High-Resolution DOA Estimation in the underwater radiated noise based on Sparse Bayesian Learning

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ABSTRACT
Wideband direction-of-arrival (DOA) estimation is a practical issue frequently occurring in passive sonar application. It is considered the fact that the targets only occupy a few directions, and it would be sparse in the entire angular domain. The DOA estimation problem is reformulated to be sparse reconstruction in the sparse Bayesian learning framework. It is further noted that signals in different spectrum bands show a strong joint group sparsity and correlation across subbands. In this paper, a novel DOA estimation method is proposed by jointly exploiting the group sparsity and correlation structure. Compared to the conventional sum-delay beamformer, the proposed method has the advantage of increased resolution. The simulation results verify the superiority over the existed CS based DOA estimation method.
Keywords: Direction-of-arrival (DOA) estimation, sparse Bayesian learning, radiated noise

1. INTRODUCTION

Direction-of-arrival (DOA) estimation of multiple sources close to each other is an important issue in passive sonar applications. In the underwater environment, the source signals radiated by ocean vessels, such as ships and submarines, at different directions have a wide frequency spectrum ranging from a few dozens of hertz to several kilo hertz. The number of methods have been proposed to estimate their DOAs, such as incoherent signal subspace method (ISSM) [1] and coherent signal subspace method (CSSM) [2]. Most of those methods decompose the incident wideband signal into narrowband components, and then realize wideband DOA estimation with incoherent or coherent techniques. The coherent spectral focusing-based methods, such as Weighted Average of Signal Subspaces (WAVES) method [3], Test of Orthogonality of Projected Subspaces (TOPS) [4], were demonstrated to surpass their incoherent counterparts [2], and thus have attracted more research interest. However, there are two significant disadvantages within this kind of methods. For example, DOA pre-estimates of the incident signal are required for spectral focusing, and this precision of pre-estimate incident has much effect on the DOA estimation performance. On the other hand, the prior information of the incident signal number is requisite, due to construction of the signal subspace, but it may not be available, especially in the noncooperative scenarios. For these reason, the delay-sum beamformer [5] is sometimes preferred in practical sonar applications. However, this method is susceptible to spatial aliasing and render poor resolution, which is confined to the Rayleigh resolution [5].

The technique of sparse representation (SR) provides a new perspective for DOA estimation, due to the spatial sparsity that targets of interest only occupy a few directions in the entire angular domain [6–11]. Methods of this category recover the spatial distribution of the incident signals by directly representing the array output on an over-complete dictionary under sparsity constraint. Several literatures, such as basis pursuit denoising (BPDN) [6], orthogonal matching pursuit (OMP) [7] and l_1-SVD [8], have directly addressed sparsity-based DOA estimation. However, the BPDN method has to choose a proper regularization parameter to balance the noise level and sparsity of the signal, which is difficult in practice, and the prior knowledge of the target number is requisite in both the OMP method and the l_1-SVD method. Several improved SR-based methods were proposed to acquire better estimation performance by exploiting the group sparsity that the signals in different targets from different directions share the same spectrum [9, 10].

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In this paper, a novel wideband DOA estimation method is proposed by jointly exploiting group sparsity and their correlations of Fourier coefficients in the framework of sparse Bayesian learning (SBL). In the proposed method, we firstly divide wideband signal into multiple narrow sub-bands by the Fourier Transform (FT), and exploit the fact that the spatial group sparsity is shared across each subband to improve the DOA estimation method. Furthermore, since the cavitations noise, which has continuous spectrum, is an important component in the radiated noise [12], a Toeplitz matrix based on the auto-aggressive model is introduced to model and learn the correlation of these Fourier coefficients. The proposed method is capability of automatically learning the spatial sparsity of the signal and avoiding the requirement of the prior knowledge. Simulation results verify the effectiveness of the proposed method.

Notations: We use lower-case (upper-case) bold characters to denote vectors (matrices). $p(\cdot)$ denotes the probability density function (pdf) and $p(x|-)$ denotes the conditional pdf of random variable x, given other parameters. $\mathcal{CN}(x|a,b)$ denotes that random variable x follows a complex Gaussian distribution with mean a and variance b. In addition, $(\cdot)^T$ denotes transpose, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix, and $\circ$ denotes element-wise (Hadamard) multiplication. $\bigotimes$ denotes the Kronecker product. $\mathrm{Tr}(\cdot)$ is the trace of a matrix.

2. DATA MODEL FOR UNIFORM LINEAR HYDROPHONE ARRAY

Supposing a uniform linear array consisting of M hydrophone sensors, with inter-element spacing d, that receives underwater acoustic noises radiated by K sources. The received signal $y_m(t)$ in the mth sensor can be expressed as

$$y_m(t) = \sum_{k=1}^{K} s_k(t - \tau_{mk}) + e_m(t), \tag{1}$$

where $e_m(t)$ is an additive noise. Noted that the $s_k(t)$ is a noise signal radiated by the kth source, which contains machinery noise, propelled noise and flow noise in a passive sonar system. Therefore, it is a typical wideband signal. τ_{mk}, which represents the time delay from the kth source to the mth hydrophone sensor, can be written by

$$\tau_{mk} = \frac{d_m \cos\theta_k}{v}, \tag{2}$$

where $d_m = (m-1)d$ is the location of the mth sensor, when the first sensor is regarded as the reference. θ_k is the direction of the kth wideband source and v is the speed of the acoustic wave.

The observed signal $y_m(t)$ at the mth sensor is decomposed into N narrowband components using Discrete Fourier Transform (DFT). The resulting narrowband components corresponding to any one of the sub-intervals may be expressed as,

$$y_m(f_n) = \sum_{k=1}^{K} s_k(f_n)\exp(-j2\pi f_n \tau_{mk}) + e_m(f_n), \quad n \in (1, \cdots, N) \tag{3}$$

where $f_n = nF_s/N$ with the sampling rate F_s. Arranging M narrowband signals with the frequency of f_n, we have

$$\mathbf{y}(f_n) = \mathbf{\Phi}(f_n)\mathbf{s}(f_n) + \mathbf{e}(f_n), \tag{4}$$

where $\mathbf{y}(f_n) = [y_1(f_n), \cdots, y_M(f_n)]^T \in \mathcal{C}^M$ is a received signal vector in the nth subband, $\mathbf{s}(f_n) = [s_1(f_n), \cdots, s_K(f_n)]^T \in C^K$ is a coefficient vector in the nth subband from the kth source, and $\mathbf{e}(f_n) = [e_1(f_n), \cdots, e_M(f_n)]^T \in C^M$ is an additive noise vector. $\mathbf{\Phi}(f_n) \in \mathcal{C}^{M\times K}$ can be expressed by

$$\mathbf{\Phi} = [\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_K] \tag{5}$$

$$\boldsymbol{\phi}_k = \left[\exp\left(-j2\pi f_n d_1 \cos\theta_k/v\right), \cdots, \exp\left(-j2\pi f_n d_M \cos\theta_k/v\right)\right]^T. \tag{6}$$

Considering the fact that targets only occupy a few directions, thus the received signal can be regarded to be sparse in the entire angular domain. The signal in the nth subband can be rewritten in sparse representation,

$$\mathbf{y}(f_n) = \mathbf{\Psi}(f_n)\mathbf{w}_n + \mathbf{e}(f_n), \tag{7}$$

where $\mathbf{\Psi}(f_n) \in \mathcal{C}^{M\times L}$ is an over-complete dictionary, and can be expressed by,

$$\mathbf{\Psi}(f_n) = [\boldsymbol{\psi}_1(f_n), \cdots, \boldsymbol{\psi}_L(f_n)] \tag{8}$$

$$\boldsymbol{\psi}_k(f_n) = [\exp(-j2\pi f_n d_1 \cos\theta_l/v), \cdots, \exp(-j2\pi f_n d_M \cos\theta_l/v)]^T, \tag{9}$$

where $\boldsymbol{\psi}_k$ is the steering vector from spatial direction θ_l, and $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_L]^T$ is the discrete angular grid. $\mathbf{w}_n \in \mathcal{C}^L$ represents the corresponding coefficient vector in the nth subband from L discrete directions.

Since the number of the targets is much smaller than that of the angles considered with $K \ll L$, coefficient vector $\mathbf{w}(f_n)$ is always sparse. On one hand, it is further assumed that targets from different directions share the spectrum. therefore $\{\mathbf{w}(f_n)\}_{n=1}^N$ are jointly sparse, where the elements of the same rows of $\{\mathbf{w}(f_n)\}_{n=1}^N$ tends to be zeros or non-zero simultaneously. On the other hand, It is considered that the received noise signal contains continuous spectrum caused by the cavitations, thus coefficients $\{\mathbf{w}(f_n)\}_{n=1}^N$ are correlated across N subbands. The characteristic would be exploited in the proposed method.

3. PROBABILISTIC MODELS FOR DOA ESTIMATION

In this section, the problem of DOA estimation is reformulated as recovering the sparse coefficients in the sparse Bayesian learning framework. According to [13–15], the detailed probabilistic models are given below.

Let $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_N] \in \mathcal{C}^{L\times N}$ is a coefficient matrix, and $\mathbf{w}_{l\cdot} = [w_{l,1}, \cdots, w_{l,N}]$ is a coefficient vector of the lth row in $\mathbf{W}$. To model the group sparse and coefficient correlation across subbands, the block sparse Bayesian learning framework [13] suggests to use the parameterized Gaussian distribution,

$$p(\mathbf{w}_{l\cdot}; \gamma_l, \mathbf{R}_l) = \mathcal{CN}(\mathbf{w}_{l\cdot}|0, \gamma_l \mathbf{R}_l). \tag{10}$$

with unknown deterministic parameters γ_l and $\mathbf{R}_l$ with $l = \{1, \cdots, L\}$. γ_i is a nonnegative parameter controlling the group-sparsity of $\mathbf{w}_{l\cdot}$. When $\gamma_l = 0$, $\mathbf{w}_{l\cdot}$ becomes zero, and suggests no target in the lth direction. During the learning procedure most γ_m tend to be zero, due to mechanism of automatic relevance determination [16]. Thus sparsity at the group level is encouraged. $\mathbf{R}_l \in \mathcal{R}^{N\times N}$ is a positive definite matrix, capturing the intra-group correlation structure. If we fix $\mathbf{R}_l = \mathbf{I}_N$ for $\forall l$ and ingores the correlation within the group, the model is reduced to those in [14, 16]. Following the method in [13], a Toeplitz matrix based on auto-aggressive model is introduced to model this correlation structure, and it is given by

$$\mathbf{R} = \text{Toeplitz}\left([1, r_n, \cdots, r_n^{N-1}]\right), \tag{11}$$

where Toeplitz is a MATLAB command expanding a real vector into a symmetric Toeplize matrix. Thus the correlation level of the intra-group correlation is reflected by the value of r_n. r_n is empirically calculated [13].

Supposing that the additive noise follows a complex white Gaussian distribution with precision β_0, the likelihood of the signal model in (7) can be expressed by

$$\mathbf{y}(f_n) \sim \mathcal{CN}(\mathbf{y}(f_n)|\mathbf{\Psi}(f_n)\mathbf{w}(f_n), \beta_0^{-1}\mathbf{I}_M) \quad n \in (1, \cdots, N) \tag{12}$$

Let $\mathbf{Y} \in \mathcal{C}^{M\times N}$ is a collection of frequency data $\mathbf{Y} = [\mathbf{y}(f_1), \cdots, \mathbf{y}(f_N)]$, and $\bar{\mathbf{y}}$ represents a rearranged vector as

$$\bar{\mathbf{y}} = \text{Vec}(\mathbf{Y}^T), \tag{13}$$

where Vec$(\cdot)$ is the vectorization of a matrix along columns. Let $\bar{\mathbf{w}}$ is a rearranged vector of the frequency coefficients, and is expressed as

$$\bar{\mathbf{w}} = \text{Vec}(\mathbf{W}^T) = [\mathbf{w}_{1\cdot}, \cdots, \mathbf{w}_{M\cdot}]^T. \tag{14}$$

Eq. (12) can be rewritten by

$$\bar{\mathbf{y}} \sim \mathcal{CN}(\bar{\mathbf{y}}\,|\bar{\mathbf{\Psi}}\bar{\mathbf{w}}, \beta_0^{-1}\mathbf{I}_{MN}), \tag{15}$$

where $\bar{\mathbf{\Psi}} \in C^{MN\times NL}$ is a matrix reconstructed by the matrixes $\{\mathbf{\Psi}_n\}_{n=1}^N$.

For tractable inference of β_0, a Gamma prior with hyperparameters a and b is further imposed on β_0

$$\beta_0 \sim \text{Gamma}(\beta_0|a, b). \tag{16}$$

4. POSTERIOR INFERENCE FOR PROBABILISTIC MODELS

Assume that the parameter β_0 and $\{\gamma_l\}_{l=1}^{L}$ are known, the posterior distribution for $\bar{\mathbf{w}}$ can be evaluated analytically based on the Bayes'rule as,

$$p(\bar{\mathbf{w}}\,|\bar{\mathbf{y}},\{\gamma_l,\mathbf{R}_l\},\beta_0\,) = \mathcal{CN}(\bar{\mathbf{w}}\,|\boldsymbol{\mu},\boldsymbol{\Sigma}\,) \tag{17}$$

$$\boldsymbol{\mu} = \boldsymbol{\Sigma}\bar{\boldsymbol{\Psi}}^H\beta_0\bar{\mathbf{y}} \tag{18}$$

$$\boldsymbol{\Sigma} = (\boldsymbol{\Lambda}^{-1} + \bar{\boldsymbol{\Psi}}^H\beta_0\bar{\boldsymbol{\Psi}})^{-1}, \tag{19}$$

where $\boldsymbol{\Lambda}$ is a a block diagonal matrix with the lth principal diagonal given by $\gamma_l\mathbf{R}_l$. Maximizing the likelihood by marginalizing the coefficients $\bar{\mathbf{w}}$, we acquire,

$$p(\bar{\mathbf{y}}\,|\{\gamma_l,\mathbf{R}_l\},\beta_0\,) = \mathcal{CN}(\bar{\mathbf{y}}\,|\mathbf{0},\mathbf{C}\,) \tag{20}$$

$$\mathbf{C} = \beta_0^{-1}\mathbf{I} + \bar{\boldsymbol{\Psi}}\boldsymbol{\Lambda}\bar{\boldsymbol{\Psi}}^H. \tag{21}$$

The updated rules for parameters γ_l, $\mathbf{R}_l$ and β_0 are derived using the Type II Maximum Likelihood [13, 14, 16] method, which has the following cost function,

$$\mathcal{L}\left(\{\gamma_l,\mathbf{R}_l\},\beta_0\right) = \log|\mathbf{C}| + \bar{\mathbf{y}}^H\mathbf{C}^{-1}\bar{\mathbf{y}}. \tag{22}$$

The parameter β_0^{-1} is the noise variance in our model, it can be estimated by [16],

$$\beta_0 = \frac{MN}{\text{Tr}(\boldsymbol{\Sigma}\bar{\boldsymbol{\Psi}}^H\bar{\boldsymbol{\Psi}}) + \|\bar{\mathbf{y}} - \bar{\boldsymbol{\Psi}}\boldsymbol{\mu}\|_2^2}. \tag{23}$$

According to [13], a different $\mathbf{R}_l$ to each group can result in overfitting. When groups have the same size, an effective strategy to avoid the overfitting is parameter averaging, i.e., constraining $\mathbf{R}_l = \mathbf{R}$ $(\forall l)$. Using this constraint, the learning rule for $\mathbf{R}$ can be derived as follows,

$$\mathbf{R} = \frac{1}{L}\sum_{l=1}^{L}\frac{\boldsymbol{\Sigma}_w^l + \boldsymbol{\mu}_w^l(\boldsymbol{\mu}_w^l)^H}{\gamma_l}, \tag{24}$$

where $\boldsymbol{\mu}_w^l \in \mathcal{C}^{N\times 1}$ is the corresponding lth group in $\boldsymbol{\mu}$, and $\boldsymbol{\Sigma}_w^l \in \mathcal{C}^{N\times N}$ is the corresponding lth principal diagonal group in $\boldsymbol{\Sigma}$ in (19).

Following the Expectation Maximizatin (EM) method [13, 16], we can derive the learning rules for γ_l,

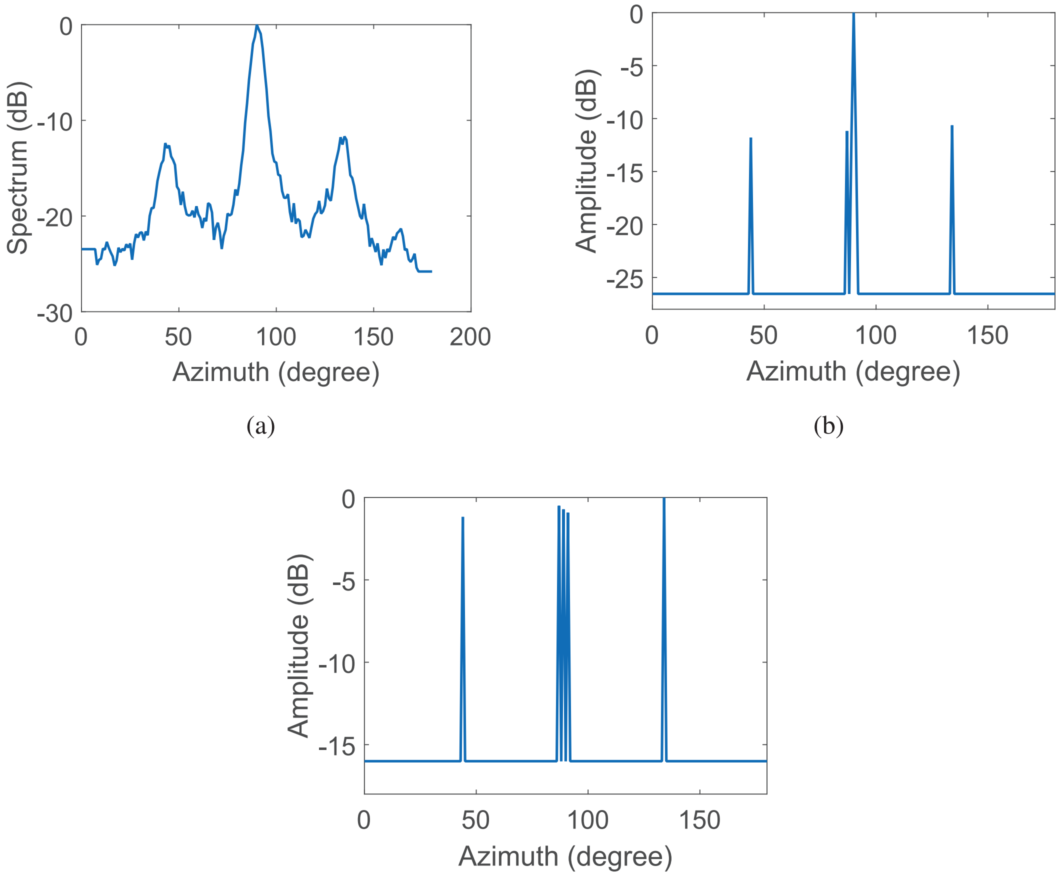
$$\gamma_l = \frac{1}{N}\text{Tr}\left[\mathbf{R}\left(\boldsymbol{\Sigma}_w^l + \boldsymbol{\mu}_w^l(\boldsymbol{\mu}_w^l)^H\right)\right].\quad \forall l \tag{25}$$

Note that β_0, $\mathbf{R}$ and γ_l are function of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, while $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are function of β_0, $\mathbf{R}$ and γ_l. This suggests an iterative algorithm, which iterates (18), (19) and (23), (24) and (25) , until a convergence criterion is satisfied.

5. SIMULATION RESULTS

In this section, simulations is performed to verify the proposed method. Suppose the data is collected from a uniform linear array with 32 sensors with spatial interval of 0.6 m. The sampling rate $F_s = 5000$ and time duration $T = 0.1s$ are used, and thus The sampling number is 500. To model the cavitations noise, The wideband signal is generated by using the Gaussian noise thought the response function, which is $\exp(2\pi F_s|\tau|)(\cos(2\pi f_0\tau) + \sin(2\pi f_0|\tau|))$ with $f_0 = 1000$ Hz. The discrete angular vector is predefined as $\theta = [0°, 1°, \cdots, 180°]^T$. The frequency band from 100 Hz to 1000 Hz is used in the simulation. The additive noise is considered and SNR =10 in the simulations.

The results of the proposed method are compared with that of the conventional delay-sum beamformer, where the DOA is estimation by matching the data with the steering vector of different directions $P_n(\theta) = \boldsymbol{\psi}(f_n)^H\mathbf{y}(f_n)$, and we acquire the incoherent DOA spectrum estimation as $P(\theta) = \sum_{n=1}^{N}\|P_n(\theta)\|^2$. To show the superiority of the proposed method on the high resolution, we use five targets including three targets



(a) (b) (c)

Figure 1: Comparisons of simulation results. (a) Result based on sum-delay beamformer. (b) Result based on SR with group sparisty.(c) Result based on proposed method.

close to each other, whose directions are $88°$,$90°$ and $92°$ respectively, and two targets far away each other, whose directions are $45°$ and $135°$.

Fig. 1(a) shows the result based on the sum-delay beamformer. It is observed that three targets close to each other can be distinguished due to the poor resolution in the conventional beamformer technique; on the other hand, it is obvious that the sidelobes are quite high. By utilizing the sparse reconstruction method with the group sparsity [10], we have the reconstruction result as Fig. 1(b). It is easy to acquire the accurate estimations for these two targets respectively located at $45°$ and $135°$. With respect to three close targets, two of them respectively located at $88°$ and $90°$ are correctly estimated, however, the target located at $92°$ are missing in the sparse reconstruction. When the correlation structure of continuous spectrum is exploited, we have the result in Fig. 1(c). Similar to the SR based method [10], The propose method does not have the sidelobes. Furthermore, it is observed that the proposed method correctly estimate the target directions, and successfully distinguish these close targets by exploiting the correlation structure learning. In addition, the proposed method is developed in the nonparametric Bayesian framework, and it has the capability of automatically inferring the number of the targets and avoiding the tedious tuning parameters.

6. CONCLUSIONS

This paper reformulates the wideband direction-of-arrival (DOA) estimation problem as sparse reconstruction in the sparse Bayesian learning (SBL) framework. Signals are hierarchically modeled to exert the sparsity by Gaussian-Gamma distribution. By exploiting the factors that signals in different spectrum bands show a strong joint group sparsity and Fourier coefficients show high correlation across subbands in the continuous spectrum, we develop a novel DOA estimation method in the nonparametric SBL framework. The proposed method is a data-driven learning process and does not need any prior information, such as the number of targets. The simulation results verify the superiorities over the state-of-the-art DOA estimation methods.

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