Parallel fast-array recursive least squares filters for active noise control with on-line system identification

A.P. Berkhoff[a,b,1], H.J. Meijer[a], and S. van Ophem[c]
[a]Applied Mechanics Group, Faculty of Engineering Technology
Universitity of Twente, 7522NB Enschede, The Netherlands
[b]TNO Technical Sciences, Acoustics and Sonar, 2597AK The Hague, The Netherlands
[c]Noise and Vibration Research Group, Division PMA, KU Leuven, 3001 Heverlee, Belgium

ABSTRACT

Rapid convergence, fast tracking, and low computational cost are desirable characteristics for active noise control applications with rapidly changing transfer paths. Kalman filters or Recursive Least Squares filters with a forgetting factor lower than unity can give a significantly improved tracking performance but have been found to be numerically unstable for the efficient fast array form of the algorithm. A convex mixing fast-array filter can provide numerical stability. This is achieved by running two finite length growing memory recursive least squares filters in parallel and using a convex combination of the two filters when the control signal is calculated. A reset of the filter parameters with proper re-initialization is enforced periodically. In previous versions it is assumed that deviations of the secondary path from the nominal value are small. If these deviations become significant then the model of the secondary path has to be updated. In this paper we present results of a version of the algorithm with on-line adjustment of the secondary path model. One adaptive filter is used to minimize the error signal while the other adaptive filter minimizes the modeling error of the secondary path.

Keywords: Active noise control, Adaptive control, On-line system identification

1. INTRODUCTION

The reasons for slow convergence and poor tracking of adaptive controllers for active noise control have been discussed by several authors [1, 2]. Kalman filters and Recursive Least Squares filters with a forgetting factor can improve these aspects as compared to Least Mean Squares based algorithms, provided the solution is stable and sufficiently efficient. In this paper the controller is combined with on-line system identification of the secondary path, such as described in Ref. [3].

For the control filter producing the secondary path cancellation signal, convergence is determined by the eigenvalue spread of a correlation matrix defined by the filtered reference signal, which is determined by filtering the reference signal with the secondary path. If the secondary path has a non-smooth frequency response then the eigenvalue spread can be significant and standard least mean squares algorithms may lead to insufficient performance, In this paper, the Single Input Single Output (SISO) Active Noise Control (ANC) algorithm presented in Ref. [4] is used. This algorithm has a rate of convergence and tracking performance similar to that of a fast array sliding window Recursive Least Squares filter, but without the numerical error growth. For the part of the algorithm that implements the on-line system identification, the eigenvalue spread of the governing correlation matrix is usually much smaller because it is determined by the identification noise added in the controller. The added noise should be small as compared to the primary noise. Because the robustness for low signal to noise ratios is more important than the ability to handle large eigenvalue spreads the on-line system identification method described in this paper is based on a standard least mean squares algorithm.

[1]email: a.p.berkhoff@utwente.nl

2. METHODS

2.1 Modified filtered-RLS and on-line system identification

In this paper a Single Input Single Output Active Noise Control system with a modified structure and on-line system identification is considered . The modified filter structure is well known in the context of ANC and has been applied both to filtered-reference LMS and RLS algorithms [1, 4]. The goal of the adaptive filter in the modified structure is to find a set of Finite Impulse Response (FIR) filter coefficients $c_i \in \mathbb{R}^{n_w}$, which minimize the modified error ε_i. This modified error is calculated by subtracting from the estimated disturbance $\hat{d}_i$ the output of the adaptive filter $\tilde{y}_i$:

$$\varepsilon_i = \hat{d}_i - \tilde{y}_i. \tag{1}$$

The output of the adaptive filter is obtained by multiplying the filtered reference signal r_i with the filter coefficients c_i:

$$\tilde{y}_i = r_{n_w,i}^T c_i, \tag{2}$$

in which $r_{n_w,i}$ is a vector with the last n_w values of the filtered reference signal:

$$r_{n_w,i} = \begin{bmatrix} r_i & r_{i-1} & \cdots & r_{i-n_w+1} \end{bmatrix}^T. \tag{3}$$

The filtered reference signal is calculated by filtering the measured reference signal with the estimated secondary path state space model

$$\theta_{i+1}^r = A_s \theta_i^r + B_s x_i, \tag{4}$$

$$r_i = C_s \theta_i^r + D_s x_i, \tag{5}$$

in which θ_i^r is the internal path state and $A_s,\ B_s,\ C_s$ and D_s are the estimated secondary path state matrices. The estimated value $\hat{e}_i$ of the error signal without identification noise is obtained by subtracting the estimated identification noise $\hat{w}_i$ from the measured error e_i:

$$\hat{e}_i = e_i - \hat{w}_i. \tag{6}$$

The estimated value $\hat{d}_i$ of the disturbance is calculated by adding the estimated output $\hat{y}_i$ of the secondary path $\hat{S}(z)$ to $\hat{e}_i$:

$$\hat{d}_i = \hat{e}_i + \hat{y}_i. \tag{7}$$

The estimated output of the secondary path is determined by filtering the control signal u_i with the estimated state space model of the secondary path

$$\hat{\theta}_{i+1} = A_s \hat{\theta}_i + B_s u_i, \tag{8}$$

$$\hat{y}_i = C_s \hat{\theta}_i + D_s u_i. \tag{9}$$

In the modified filtered RLS scheme the control signal u_i is calculated by filtering the reference signal x_i with the adaptive filter:

$$u_i = x_{n_w}^T c_i, \tag{10}$$

$$x_{n_w} = \begin{bmatrix} x_i & x_{i-1} & \cdots & x_{i-n_w+1} \end{bmatrix}^T. \tag{11}$$

2.2 Mixed windowed RLS

For the adaption of the filter coefficients a filter is used which behaves like a constant length finite memory RLS algorithm with a linear calculation complexity $O(n_w)$, equivalent to the Chandrasekhar form of the sliding window RLS filter [5], but does not exhibit the round-off error propagation. To achieve this, a convex mixing approach [4] is used to emulate the sliding window RLS filter. Instead of a single control filter c_i used to minimize ε_i, two sets of control coefficients $c_{1,i}$ and $c_{2,i}$ are used to produce outputs $\tilde{y}_{1,i}$ and $\tilde{y}_{2,i}$ which minimize $\varepsilon_{1,i}$ and $\varepsilon_{2,i}$, respectively. A control filter $c_{mix,i}$ is used to produce the actual control output u_i.

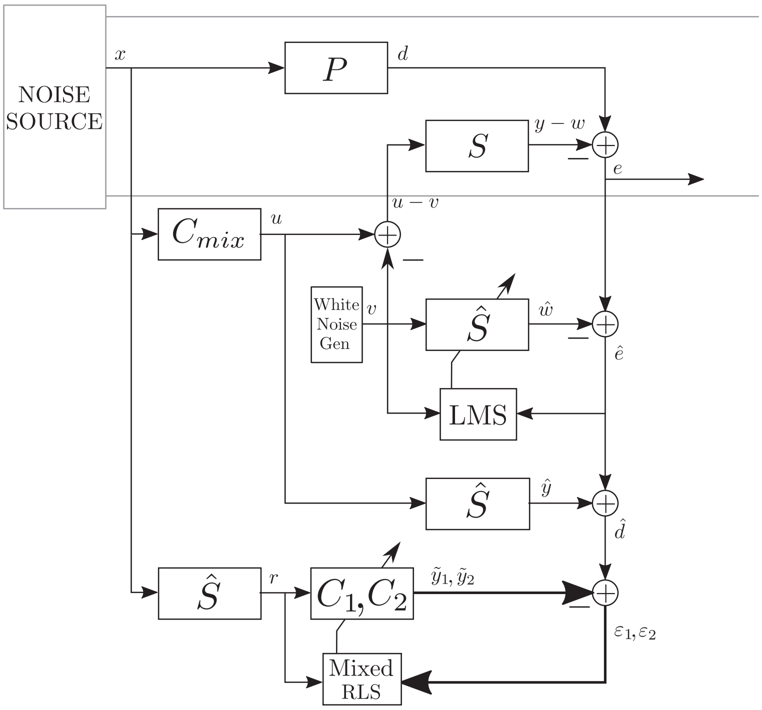

Figure 1: Block diagram of a feedforward active noise control system based on a Recursive Least Squares mixing scheme and Least Mean Squares on-line system identification.

The latter filter consists of a convex combination of $c_{1,i}$ and $c_{2,i}$, as described below. A block diagram of this system is shown Fig. 1.

Convex combinations of filters have been a popular topic in recent years, see Refs. [6], [7] and [8]. An example of convex filters in the context of ANC is given by Ferrer [9]. The main difference between these approaches and the filter in this paper, as described in Ref. [4], is the way the convex combination is applied. The optimal mixing parameters, which give the lowest mean square error (MSE), then will be determined by an extra adaptive filter.

The proposed implementation uses two filters with identical filter parameters and predetermined time-varying mixing coefficients. This means that not necessarily the convex combination with the lowest MSE will be found. Instead, it simulates a filter with a constant memory length, such as the sliding window RLS filter. Two parallel growing memory filters are mixed in such a way, that the total available information used for calculating the least squares solution will be equal at every time instance. Firstly, the equations for a recursive update of the mixed solution are presented. The mixing parameters α_i and β_i are constrained by $0 \leq \alpha_i \leq 1$, $0 \leq \beta_i \leq 1$, and sum up to unity:

$$\alpha_i + \beta_i = 1, \ \forall i. \tag{12}$$

A possible choice for the mixing parameters can be found in Fig. 2 [4]. The first filter will be activated at time instance U and the second filter will be activated after $V = U + W/2$ iterations. The filters are defined by the data matrices $H_{1,i}$, $H_{2,i}$ and measurement vectors $y_{1,i}$, $y_{2,i}$:

$$H_{1,i} = \begin{bmatrix} r^T_{n_w,U} \\ r^T_{n_w,U+1} \\ \vdots \\ r^T_{n_w,i} \end{bmatrix}, \ H_{2,i} = \begin{bmatrix} r^T_{n_w,V} \\ r^T_{n_w,V+1} \\ \vdots \\ r^T_{n_w,i} \end{bmatrix}. \tag{13}$$

$$y_{1,i} = \begin{bmatrix} \hat{d}_U \\ \hat{d}_{U+1} \\ \vdots \\ \hat{d}_i \end{bmatrix}, \ y_{2,i} = \begin{bmatrix} \hat{d}_V \\ \hat{d}_{V+1} \\ \vdots \\ \hat{d}_i \end{bmatrix}. \tag{14}$$

The cost functions of the parallel RLS filters are:

$$\min_{c_{1,i}}[c_{1,i}^T \Pi c_{1,i} + \|y_{1,i} - H_{1,i}c_{1,i}\|^2], \tag{15}$$

$$\min_{c_{2,i}}[c_{2,i}^T \Pi c_{2,i} + \|y_{2,i} - H_{2,i}c_{2,i}\|^2], \tag{16}$$

in which the matrix $\Pi \in \mathbb{R}^{n_w \times n_w}$ is a positive definitive regularization matrix. In Ref. [4] it is shown that the resulting update equations are:

$$\begin{aligned} c_{mix,i} = \alpha_i(c_{1,i-1} + K_{1,i}R_{1,i}^{-1}\varepsilon_{1,i}) + \\ \beta_i(c_{2,i-1} + K_{2,i}R_{2,i}^{-1}\varepsilon_{2,i}). \end{aligned} \tag{17}$$

in which $K_{1,i}$, $K_{2,i}$ are Kalman gains, $\varepsilon_{1,i}$, $\varepsilon_{2,i}$ are the innovations and $R_{1,i}$, $R_{2,i}$ are the expected values of the innovations.

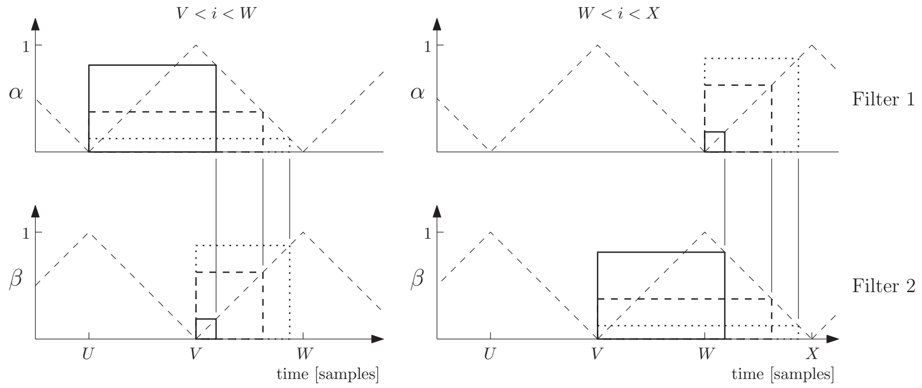

Figure 2: Mixing parameters α_i and β_i to combine two parallel Recursive Least Squares filters.

The parameters in Eq. (17) will be updated with two parallel growing memory (forgetting factor $\lambda = 1$) fast array RLS algorithms. The complexity of these algorithms grows linearly with n_w. The filter parameters of the two filters are calculated completely in parallel and no interaction takes place between the filters. Both filters are reset every time a window length W has passed. A complete description of the algorithm can be found in Ref. [4].

3. RESULTS

The performance of the algorithm was evaluated using a model of a loudspeaker in a duct. A sample rate of 2 kHz was used with 250 Finite Impulse Response coefficients for the secondary path model and 250 Finite Impulse Response coefficients for the control filters. The initial rms value of the identification noise is at a relatively high level of 0.8 for times between 1 s and 3.5 s, whereas the rms value for times later than 3.5 s is 0.1. The controller is switched on at time 1 s. The primary path is modified at time 11 s. All initial FIR coefficients of the controller and secondary path model are zero. The resulting error signal is shown in Fig. 3, the modeling error $\hat{e}$ is shown in Fig. 4 and the secondary path coefficients in Fig. 5. The reduction of the error signal obtained between 1 s and 11 s starting from zero, i.e., convergence, and the reduction of the error signal obtained between 11 s and 16 s after the change of the primary path at 11 s, i.e., tracking were found to be considerably higher than obtained with a standard least mean square adaptive control filter. It was also found that in this application the convergence time of a least mean square adaptive filter for modeling the secondary path was found to be sufficiently small.

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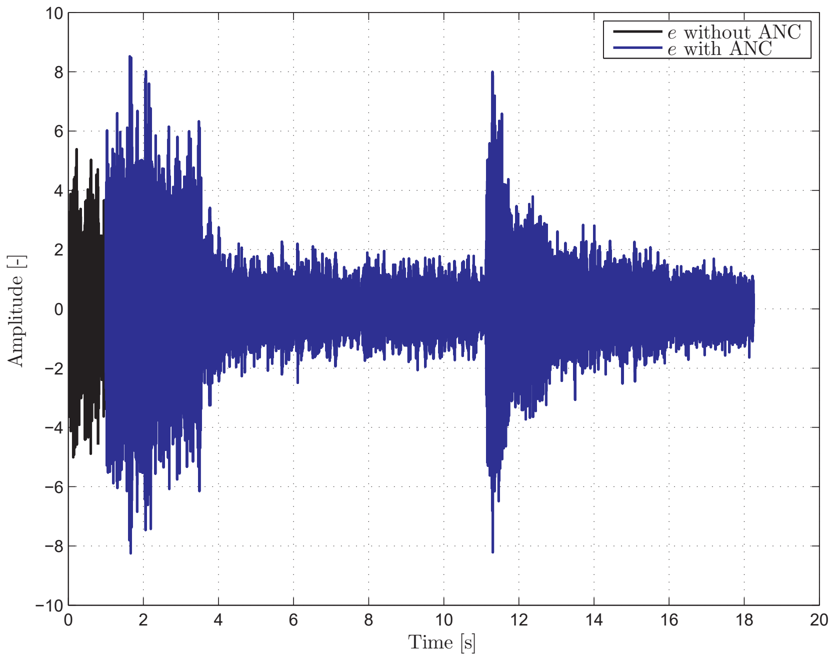

Figure 3: Error signal $e = d - y + w$ with controller switched on at 1 s and modification of the primary path at 11 s. The rms value of the identification noise v is 0.8 from 1 s to 3.5s and 0.1 after 3.5 s.

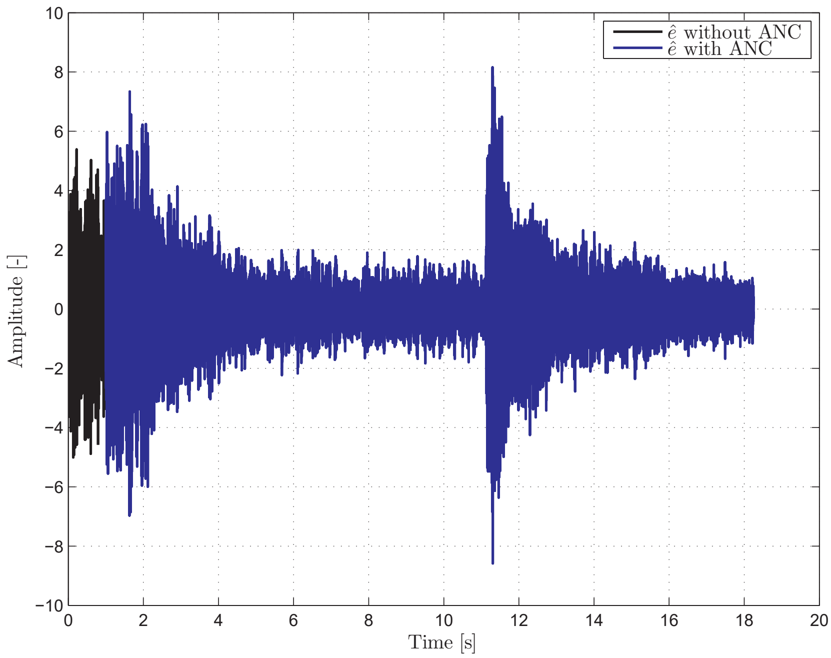

Figure 4: As Fig. 3, modelling error signal $\hat{e} = d - y + w - \hat{w}$.

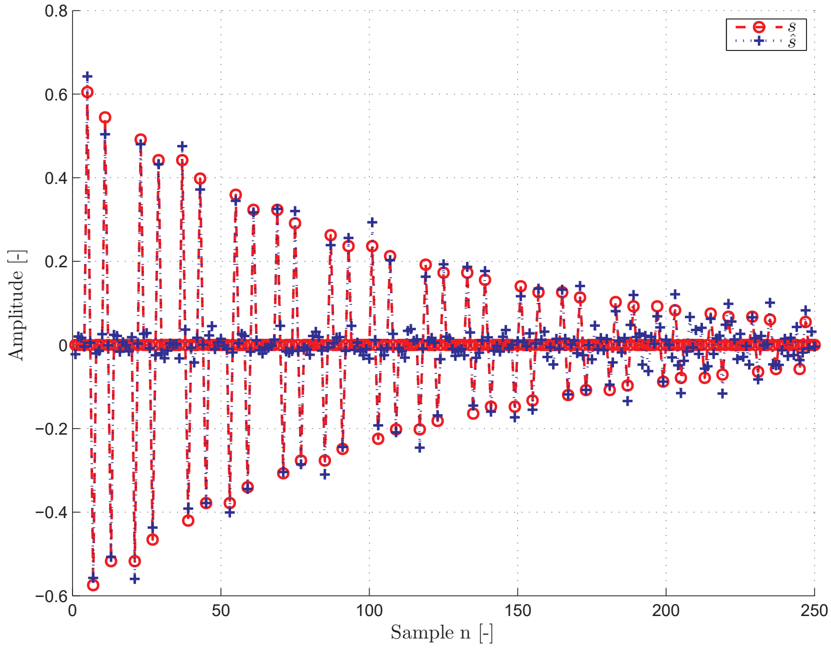

Figure 5: Impulse response of the secondary path and the estimated impulse response using on-line system identification corresponding to Fig. 3.

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