Influence of the ground/structure interaction on the calculation of the force at the wheel/rail contact

Loïc GRAU¹; Astrid PIERINGER²; Bernard LAULAGNET³; Wolfgang KROPP⁴

¹ ACOUPHEN, 33 Route de Jonage, 69891 Pusignan Cedex France
², ⁴ Applied Acoustics / CHARMEC, Department of Civil and Environmental Engineering
Chalmers University of Technology, Gothenburg, Sweden
³ Laboratoire Vibrations et Acoustique, Campus Lyon Tech La Doua – INSA Lyon
25 Bis avenue Jean Capelle, 69621 Villeurbanne, France

ABSTRACT

The prediction of ground vibration from railway traffic represents a major challenge for railway operators, especially with regard to the increasing number of new lines built close to residential buildings. In this context, it becomes essential to have a model that accounts on the one hand for ground/structure interaction and on the other hand for wheel/rail interaction. In this paper, such a model is developed by combining two existing models. The model for ground/structure interaction, SIPROVIB, is an analytical model of a slab with Kirchhoff-Love hypothesis coupled to the ground in 3D. The model for wheel/rail interaction is a computationally efficient time-domain model, where vehicle and track are represented by pre-calculated Green's functions. The wheel/rail contact is modelled as 3D, non-linear and non-Hertzian. Both models are combined by replacing the track Green's function by a Green's function representing the ground and slab coupled to a simplified rail model. Numerical results showed that the influence of slab and ground on the dynamic wheel/rail contact forces increases for thinner slabs and softer grounds, but is generally of secondary importance. Deviations in contact force did not exceed 2 dB for frequencies up to 200 Hz.

Keywords: Ground/slab interaction, Wheel/rail interaction, Contact force
I-INCE Classification of Subjects Numbers: 11.7.2, 13.4.1, 13.4.3, 43, 76.1.2

1. INTRODUCTION

Beside air-borne noise, railway-induced vibration is one of the major problems for the resident when it comes to the construction of a new railway line or to extension of an existing line. Prediction models can help to identify and design adequate mitigation measures and by this means prevent annoyance. For this reason, models have been developed these last two decades in order to estimate railway-induced ground vibration propagating to the foundation of buildings. As most of the time it is too computationally demanding to build the whole problem from source to receiver into a single model, the problem is generally divided into subproblems. In such a subproblem, we can be interested either in the dynamic wheel/rail interaction or the ground/slab interaction, or the ground/building interaction, or the acoustic radiation into rooms. The aim of the present paper is to investigate whether the common practice to uncouple the subproblem of wheel/rail interaction from the subproblem of ground/slab interaction is justifiable.

In models for the dynamic wheel/rail interaction, the rail support is often modeled by one or several layers of linear springs and dampers, with spring and damping coefficients independent of frequency. For rail supported by rail pads and sleepers on ballast, a two-layer support model is common, where the lower layer represents both the properties of the ballast and the underlying ground, see e.g. (1). According to (2), for track models for slab track a single layer of resilience

¹ loic.grau@acouphen.fr
² astrid.pieringer@chalmers.se
³ bernard.laulagnet@insa-lyon.fr
⁴ wolfgang.kropp@chalmers.se
under the rail is sufficient, since the concrete slab generally has such a high impedance that it vibrates much less than the rail. This latter hypothesis is tested in the current work by combining a wheel/rail interaction model with a model for the slab/ground interaction.

The model for wheel/rail interaction, which has been presented in (3), is a computationally efficient time-domain model, where vehicle and track are represented by pre-calculated Green's functions. The wheel/rail contact is modelled as three-dimensional (3D), non-linear and non-Hertzian.

The ground/slab interaction is based on a 3D analytical model recently developed (4). This model called SIPROVIB has been experimentally validated for a tramway slab in the frequency range [4Hz ;250Hz] and with a numerical model (BEM-FEM) called MEFISSTO. SIPROVIB can account for the stratified ground with different kinds of excitation on the slab. In order to combine the wheel/rail interaction model with SIPROVIB, a punctual excitation along the slab will be considered.

2. Problem formulation

The problem under consideration consists of a tramway running over slab track, see Figure (1). Dynamic contact forces are generated at the wheel/rail interface due to irregularities ("roughness") of the profiles of the wheel and rail running surfaces. This leads to vibrations of the wheels and wave propagation in the rails, the slab and the ground. Another excitation mechanism for ground vibrations is the moving axle load associated with each wheelset. The frequency range of interest to cover both ground vibrations and ground-borne noise is approximately from 4 Hz to 250 Hz (2).

![Figure 1: Overview of the problem under consideration](image)

The track includes rails attached to a concrete slab via a resilient layer (rail fasteners involving flexible rail pads). One slab is included which is coupled to the top surface of a homogeneous semi-infinite half-space representing the ground. The vehicle model could include bogies and the car body, but these sprung masses are neglected here since the primary and secondary suspension isolate the bogie and the car body from the wheelset at frequencies of more than a few Hertz (5). A further simplification is that the problem is assumed symmetrical with regard to the symmetry plane of the track and only one of the two rails is considered. Finally, only one wheel including its primary suspension is considered in this study. This simplification may affect the results below about 20 Hz, since multiple wheels exciting the rail cannot be treated anymore as incoherent sources in this frequency range (2).
2.1 SIPROVIB: Ground plate interaction

2.1.1 Plate equation of motion

In the frequency range of interest i.e. [4Hz; 250Hz], a tramway slab can be modelled under Kirchhoff hypothesis. The slab has finite length and width \((L_x, L_y)\). It is coupled to the ground over its complete surface \(S\) and the stress applied by the ground on the slab is \(\sigma_p(x, y)\). The equation of motion for the slab is:

\[
D^* \Psi^4 w(x, y) - \omega^2 \rho hw(x, y) = F_e \delta(x - x_e)\delta(y - y_e) + \sigma_p(x, y)
\]

(1)

where \(D^* = E(1 + \eta)\), \(w(x, y)\), \(\rho\) and \(h\), are respectively the flexural rigidity, the vertical displacement, the mass density and the thickness of the slab. The slab is excited by a punctual force of amplitude \(F_e\), which is located at the position \((x_e, y_e)\).

In order to solve the slab equation of motion, the vertical displacements are developed in Fourier series. For regularization consideration, the force is also developed in Fourier series.

\[
\begin{align*}
\{ w(x, y) &= \sum_{nm} a_{nm}(\omega) \phi_{nm}(x, y) \\
F(x, y) &= \sum_{nm} F_{nm}\phi_{nm}(x, y)
\end{align*}
\]

(2)

where \(a_{nm}(\omega)\) is the modal amplitude and \(F_{nm}\) is the modal force amplitude.

The slab boundary conditions are guided boundary conditions. These boundary conditions state that the rotation and the shear force are null at the boundaries. Although these boundary conditions are not the real slab boundary conditions, it has been shown that they give a good approximation of the slab displacement in a first approximation (4). So the modal shape of the slab is \(\phi_{nm}(x, y) = \cos\left(\frac{m\pi}{L_y} y\right)\cos\left(\frac{n\pi}{L_x} x\right)\). Concerning the modal force amplitude, we have in the case of a punctual force applied on the slab:

\[
F_{nm} = \begin{cases} \\
\frac{2}{L_x} \phi_{nm}(x_e, y_e) & \text{if } n = 0 \text{ and } m = 0 \\
\frac{2}{L_y} \phi_{nm}(x_e, y_e) & \text{if } n \neq 0 \text{ or } m \neq 0 \\
\frac{4}{L_x L_y} \phi_{nm}(x_e, y_e) & \text{if } n \neq 0 \text{ and } m \neq 0
\end{cases}
\]

(3)

Inserting solutions (2) into equation (1) leads to:

\[
\sum_{nm} \left( (D^* k_{nm}^4 - \omega^2 \rho h)a_{nm}(\omega) - F_{nm}\phi_{nm}(x, y) \right) \phi_{nm}(x, y) = \sigma_p(x, y)
\]

(4)

It can be reminded that the unknowns of the problem are the modal amplitudes \(a_{nm}(\omega)\). In the next section, we introduce the ground equation of motion to consider for coupling.

2.1.2 Ground equation

The ground is an unbounded, homogeneous, linear elastic and isotropic continuum. The equation of motion to consider is the Navier equation (6). In the absence of body forces, it can be written as:

\[
\mu \Delta \ddot{u} + (\lambda + \mu) \text{grad} (\text{div} (\ddot{u})) + \rho \omega^2 \ddot{u} = 0 .
\]

(5)

where \((\mu, \lambda)\) are the Lamé’s elastic constants, \(\rho\) is the ground mass density and \(\ddot{u}\) is the ground
displacement vector. It is possible to uncouple the ground displacement vector in the Navier equation using the Helmholtz’s decomposition (6). This leads to the following potential vector \( \Psi \) and scalar \( \phi \) equation:

\[
\begin{align*}
\Delta \phi + k_p^2 \phi &= 0 \\
\Delta \Psi + k_z^2 \Psi &= 0
\end{align*}
\]

where \( k_p = \frac{\omega}{c_p} \) and \( k_z = \frac{\omega}{c_z} \) are the dilatational and the shear wavenumbers, respectively.

In the equation (5), it is possible to identify the two propagating waves that is to say the dilatational wave \( c_p \) and the shear wave \( c_z \).

The media being unbounded in the x- and y- direction and semi-infinite in the z-direction, a 2D spatial Fourier transform is performed on the potential scalar and vector. A solution of the potential can then be expressed as:

\[
\begin{align*}
\phi(x, y, z) &= A e^{-j k_1 z} \\
\psi_x(x, y, z) &= B e^{-j k_2 z} \\
\psi_y(x, y, z) &= C e^{-j k_2 z} \\
\psi_z(x, y, z) &= D e^{-j k_2 z}
\end{align*}
\]

where \( k_1 = \sqrt{k_p^2 - k_z^2 - k_y^2} \) and \( k_2 = \sqrt{k_p^2 - k_x^2 - k_y^2} \) are the dilatational and shear reduced wavenumbers, respectively.

In order to obtain an expression for the potential amplitudes, \( A, B, C \) and \( D \), we are using the ground boundary conditions. The ground is free at the top surface except under the slab for the normal stress. This leads to:

\[
\begin{align*}
\sigma_{x x}(x, y, 0) &= 0 \\
\sigma_{y y}(x, y, 0) &= 0 \\
\sigma_{z z}(x, y, 0) &= \begin{cases} 0 & \text{if } (x, y) \in \mathbb{R}^2 - S \\ \sigma_p(x, y) & \text{if } (x, y) \in S \end{cases}
\end{align*}
\]

In the Fourier domain, the boundary conditions are still zero for the tangential stress. However the normal stress, \( \sigma_p(k_x, k_y) \), refers to the stress from the slab.

\[
\begin{align*}
\sigma_{x z}(k_x, k_y, 0) &= 0 \\
\sigma_{y z}(k_x, k_y, 0) &= 0 \\
\sigma_{z z}(k_x, k_y, 0) &= \sigma_p(k_x, k_y)
\end{align*}
\]

With these boundary conditions, it is possible to give an analytical expression of the potential amplitudes \( A, B, C \) and \( D \). Using the definition of the Helmholtz decomposition in the Fourier domain, the ground displacement in the z-direction reads:

\[
\sigma_z(k_x, k_y, 0) = N(k_x, k_y) \sigma_p(k_x, k_y),
\]

where \( N(k_x, k_y) = \frac{j k_1 k_2}{4 \mu k_1 k_2 (k_x^2 + k_y^2) - (\lambda k_1^2 + 2 \mu k_2^2)(k_x^2 + k_y^2 - k_2^2)} \) is valid for a semi-infinite unbounded ground.

As the ground displacement is known in the Fourier domain, it is now possible to obtain a linear system in order to obtain the unknowns of the problem \( a_{nm} (\omega) \).
2.1.3 Linear system to solve

The continuity of the ground displacement with the slab can be written:

\[
\frac{1}{4\pi^2} \int_{-\infty}^{\infty} a_z(k_x, k_y, 0) e^{i(k_x x+k_y y)} dk_x dk_y = \sum_{nm} a_{nm}(\omega) \phi_{nm}(x,y) \quad \forall (x,y) \in S
\]  

(11)

In expression (9), the normal stress in the Fourier domain is replaced by equation (3). Then it is possible to obtain the ground displacement depending on the modal amplitude of the slab. After an orthogonality process, the linear system for the modal amplitude is:

\[
\sum_{nm} (M_{nm} \omega^2_{nm} - \omega^2) a_{nm}(\omega) - F_{nm}) \gamma_{nmrs} = S_{rs} a_{rs}(\omega)
\]  

(12)

where

\[
\gamma_{nmrs} = \int_{-\infty}^{\infty} N(k_x, k_y) G_{nm}(k_x, k_y) \tilde{G}_{rs}(k_x, k_y) dk_x dk_y
\]

is the ground cross-modal mobility of one slab mode (n,m) acting on another slab mode (p,q). These terms refer to the influence of the ground coupling on the slab.

As we are now able to determine the unknowns of the problem, it is possible to calculate the receptance of the slab, which is needed for the wheel/rail interaction model presented in the next section. The receptance \( G_s(x_s, y_s, x_r, y_r) \) giving the slab displacement at a position \( (x_r, y_r) \) due to a punctual force located at a position \( (x_s, y_s) \) applied on the plate is obtained from Equation (2):

\[
G_s(x_s, y_s, x_r, y_r) = \frac{w(x_s, y_s)}{F(x_s, y_s)}
\]  

(13)

This receptance is considered as an input data for the wheel/rail interaction model instead of considering a constant stiffness. As the slab displacement depends on the ground properties, we will consider different kinds of ground. In the present paper, we have modeled only one layer in the ground model because the slab displacement is mainly influence by the first layer. In some cases, stratification could influence the slab displacement, which has not been considered in this study.

2.2 Wheel / rail interaction

The vertical wheel/rail interaction model, which has been presented in (3) is formulated in the time domain. This allows the inclusion of non-linearities occurring in the contact area. Wheel and track are represented by impulse response functions (Green's functions) that are calculated from the wheel and track receptances at the contact point by inverse Fourier transform. Instead of performing a time integration of the differential equations describing wheel and track dynamics, the wheel and rail displacements \( w_W \) and \( w_T \) at the contact point are obtained by convoluting the time series of the normal wheel/rail contact forces \( F_c(t) \) with the pre-calculated Green's functions \( g_W(t) \) and \( g_W^{x_0}(t) \) representing wheel and track, respectively:

\[
w_W(t) = -\int_{0}^{t} F_c(\tau) g_W'(t-\tau) \, d\tau
\]

(14)

\[
w_R(t) = \int_{0}^{t} F_c(\tau) g_W^{x_0}(t-\tau) \, d\tau
\]

The Green's functions for the track (called moving Green's functions) include the motion of the contact point along the rail. For vertical excitation of the rail at the position \( x_0 \) at time \( t_0 = 0 \), the function \( g_R^{x_0}(t) \) describes the vertical displacement response at the railhead at a point moving at train speed \( v \) away from the excitation. The train speed is assumed constant. The discrete version of the moving Green's function \( g_R^{x_0}(t) \) is constructed from (ordinary) Green's functions \( g_R^{x_0}(t) + \alpha(t) \) where the superscripts specify the excitation point \( x_0 \) and the response point \( x_0 + \alpha \) on the rail. The usage of pre-calculated Green's functions leads to a computationally efficient algorithm allowing
the inclusion of a detailed, more computationally demanding contact model. In addition, the approach is versatile. Any linear wheel and track model represented by Green's functions can easily be included without changing the algorithm.

2.2.1 Contact model

The wheel-rail contact model is an implementation of Kalker’s variational method based on the algorithm NORM (7). This is a three-dimensional and non-linear contact model that relies on the assumption that wheel and rail can be locally approximated by elastic half-spaces. The model considers the three-dimensional running surfaces of wheel and rail including the surface roughness on several parallel lines in rolling direction. The combined wheel/rail roughness is the excitation of the wheel/track system in from of a relative displacement input giving rise to dynamic contact forces at the wheel/rail interface and wheel and track vibrations. Details about the implementation are found in (8). In this paper, wheel and rail surface are modeled as cylinders. Measured roughness data from a wheel with sinter block brakes from (9) are used as excitation. These data were available with a spacing of 2 mm across the width of the running surface, and sampled at 0.5 mm around the perimeter of the wheel.

2.2.2 Wheel model

The wheel is modeled as a single degree-of-freedom system containing half the wheelset mass $M_w$ and the primary suspension stiffness $k_w$ and damping $c_w$. The vehicle system above the primary suspension of the wheelset is simplified to a static preload, $P$. In the low-frequency range that is of interest in this paper (below 250 Hz), the behavior of the wheel is mass controlled (2) and the influence of wheel modes can be neglected.

2.2.3 Track model and coupling to the slab/ground model

In (10), the track has been modeled as one continuously supported rail built with waveguide finite elements (WFE) using the software package WANDS (11). The support consists of a continuous rail pad on rigid ground. In order to facilitate the coupling of the rail model to the slab/ground model described in Section 2.1, the rail model is strongly simplified. The WFE model is replaced by a series of springs with complex spring constants $k_{eq}(\alpha)$ depending on the distance $\alpha$ between excitation and receiver point on the rail.

![Simplified rail model](image)

Figure 2  Simplified rail model: Rail and railpad are replaced by a series of springs with spring constants $k_{eq}(\alpha)$ depending on the distance $\alpha$ between excitation and receiver point on the rail. The springs are coupled to the slab/ground model with receptance $G_S$.

The rail receptance obtained with the WFE model at frequency $f = 0$ Hz is taken as approximation for $1/k_{eq}$. This procedure corresponds to replacing the point and transfer receptances of the rail by horizontal lines and is only reasonable at low frequencies. Figure 1 shows as examples the simplified point receptances of the rail in comparison to the original ones for a soft and a stiff rail pad.
Figure 3 Simplified vertical point receptance of the continuously supported rail in comparison to the original receptance calculated with the WFE model: rail on soft rail pad (left figure); rail on stiff rail pad (right figure).

The coupling of the simplified rail model and the slab/ground model is carried out as follows. Considering the system in Figure (2), the equations of motion can be written assuming harmonic motion

\[
\begin{align*}
F_c &= k_{eq}(w_R - w_S) \\
F_c &= \frac{w_S}{G_S}
\end{align*}
\]  

(15),

where \(w_R\) and \(w_S\) are the displacements at the top of the rail and the top of the slab, respectively, and \(G_S\) is the slab/ground receptance from Equation (13). All arguments have been omitted in (15) for simplicity. Eliminating \(w_S\) gives

\[
\frac{w_R}{F_c} = \frac{1}{k_{eq}} + G_S.
\]  

(16)

These combined rail/slabb/ground receptances are used to calculate the moving Green’s functions representing the track in Equations (14).

3. Numerical results

In this section, we present the numerical results of the influence of the ground/slabb interaction on the calculation of the wheel/rail contact force.

The slab and the ground characteristics considered in SIPROVIB model are listed in Table 1 while the rail, railpad and wheel properties considered in the wheel/rail interaction model are listed in Table 2.

| Characteristics | Slab | | Characteristics | Ground |
|----------------|------||----------------|--------|
| \(L_x\), m     | 28   | | \(C_s\), m.s\(^{-1}\) | 160 | 800 |
| \(L_y\), m     | 6.6  | | \(\eta_s\) | 3% | 3% |
| \(h\), m       | 0.4 or 0.6 | | \(C_p\), m.s\(^{-1}\) | 600 | 1800 |
| \(E\), Pa      | 2.5.10\(^{10}\) | | \(\eta_p\) | 3% | 3% |
| \(\rho\), kg.m\(^{-3}\) | 2500 | | \(\rho\), kg.m\(^{-3}\) | 1200 | 2500 |
| \(v\)          | 0.3  | | \(\eta\) | 0.03 |
Table 2 Rail, railpad and wheel characteristics

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Rail</th>
<th>Rail pad</th>
<th>Characteristics</th>
<th>Wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus $E$, MPa</td>
<td>210.10$^4$</td>
<td>4.80 or 21.6</td>
<td>Half wheelset mass $M_W$, kg</td>
<td>350</td>
</tr>
<tr>
<td>density $\rho$, kg.m$^{-3}$</td>
<td>7800</td>
<td>10</td>
<td>Primary stiffness $k_W$, MN.m$^{-1}$</td>
<td>0.4</td>
</tr>
<tr>
<td>Poisson ratio $\nu$</td>
<td>0.3</td>
<td>0.45</td>
<td>Primary damping $c_w$, kN.s.m$^{-1}$</td>
<td>6.4</td>
</tr>
<tr>
<td>damping loss factor $\eta$</td>
<td>0.01</td>
<td>0.25</td>
<td>Wheel radius $r_W$, m</td>
<td>0.30</td>
</tr>
<tr>
<td>rail head radius $r_e$, m</td>
<td>0.30</td>
<td></td>
<td>Static preload $P$, kN</td>
<td>44.1</td>
</tr>
<tr>
<td>lateral location on slab $y_w$, m</td>
<td>1.65</td>
<td></td>
<td>Speed $v$, km.h$^{-1}$</td>
<td>50</td>
</tr>
</tbody>
</table>

The railpad data corresponds to a soft railpad with vertical stiffness 102 MN.m$^{-2}$ and a more stiff pad with vertical stiffness 459 MN.m$^{-2}$.

3.1 Receptances of the slab/ground model

Figure 4 shows the vertical slab displacement at the longitudinal position $x_r=0$ m, 6 m and 12 m for an excitation at $x_e=0$ m both for a soft ground and a stiff ground. It can be noticed that the slab displacement depends on frequency with a fluctuation of approximately 20 dB in the frequency range up to 200 Hz for the soft ground. For the stiff ground, the slab displacement is much lower and the fluctuation with frequency is reduced. This first results indicates that the influence of the ground/slab model should be stronger in the case of a soft ground.

Figure 4 Vertical point and transfer receptances of the slab/ground (SG) model for excitation ($x_e$) and response ($x_r$) point on the slab surface: a) 0.4 m thick slab on soft ground, b) 0.4 m thick slab on stiff ground

3.2 Combined receptances of the rail/slab/ground model

The combined receptance of the rail/slab/ground model, which is shown in Figure 5 for a soft railpad, is dominated by the receptance of the supported rail (see Figure 3). Consequently, only a small influence of the slab/ground model is seen at low frequencies for the soft ground in the point receptance and no influence is visible for the stiff ground. Figure 5 also shows the wheel receptance. The rail displacement is lower than the wheel displacement up to about 80Hz. The difference reaches 50dB at the resonance frequency of the wheel suspension at 5 Hz. Above 80Hz, the rail
displacement is higher than the wheel displacement and reaches a 20 dB difference at 200Hz.

Figure 5 Vertical point and transfer receptances for the rail/slab/ground (RSG) model including a soft rail pad; excitation \((x_E)\) and response \((x_R)\) point on the rail head: 0.4 m thick slab on soft ground (left figure), 0.4 m thick slab on stiff ground (right figure); vertical point receptance of the wheel for comparison

In the case of the stiff railpad, the response of the supported rail (Figure 3) is lower than in the case of the soft railpad. This also becomes apparent in the combined rail/slab/ground receptance, see Figure 6. The influence of the soft ground/slab model is now slightly more visible in the combined point receptance at low frequencies, while still no influence is seen from the stiff ground/slab model. Another consequence is that the wheel response is now dominating over the rail response up to 135 Hz instead of the 80 Hz found for the soft railpad.

Figure 6 Vertical point and transfer receptances for the rail/slab/ground (RSG) model including a stiff rail pad; excitation \((x_E)\) and response \((x_R)\) point on the rail head: 0.4 m thick slab on soft ground (left figure), 0.4 m thick slab on stiff ground (right figure); vertical point receptance of the wheel for comparison

3.3 Wheel/rail contact forces for roughness excitation

Calculations of the dynamic contact force have been carried out with the wheel/rail interaction model considering the soft and stiff rail pad, the soft and stiff ground and the two different slab thicknesses specified in Tables 1 and 2.

Measured spatial wheel roughness from a wheel with sinter block brakes from (10) has been used as roughness excitation in the simulations. The spectrum of these data is shown in Figure 7.
The results from the wheel/rail interaction model for the case of a soft railpad are shown in Figure 8 in form of the third-octave band spectrum of the contact force. Beside the spectrum for the case when the track is represented by the simplified rail model only, the deviations in dB are given when the different grounds and slabs are included. There is no influence of the ground or slab below 63 Hz, which is a consequence of the greater wheel receptance compared to the track receptance. Above 63 Hz, the influence of the slab/ground model on the contact force is up to 1.5 dB. For the same ground, the contact force difference is higher when the slab thickness is reduced. For the same slab thickness, the contact force difference is higher for a softer ground compared to a stiffer ground.

Figure 8 Vertical dynamic wheel/rail contact force due to roughness excitation; soft rail pad; track represented by the simplified rail only model (R) (left figure); difference in dB in contact force level between the track represented by the rail/slab/ground (RSG) model and the rail only model, $L_{F,RSG} - L_{F,R}$, for different grounds and slab thicknesses (right figure)

An influence of the slab/ground model on the contact force is observed when the wheel displacement becomes of the same order as the plate displacement. As the wheel displacement depends mainly on the rail pad, the case of a stiff pad has also been considered in order to validate the previous analysis.

Figure 9 presents the dynamic wheel/rail contact forces for the case of a stiff rail pad. As previously, an influence of the ground/slab model is observed when the track receptance reaches the
same order of magnitude as the wheel receptance. As this occurs at a higher frequency for the stiff railpad, deviations in the contact force are observed mainly at higher frequencies above 125 Hz. Maximum deviation is about 2dB.

Figure 9 Vertical dynamic wheel/rail contact force due to roughness excitation; stiff rail pad; track represented by the simplified rail only model (R) (left figure); difference in dB in contact force level between the track represented by the rail/slab/ground (RSG) model and the rail only model, $L_{F,RSG}-L_{F,R}$, for different grounds and slab thicknesses (right figure)

4. CONCLUSIONS

A model for wheel/rail interaction has been coupled to a model for slab/ground interaction and the influence of the slab and ground on the dynamic wheel/rail contact forces has been studied in the frequency range up to 200 Hz.

The study showed that the ground/slab interaction has a second order influence on the calculation of the contact force. Deviations in the contact force occurred only in the frequency range where the track receptance was higher than the wheel receptance. The deviations increased with the slab thickness becoming smaller and the ground becoming softer, but stayed below 2 dB in all studied cases.

These results indicate that the approximation of uncoupling the ground/slab model and the wheel/rail interaction model is a good approximation if we intend to use this force for the prediction of railway-induced vibrations in a building. This is especially true with regard to the high amount of uncertainty in the input data (e.g. the ground properties) in such a model.

REFERENCES

5. Knothe KL, Grassie SL. Modelling of railway track and vehicle/track interaction at high frequencies.