a big influence of the 80 Hz resonance. The bigger dynamic range used to estimate the reverberation time, the bigger the error. It can be observed how the distance between the expected value (0.4 s) and the estimation at the 63 Hz band is bigger than the in the case of the EDT. It could be then suggested that in this cases, the estimation of the reverberation time should be done using short evaluation ranges: EDT or $T_{10}$ should be used in stead of $T_{20}$ of $T_{30}$.
Figure 5: Case 2: Acoustic decay at the input and the output of the filters with central frequencies 63, 80 and 100 Hz. Two decaying modes with resonances $f_1 = 63$ Hz, $f_2 = 80$ Hz and $T_{60}(f_1) = T_{60}(f_2) = 1.2$ s.

Table 2: Estimation of the reverberation time (in seconds) from the acoustic decays (case 2).

<table>
<thead>
<tr>
<th>Freq. Band</th>
<th>63 Hz</th>
<th>80 Hz</th>
<th>100 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDT</td>
<td>0.4529</td>
<td>1.1989</td>
<td>1.2011</td>
</tr>
<tr>
<td>$T_{10}$</td>
<td>0.4791</td>
<td>1.1998</td>
<td>1.2009</td>
</tr>
<tr>
<td>$T_{20}$</td>
<td>0.6228</td>
<td>1.1989</td>
<td>1.2043</td>
</tr>
<tr>
<td>$T_{30}$</td>
<td>0.8569</td>
<td>1.1941</td>
<td>1.2262</td>
</tr>
</tbody>
</table>

Figure 6: Case 2: Signal at the output of the 63 Hz 1/3 octave band filter.

From this analysis we can conclude that even using a wavelet filter bank, there is a noticeable influence of modes with low damping with resonance frequencies outside the frequency band at low frequencies. In order to perform good reverberation time measurements, a band limited signal should be used to avoid the excitation of resonant modes outside the band of interest.
4. MEASUREMENTS

In order to check the results, some measurements of reverberation time have been performed at the listening room at DTU and at a hemianechoic chamber at Vigo University. The measurements have been carried out using both, balloons and tone burst. The figure 8 shows the frequency response measured at one of the measurement points. It should be noticed the high amplitude resonance in the 63 Hz freq. band. This peak suggest a possible influence of his reverberation time on adjacent bands. The figure 9 compares the decay obtained at one of the measurement points at the listening room at DTU: the red lines shows the decay obtained using a tone burst while the blue one shows the decay obtained using balloons. It is then clear, that the double slope of the decay at the 80 Hz is due to the influence of decaying modes outside the 80 Hz band. The table 3 reveals this influence in the 80 Hz. We compare the measurements using a wavelet filter bank with the results obtained using the inverse filtering technique. Inverse filtering (time reversing the signal before filtering) has been shown to be more precise than the conventional direct filtering when the conventional digital Butterworth filters are used [9]. There is no significant differences between the estimation of the reverberation time using the wavelet transform and the results obtained using the band pass filters with the time-reversed filtering. It is clear from the figure 9 that the measurements are more robust using tone bursts: there is not a significant change of the slope of the decay.

Table 3: Results of the estimation of reverberation time at one of the measurement points at the listening room of DTU.

<table>
<thead>
<tr>
<th>Method</th>
<th>Inverse Filtering</th>
<th>Wavelet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>Balloon</td>
<td>Tone Burst</td>
</tr>
<tr>
<td>EDT</td>
<td>1.2918</td>
<td>1.0141</td>
</tr>
<tr>
<td>$T_{10}$</td>
<td>0.9577</td>
<td>1.0831</td>
</tr>
<tr>
<td>$T_{20}$</td>
<td>1.3943</td>
<td>1.1745</td>
</tr>
<tr>
<td>$T_{30}$</td>
<td>1.4241</td>
<td>1.0966</td>
</tr>
</tbody>
</table>
The CTW filter bank allowed to measure a reverberation time as low as 0.08 s at 63 (BT=0.08) at the Hemianechoic chamber at the University of Vigo revealing to be more accurate in the estimation of the EDT in agreement with Lee [11].

5. CONCLUSIONS

Previous jobs on the use of the Continuous Wavelet Transform [10, 11] showed that this technique could be a good alternative to traditional filtering techniques to improve the accuracy of the estimation of reverberation time. At low frequencies, where the modal density is low, modes whose resonance frequency lies outside the band could influence the measurements due to the non-ideal performance of the filters. It has been shown in this paper that:

- In the case of modes with similar reverberation times there is no noticeable influence.
- In the case of modes with different attenuation, the acoustic decays show a double slope, leading to an overestimation of the reverberation time of the bands with more damped modes. In this case, the evaluation using short dynamic ranges \( EDT \) and \( T_{10} \) gives more precise results than using \( T_{20} \) or \( T_{30} \).
- The use of CTW 1/3 octave filter banks is as sensitive as the usual filtering techniques to the influence of modes with their resonance frequencies outside the band of interest.
- It is recommended to use band limited signals, as tone bursts, to avoid the influence of modes with their resonance frequencies outside the band of interest.
REFERENCES


