Experimental study of active sound transmission control into enclosed spaces

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ABSTRACT
The transmission of sound into enclosures, such as an aircraft cabin, generally occurs through thin-walled panels. In order to actively reduce the sound radiation of vibrating structures, far-field sound power is often the quantity to be suppressed. In reality, the interior sound field interacts with the structural vibration and an exchange of energy occurs. To account for the interaction, the enclosure dynamics need to be included into the controller formulation. This study examines different active vibration control (AVC) and active structural-acoustic control (ASAC) concepts by experimental means. A rectangular aluminium plate in contact to an acoustic enclosure is used for a comparative evaluation of control performance. Structural sensors and actuators are used, for the purpose of suppressing the acoustic response inside the enclosure. For studies regarding ASAC, additional radiation filters of interior sound radiation are designed in order to incorporate the enclosure dynamics in the control path. A reduction of sound radiation is estimated by microphone measurements inside the enclosure and compared for the different control concepts.

Keywords: active control, ASAC, fluid-structure interaction

1. INTRODUCTION
A common cost function for the active control into enclosed spaces is the acoustic potential energy (APE). It is defined as the volume average of the squared pressure magnitude. The active control of global interior sound radiation can be achieved by a reduction of the interior radiation modes. The interior radiation modes are orthogonal vibration modes of the structure, such that the contribution to the APE in the fluid from each one is uncoupled from any other.

The interior radiation modes are usually derived from the eigenvectors of the uncoupled systems [1–3]. These studies are limited to analytical or numerical investigations as the uncoupled structural eigenvectors are challenging to obtain in practice. The radiation modes resulting from an orthogonal matrix decomposition often depend on frequency, which poses additional difficulties for the active control implementation. For experimental investigations regarding the sound radiation into enclosed spaces, the active control of local structural error sensors [4–6] as well as local acoustic error sensors [7] have been employed on complex structures. On the other hand, the active control of radiation modes has not been considered in experimental studies. In summary, most research regarding the active control of structural sound radiation into enclosed spaces investigates the reduction of local accelerometer or microphone measurements. Few experimental studies take into account the radiation modes of the structure. To the authors’ knowledge the active reduction of the radiation modes have not yet been compared to accelerometer control. A comparative study of the local as well as global structural control (AVC) to radiation mode control (ASAC) is the aim of this paper.

A rectangular plate coupled to a cuboid cavity is considered for experimental studies regarding the active sound transmission control. The experimental setup is described in section 2. The normal structural velocity is measured using a laser scanning vibrometer (LSV) and appended to an identified model of the control plant. The process of identification as well as synthesis of feedback controllers for three different control concepts are presented in section 3. Section 4 presents the control results for the three control concepts, whereas section 5 concludes the study and outlines future research activities.

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2. EXPERIMENTAL SETUP

The experimental plate-cavity system as well as the measurement equipment and the active elements used throughout this study are presented in this section. An exterior view of the experimental plate-cavity system is shown in figure 1a. The plate is made of 2 mm thick aluminium with the dimensions \((L_x \times L_y) = (0.6 \times 0.8) \text{ m}^2\). The plate edges are fixed in an aluminium frame of 15 mm thickness.

![Plate-cavity system](image)

(a) Exterior view  
(b) Interior view

Figure 1: Experimental plate-cavity system

The hardware components for the active control system as well as additional measurements are listed in table 1. The disturbance is induced by a primary shaker, while two secondary shakers are used for the active control of the sound radiation. During the experimental identification process, broadband uncorrelated signals for the primary and secondary shakers are generated by a dSPACE\textsuperscript{R} rapid control prototyping system. The low-pass filtered accelerometer outputs are measured and the transfer functions estimated. The positions of the primary and secondary shakers as well as the sensors on the plate are summarized in figure 2. For the primary shaker, the position is chosen not to coincide with a nodal line in the frequency range up to 500 Hz, so that all modes of the plate can be excited. The positions of the secondary actuators were chosen taking into account controllability and observability criteria.

<table>
<thead>
<tr>
<th>Device</th>
<th>Type</th>
<th>Additional informations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometers</td>
<td>PCB 352A24</td>
<td>Mass ( m = 0.8 \text{ g} ), grid of ( 6 \times 4 ), distributed equally spaced across the plate</td>
</tr>
<tr>
<td>Primary Shaker</td>
<td>LDS V201</td>
<td>-</td>
</tr>
<tr>
<td>Control Shaker</td>
<td>Visaton EX45</td>
<td>Mass ( m = 60 \text{ g} )</td>
</tr>
<tr>
<td>Real-Time system</td>
<td>dSPACE\textsuperscript{R} DS1006</td>
<td>Sampling frequency ( F_s = 1 \text{ kHz} )</td>
</tr>
<tr>
<td>Low-pass filters</td>
<td>Kemo Card Master 255G</td>
<td>Cut-off frequency ( F_c = 1 \text{ kHz} ), Attenuation 24 dB/Octave</td>
</tr>
<tr>
<td>Laser scanning vibrometer</td>
<td>OFV-055 (scan unit), OFV-3001-S (controller)</td>
<td>Grid of ( 13 \times 15 ) scanning points equally spaced across the plate</td>
</tr>
<tr>
<td>Microphones</td>
<td>PCB T130D21</td>
<td>Grid of ( 4 \times 8 ), shifted in six equally spaced positions along the cavity depth</td>
</tr>
</tbody>
</table>

Figure 1b shows an interior view of the acoustic cavity. It has a depth of \( L_z = 0.42 \text{ m} \). A total of 192 pressure measurements is used for the calculation of the APE. The microphones are not part of the control system, as they are solely used for the evaluation of the APE inside the enclosure.
3. SYSTEM IDENTIFICATION AND CONTROLLER SYNTHESIS

The methods used for the system identification and the controller design are presented in this section. The identification of plant models and their extension with additional velocity outputs as well as radiation filter sensing are introduced and discussed. Based on three different plant models, the applied feedback control synthesis is described. Each plant model is chosen in order to suppress a certain cost function.

3.1 Extended plant model

The synthesis of feedback controllers necessitates accurate models of the plant to be controlled. For this study, a linear time-invariant discrete-time state-space model \( G(z) \) is used, which is described at discrete time steps \( n \in \mathbb{N}_0 \) as

\[
\begin{align*}
    x(n+1) &= Ax(n) + Bu(n) \\
    y(n) &= Cx(n) + Du(n).
\end{align*}
\]

(1a) (1b)

Based on equations (1a) and (1b), the plant model \( G(z) \) can be defined as

\[
G(z) := \begin{bmatrix} A & B \\ C & D \end{bmatrix},
\]

(2)

with the complex variable \( z = \exp(j\omega T) \). Here, \( \exp \) describes the exponential function, \( j = \sqrt{-1} \) the complex unit, \( \omega \) the angular frequency and \( T \) the sampling period. For the control system shown in figure 1a, \( u \in \mathbb{R}^{n_u} \) are the primary and secondary shaker inputs and \( y \in \mathbb{R}^{n_y} \) are the accelerometer outputs. In equations (1a) and (1b), \( x \in \mathbb{R}^{n_x} \) is the state vector. The matrices \( A, B, C \) and \( D \) of the state space model are identified using measured data of \( u \) and \( y \). For the system identification, the subspace identification method is used, the theory of which can be found e.g. in the textbook of Katayama [8]. A comparison of the singular values of the identified plant using 150 states with the corresponding transfer functions is shown in figure 3a. This number of states is sufficient to model the plant dynamics accurately.

For the efficient active control of acoustic quantities, e.g. sound power or potential energy, over a wide frequency range a finer grid of measured data is needed than supplied by accelerometer measurements. In order to include additional velocity outputs in the control model, an extended plant [9] is identified, based on laser scanning vibrometer measurements. The identification process of the extended plant takes place as follows. Every shaker (primary and secondary) is excited individually as the point velocities are measured by the laser scanning vibrometer sequentially. The resulting complex frequency response function (FRF) matrix \( H(z_k) \) links the three actuators to the 195 LSV measurement points. In order to include these points in the plant \( G \), additional state-space matrices \( C_{ext} \) and \( D_{ext} \) are calculated. Fixing the system matrix \( A \) and the input matrix \( B \) of the original plant model, yields the least-squares problem

\[
H(z_k) = \begin{bmatrix} C_{ext} & D_{ext} \end{bmatrix} \begin{bmatrix} [Ez_k - A]^{-1}B \\ E \end{bmatrix}, \text{ for } k = 1,2,...,N,
\]

(3)
where \(E\) is the identity matrix. Equation (3) needs to be solved for the unknown matrices \(C_{ext}\) and \(D_{ext}\) for all \(N\) discrete frequency points. Appending the velocity outputs to the plant in equations (1a) and (1b) results in the extended plant \(G_{ext}(\omega)\)

\[
\begin{bmatrix}
  y_{ext}(n) \\
  y(n)
\end{bmatrix} = \begin{bmatrix} C_{ext} & D_{ext} \\
  C & D
\end{bmatrix} \begin{bmatrix}
  x(n+1) \\
  x(n)
\end{bmatrix} + \begin{bmatrix} A \\
  B
\end{bmatrix} u(n),
\]

with

\[ G_{ext}(\omega) = \begin{bmatrix} A & B \\
  C_{ext} & D_{ext}
\end{bmatrix}. \] (5)

The extended plant \(G_{ext}(\omega)\) allows the real-time estimation of the additional point velocities \(y_{ext}(n)\) by measuring the accelerometer outputs \(y\). In figure 3b, the singular values of the additional part of the extended plant are compared to the singular values of the FRF matrix \(H(\omega_k)\). A reasonable agreement is indicated.

**Figure 3:** Comparison of singular values of identified to the measured plants

### 3.2 Interior radiation filters

The interior radiation modes are to be derived as the structural distributions to the APE. The APE \(E(\omega)\) inside a volume \(V\) is defined as the volume average of the mean squared acoustic pressure \(p\) [10]

\[
E(\omega) = \frac{1}{4\rho c^2} \int_V |p(r, \omega)|^2 dV,
\] (6)

where \(\rho\) and \(c\) are the fluid density and acoustic velocity respectively. These are assumed as \(\rho = 1.204 \frac{kg}{m^3}\) and \(c = 343 \frac{m}{s}\). Here, \(\omega\) is the angular frequency and the coordinate vector \(r = (x, y, z)\) describes points in the acoustic domain \(V\). The acoustic pressure \(p(r, \omega)\) can be expressed as a linear combination of eigenvectors \(\Phi_{l,m,n}(r)\) as

\[
p(r, \omega) = \sum_{l,m,n=1}^{\infty} q_{l,m,n}(\omega) \Phi_{l,m,n}(r)\] (7)

which can be written for the rectangular enclosure as [11]

\[
\Phi_{l,m,n}(x, y, z) = \cos\left(\frac{l\pi x}{L_x}\right) \cos\left(\frac{m\pi y}{L_y}\right) \cos\left(\frac{n\pi z}{L_z}\right).
\] (8)

Here, \(l, m, n \in \mathbb{N}_0\) are the acoustic modal indices in the \(x, y\) and \(z\)-direction respectively. In equation (7), \(q_{l,m,n}(\omega)\) describes the amplitudes of each acoustic eigenvector. The acoustic eigenfrequencies of the rectangular enclosure [11] are calculated as

\[
\omega_{l,m,n} = c\pi \sqrt{\left(\frac{l}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2 + \left(\frac{n}{L_z}\right)^2}.
\] (9)
The APE is to be expressed in terms of the surrounding structural velocity \( v(r_s, \omega) \) of the points \( r_s = (x_s, y_s) \) at the structural surface. The transfer function from the structural velocity to the acoustic pressure \( p(r, \omega) \) from the Kirchhoff-Helmholtz integral equation is therefore considered [10]

\[
p(r, \omega) = j\rho\omega \int_S v(r_s, \omega)G(r|r_s, \omega)dS,
\]

(10)

with the complex unit \( j = \sqrt{-1} \). The function \( v(r_s, \omega) \) is the normal velocity on the structural boundary \( S \) in contact with the fluid and \( G(r|r_s, \omega) \) is the Green’s function [10]

\[
G(r|r_s, \omega) = \sum_{l,m,n=1}^\infty \frac{\Phi_{l,m,n}(r)\Phi_{l,m,n}(r_s)}{\Gamma_{l,m,n}(\kappa_{l,m,n}^2 + 2j\zeta_{l,m,n}\kappa_{l,m,n}k - k^2)}.
\]

(11)

In equation (11), \( k \) describes the frequency-dependent wave number \( k = \frac{\omega}{c} \) and \( \kappa_{l,m,n} \) the resonant wave number of the eigenvector with indices \( l, m, n \). The scalar \( \zeta_{l,m,n} \) describes the modal damping ratio. The volume normalization factors \( \Gamma_{l,m,n} \) according to equation (11) of the rectangular cavity modes are derived according to

\[
\Gamma_{l,m,n} = \int_V \Phi_{l,m,n}^2(r)dV = \frac{L_xL_yL_z}{8}\epsilon_l\epsilon_m\epsilon_n
\]

(12)

with

\[
\epsilon_l = \begin{cases} 2 & l = 0 \\ 1 & \text{otherwise}. \end{cases}
\]

(13)

With the inner product defined at the structural surface \( S \) as

\[
\langle f, g \rangle = \int_S f(r_s)g^*(r_s)dS,
\]

(14)

the APE can be rewritten as

\[
E(\omega) = \sum_{l,m=0}^\infty s_{l,m}(\omega)\left|\langle u_{l,m}, v(\omega) \rangle\right|^2.
\]

(15)

Here, the interior radiation modes \( u_{l,m}(x, y) \) are described as

\[
u_{l,m}(x_s, y_s) = \cos\left(\frac{l\pi x_s}{L_x}\right)\cos\left(\frac{m\pi y_s}{L_y}\right)
\]

(16)

and the radiation efficiencies \( s_{l,m}(\omega) \) as

\[
s_{l,m}(\omega) = \sum_{n=0}^\infty \frac{\rho c^2}{4\Gamma_{l,m,n}} \left|\frac{j\omega}{\omega_{l,m,n}^2 + 2j\zeta_{l,m,n}\omega_{l,m,n} \omega - \omega^2}\right|^2.
\]

(17)

The radiation modes \( u_{l,m}(x_s, y_s) \) can be derived independent of the modal index \( n \) in \( z \)-direction, as it becomes evident from equation (8), that the respective eigenvectors do not change at the structural interface \( (z = 0) \). This unaltered surface distribution is shown for two exemplary eigenvectors in figure 4. The radiation efficiencies of the radiation modes can then be regarded as the sum of efficiencies of the individual acoustic eigenvectors, as shown in figure 5. The radiation modes \( u_{l,m}(x_s, y_s) \) are orthogonal along the surface \( S \) and independent of frequency for the rectangular enclosure.

The modal damping ratios \( \zeta_{l,m,n} \) as well as the angular eigenfrequencies \( \omega_{l,m,n} \) of the fluid modes with indices \( l, m, n \) are identified experimentally. The eigenvectors are assumed to be invariant. Using the identified eigenfrequencies and modal damping ratios, the radiation efficiencies are calculated according to equation (17). A total of five radiation efficiencies dominates the considered frequency range up to 500 Hz, as depicted in figure 6a.

The square root of each radiation efficiency \( \sqrt{s_{l,m}(z)} \) is modelled as a minimum phase filter \( S_l(z) \) by means of the Robust Control Toolbox [12] in Matlab®. The singular values of the identified state-space model with the state-dimension four and the square root of the radiation efficiencies are shown in figure 6b. A good
agreement of the identified model $S_1(z)$ with the square root of the radiation efficiencies is achieved. The radiation filter $r_i(z)$ for radiation mode $i$ used for the control synthesis is calculated as

$$r_i(z) = S_i(z)u_i^T.$$ (18)

Stacking of all radiation filters $r_i(z)$ into a single system yields a rectangular transfer matrix $R(z)$. The 195 velocity outputs of the extended plant are multiplied by $R(z)$, which leads to five outputs, the signal energy of which corresponds to the APE radiated by each radiation mode.

With the five radiation filters, the APE in the enclosure is estimated based on the structural velocity. In order to get a reference for the estimation of the APE, a number of $m = 192$ acoustic pressure values $p$ inside the cavity are measured as described in section 2. The reference APE $E_r(\omega)$ obtained by the measurements is calculated as

$$E_r(\omega) = \frac{L_xL_yL_z}{4\rho c^2m}p^H p.$$ (19)

A comparison of the measurement $E_r(\omega)$ with the FRFs from the primary input to the APE for the case of the radiation filter sensing at the LSV points is shown in figure 7. A qualitative agreement of the coupled system dynamics can be observed in the considered frequency range.

### 3.3 $\mathcal{H}_\infty$ control synthesis

This section describes different control concepts as well as the synthesis of the controller $K$. Figure 8 shows the three considered control concepts. These can be summarised as follows:

- **Control concept $P_1$:** Local vibration control, performance outputs $p$ correspond to accelerometer outputs of the plant $G$.
- **Control concept $P_2$:** Global vibration control, performance outputs $p$ correspond to velocity outputs of the extended plant $G_{ext}$.

![Figure 4: Acoustic eigenvectors, parallel at structural surface](image)

![Figure 5: Radiation efficiencies of acoustic eigenvector $(0 \times 0 \times 0)$ and $(0 \times 0 \times 1)$ as well as radiation mode 1](image)
• Control concept $P_3$: Radiation mode control, performance outputs $p$ correspond to the outputs of the radiation filter $R$.

Subsequently, the frequency-dependence of the plant models is omitted for the sake of brevity. Due to the stochastic nature of the excitation, the feedback controller needs to provide broadband control performance. This is achieved by using a robust control approach [13].

The transfer functions of each plant $P$ in figure 8 can be expressed as

$$\begin{align*}
\{ p, y \} = P \left\{ d, u \right\} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \{ d, u \}. 
\end{align*}$$

In equation (20), $d$ describes the disturbance, $y$ the sensor outputs, $u$ the controller output and $p$ the performance output.

The control objective for robust control is achieved by minimising the $H_\infty$ norm of the transfer function from the disturbance $d$ to the performance output $p$. The aim of the $H_\infty$ control synthesis is to find a controller $K$ that limits the $H_\infty$ norm of the plant $P$ below a value $\gamma$:

$$\| P \|_\infty < \gamma.$$  \hfill (21)

The transfer function $P$ is defined in closed-loop form as

$$P = \begin{bmatrix} (P_{11} - P_{12}KSP_{21}) & P_{12}KS \\ -KSP_{21} & KS \end{bmatrix}$$  \hfill (22)

with the sensitivity $S$

$$S = [E + P_{22}K]^{-1}.$$  \hfill (23)
The term $P_{11} - P_{12}KSP_{21}$ in equation (22) describes the closed loop transfer path from the disturbance $d$ to the performance output $p$. The solution of the synthesis is computed in Matlab® by means of the Robust Control Toolbox [12].

4. CONTROL RESULTS

This section presents the control results of the measured plant models. The closed-loop performance is evaluated in terms of the plate vibration and cavity noise reduction for the three presented control concepts.

A comparison of the overall reduction of the control concepts $P_1$-$P_3$ is shown in table 2. Each given performance is integrated over the frequency bandwidth. The mean acceleration is evaluated for the 24 acceleration sensors, the mean velocity results from the 195 LSV scanning points. The APE is estimated from the radiation filter outputs $p$ according to concept $P_3$ in figure 8c, as a reasonable agreement with microphone measurements was observed in figure 7. The control concepts $P_1$ and $P_2$ achieve similar reductions in terms of mean squared acceleration as well as velocity. The reduction of the APE on the other hand is different for the two concepts and significantly lower compared to control concept $P_3$. The latter one achieves a higher reduction in APE while the overall acceleration remains nearly unaltered. The amount of structural velocity reduced is also less than the other two concepts. This fact suggests, that a reduction in sound radiation is achieved not only through suppression of the structural vibration but through a specific restructuring of the vibration shapes as well. This phenomenon is consistent with the active control of the far field radiated sound power [14].

<table>
<thead>
<tr>
<th>Concept</th>
<th>Mean squared acceleration</th>
<th>Mean squared velocity</th>
<th>Acoustic potential energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>2.82 dB</td>
<td>3.21 dB</td>
<td>1.63 dB</td>
</tr>
<tr>
<td>$P_2$</td>
<td>2.81 dB</td>
<td>3.48 dB</td>
<td>2.47 dB</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.01 dB</td>
<td>0.85 dB</td>
<td>3.43 dB</td>
</tr>
</tbody>
</table>

The FRF from the primary shaker to the mean squared levels of acceleration as well as velocity are shown in figure 9 and figure 10 respectively. While the concepts $P_1$ and $P_2$ result in a broadband reduction of these two measures, the concept $P_3$ leads to an increase in the structural vibration in certain narrowband frequency intervals.
The FRF from the primary shaker to the APE level is depicted in figure 11. As expected, the control concept $P_3$ results in the highest reduction compared to the other two. Especially around the acoustic resonance frequencies (e.g. 212 Hz) $P_1$ and $P_2$ achieve only negligible reductions. Compared to figure 10, at this frequency an increase in the overall structural vibration can be observed with control concept $P_3$. At 39 Hz, the concept $P_3$ achieves a slightly lesser reduction in APE than concept $P_2$. At this frequency, the structural, fundamental (1, 1) mode is resonant and a modal reduction of this mode performs equal to the radiation mode control. A modal restructuring can not be realised with concept $P_3$, because of the predominant coupling of this structural mode to the acoustic (0, 0, 0) eigenvector in the low frequency regime.
5. CONCLUSION

The active sound transmission control into an enclosed space is investigated using an experimental coupled plate-fluid system. A comparative study of different control concepts with structural sensing was conducted. Each control concept showed the highest performance with respect to the quantity it was designed to suppress. The active control of radiation mode sensing was shown to be most successful in reducing the APE in the cavity.

Future work regarding the active control of interior sound radiation will include more realistic excitations as well as increasingly complex structure-cavity systems. E.g. the disturbance path cannot be included into the plant model, when an excitation of a turbulent boundary layer is assumed. Additionally, structural stiffeners as well as curvature effects need to be taken into consideration in the case of more complex structures.

REFERENCES


