Application of NAH method for the prediction of sound radiation from a flexible box structure
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ABSTRACT
There are different direct and indirect techniques existing to know the sound radiated from a machine or structure. Direct methods such as sound intensity method, sound pressure method and indirect methods or inverse methods such as Near-field Acoustic Holography (NAH) and beam forming are popular. NAH is one of the good inverse technique to predict the vibro-acoustic properties of a sound source at lower frequencies. As the NAH technique is an ill-posed inverse problem, regularization technique is used to overcome the problem. The purpose of the present investigation is to study the applicability of NAH method for source reconstruction at structural-acoustic coupled frequency. A box structure with one wall flexible is considered as a coupled system with an acoustic source inside the box. Sound pressure distribution on a flexible wall surface is reconstructed at uncoupled and coupled frequencies by using the NAH technique based on Equivalent Source Method (ESM). The effect of the Signal-to-Noise Ratio (SNR) on the reconstruction accuracy is also studied. The reconstructed results are good at the uncoupled frequency when compared to the coupled frequency.

Keywords: Near Field Acoustic Holography, Sound radiation, Regularization. (I-INCE Classification of Subjects Number(s):75.7, 23.1)

1. INTRODUCTION
Analysis of sound radiated from a flexible structure is important for effective noise control. There are some techniques to analyze the sound source characteristics from the machine or the structure such as sound intensity method, surface contribution method and near-field acoustic holography (NAH). Among these, NAH is a good technique to reconstruct the acoustic parameters such as sound pressure, particle velocity and the sound power by using measured sound pressure in a holographic plane (1). NAH technique has high-resolution when compared to conventional holography method and it was first introduced by Maynard and Williams (1, 2). Since as planar NAH is confined only to the regular shapes, Borgiotti et al. developed methods for the complicated shaped geometries (3, 4). Later Bai developed four holography transformation algorithms for arbitrarily shaped vibrating surfaces, based on boundary element method (BEM) (5). The difficulty of implementation of NAH on large vibrating surfaces has been overcome by new patch NAH methods (6, 7). Meantime Sarkissian introduced a new method called as an equivalent source method (ESM) (8) since the BEM method requires a large computational resources for complex structures. In ESM method, the sound field is represented by a set of standard sources like a monopole and dipole. An important advantage is that the computational time required for ESM method is much lesser than the other NAH methods. Recently, Valdivia et al. developed a new near-field acoustic holography surface decomposition method, which combines the patch based and ESM based NAH methods (9).

One of the challenges in the NAH technique is to overcome the ill-posedness using regularization methods (10). The most commonly used method in NAH process is the Tikhonov regularization with L-curve parameter selection technique (11). The characterization of noise radiated from the flexible structure which encloses the sound source is of great interest to understand the coupling behavior and the noise radiation mechanism. Sound radiation characteristics of box type structures due to structural

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excitation were investigated by Lin and Pan (12) using finite element method (FEM) and boundary element method (BEM) method.

The present study aims to understand the applicability of NAH (ESM) method for structural-acoustic coupled system problems. This prediction will be helpful to understand the noise radiated from a thin structure like an engine cover or from a transformer tank or from air-conditioning cover. FEM-BEM based simulations are used to calculate the radiated sound pressure due to acoustic source excitation placed inside the box structure in the present work. The sound pressure distribution on the flexible surface is reconstructed by ESM based NAH technique at an uncoupled and coupled frequency. The effect of measurement error or background noise on the reconstruction accuracy is studied by varying the signal-to-noise ratio (SNR) or dynamic range.

2. THEORY

2.1 Near-field acoustic holography (NAH)

This section gives a brief theory about the basic principle of Near-Field Acoustic Holography. NAH is an array technique, in which the acoustic parameters can be reconstructed by measuring sound pressure with an array of microphones, in parallel and near to the sound source. Figure 1 depicts the measurement method involved in the NAH technique. The sound source is located at $Z_s$ and the sound pressure is measured on the holography plane located at $Z_h$ with distance $d$ from the source. The sound field can be reconstructed on any plane in between the source plane and holographic plane using the current technique.

![Figure 1 – Schematic diagram of NAH method with different planes.](image)

The sound radiated from the source can be measured at any set of points in terms of sound pressure, which is given as (13)

$$\{p_h\}_M = [A]_{MN} \{q_s\}_N$$  

Where $p_h$ and $q_s$ denote the measured pressure and an acoustic source strength, respectively. $A$ is the transfer matrix which relates to the known pressure to the unknown source strength. $M$, $N$ are the number of measurement points and reconstructed points. The procedure involved in the calculation of a transfer matrix (TM) varies for different NAH methods. In an Equivalent source method (ESM), transfer matrix can be obtained by free space Green’s function, for Inverse boundary element method (IBEM) transfer matrix can be obtained by using Helmholtz integral equation and in Statistically optimized NAH (SONAH) method transfer matrix can be calculated by wave components of a plane-wave expansion (elementary wave functions). As NAH methods are an ill-posed inverse problem so regularization is necessary to obtain better reconstruction results. The regularized solution can be given by the following expression,

$$q_s = (A^H A + \lambda^2 I)^{-1} A^H p_h$$  

Where, $I$ is the identity matrix, $H$ denotes Hermitian transpose, and $\lambda$ is a regularization parameter. The particle velocity can be calculated by using reconstructed sound pressure. An active intensity of the sound source can also be calculated using the reconstructed pressure and particle velocity.
2.2 **Signal-to-noise ratio (SNR)**

It is defined as the ratio of signal power to the noise power. It represents an arithmetical difference between the signal level in decibels and noise level in decibels. It can be written as (13),

\[
SNR = 20 \log_{10} \left| \frac{P_{\text{signal+noise}} - P_{\text{noise}}}{P_{\text{noise}}} \right| \tag{3}
\]

where, \(P_{\text{signal+noise}}\) is the measured signal with the presence of noise and \(P_{\text{noise}}\) is the noise signal.

2.3 **Reconstruction error**

The accuracy of the sound source parameters reconstruction can be expressed in terms of reconstruction error. The reconstruction error can be written in general form as (14)

\[
\frac{\|P_s - P_{s,\lambda}\|_2}{\|P_s\|_2} \times 100\% \tag{4}
\]

where, the vectors \(P_s\) and \(P_{s,\lambda}\) represents the measured and reconstructed sound pressure, respectively.

3. **NUMERICAL MODEL**

A schematic and numerical model of the sound pressure measurement setup for the present study is shown in Figure 2. The model consists of a rectangular box with dimension's 0.868*1.15*1 m\(^3\) and 10 gauge (2.5 mm) and the sound source is located at [0.145, 0.15, 0.15 m] inside the box structure. The material properties of the box are: Young’s modulus is 71 GPa, Density is 2700 kg/ m\(^3\) and Poison’s ratio is 0.3. The finite element / boundary element (FEM-BEM) based numerical model with coupled analysis option is used to calculate the radiated sound pressure and power from the flexible wall. The structural part is discretized by using SHELL63 elements and acoustic volume with SOLID185 elements. First, the vibration of the flexible wall structure due to acoustic source excitation is predicted by FEM Acoustics- FEM structural coupled model. The sound pressure radiated from the flexible wall surface to the surrounding field is calculated by BEM analysis. The holographic plane dimensions and spacing between measurement points are chosen based on the NAH principle and the interested frequency range.

![Holographic plane and source](image)

**Figure 2**– Representation of sound pressure measurements: (a) Schematic (b) Numerical model

The measurement (holographic) plane has a dimension of 1.0*1.3 m\(^2\) with an 11*11 number of microphones located at 0.25 m from the top plate surface of the box. The distance between microphones along x-direction is 0.1m and along y-direction is 0.13 m as shown in Figure 2(b).

4. **RESULTS AND DISCUSSION**

The sound pressure at the holographic surface (filed points) is calculated by the FEM-BEM numerical simulation, and the radiated sound power is also calculated. The calculated sound pressure is assumed as a measured data for the current study. It is used for reconstruction of acoustic parameters on a source plane by an equivalent source method (ESM). Tikhonov regularization with L-curve parameter selection method is used to solve the ill-posed inverse problem. The measured results are
compared with reconstructed results and the effect of SNR (dynamic range) on reconstruction error is studied.

4.1 Acoustic parameters

The sound pressure and sound power are measured on a holographic plane near the source surface. Figure 3 (a) shows the sound power level radiated from a flexible wall surface. It is observed from the plot that the sound radiates efficiently at 23 Hz and 196 Hz, which are uncoupled structural and uncoupled acoustic frequencies. It is also observed from the graph that there is a split in natural frequencies, which is due to the coupling of 147.5 Hz structural frequency with 147.83 Hz acoustic frequency. The 147 Hz uncoupled frequency is split into 145 Hz and 149 Hz frequencies due to a strong coupling i.e due to the energy exchange between the structural and acoustic subsystems.

![Radiated sound power level at 23 Hz and 196 Hz](image1.png)

The sound pressure is measured by a set of virtual microphones in the near field of the source at two frequencies such as uncoupled 23 Hz and coupled 149 Hz. Figure 3(b) shows the measured sound pressure level with respect to microphones. It is clearly seen that at all the microphones, the sound pressure level at 23 Hz is higher than the 149 Hz frequency. A similar behavior is observed from the sound power level plot (Figure 3(a)) where the radiated sound power at 23 Hz is more than the 149 Hz frequency.

4.2 Regularization

Regularization is important for NAH methods to avoid the amplification of the noise since the NAH methods are an ill-posed inverse problem. Tikhonov regularization with L-curve parametric selection method is the most optimal and robust method for the ESM based NAH methods (11). Therefore, L-curve method is used in the present analysis for selecting the regularization parameter. Figure 4 shows the plots of the L-curve at the interested frequencies such as 23 Hz and 149 Hz (Figure 3a).

![L-curves in ESM method for two frequencies](image2.png)

L-curve plots show that the regularization parameter ($\lambda$) for the 23 Hz frequency is 3.0821e-06 and for the 149 Hz frequency it is 0.003143.
4.3 Reconstructed parameters

The measured sound pressure with virtual microphones at the field points is used as an input to the NAH to reconstruct pressure distribution on a flexible wall surface at uncoupled and coupled frequency. To produce more realistic case, error signal (white noise) is added to the field pressure signal which is obtained by numerical simulations. The effect of noise such as measurement noise and background noise on the reconstruction accuracy is studied in terms of signal-to-noise ratio (dynamic range). Figure 5 shows a plot of reconstruction error for ensemble of microphones at 23 Hz and 149 Hz frequencies for different SNR values. It is observed from the plot that at an uncoupled frequency (23 Hz) the error is varied uniformly as SNR values change. The uniform directivity of radiated sound behavior is not influenced by noise signals. However, for the coupled frequency, the reconstruction error is not uniformly changing because the radiated sound includes the higher-order modes. It is clear from the reconstruction error graphs that the minimum value of SNR should be 40 dB to obtain an acceptable reconstruction error.

![Figure 5 - Reconstruction errors for different SNR. Left: 23 Hz, Right: 149 Hz.](image)

As it is observed from the reconstruction error plots (Figure 5) that the SNR of 50 dB is appropriate to obtain good reconstruction results. Therefore, in the present work, an SNR value of 50 dB is used for the reconstruction of the source at 23 Hz and 149 Hz frequencies. The ESM based NAH technique is used for reconstruction of the sound pressure on the source plane. The 11*11 number of virtual sources are used with a spacing of 0.0868 m in the x-direction and 0.115 m in the y-direction. The optimal retreat distance, RD (distance between the virtual plane and the actual source plane) should be from one to two times of microphone spacing (11). The RD of -0.1736 m for 23 Hz frequency and -0.0868 m for 149 Hz is used in the analysis to locate the virtual sources.

The reconstructed results obtained by ESM technique are compared with the measured results which are shown in Figure 6 and Figure 7 for 23 Hz and 149 Hz frequencies, respectively.

![Figure 6 - Measured and reconstructed sound pressure at 23 Hz frequency. Left: Measured, Middle: Reconstructed, Right: Comparison](image)

Figure 6 shows the contour plots of measured and reconstructed sound pressure at a frequency of 23 Hz. It shows that the reconstructed results are matching with the measured results. The reconstructed
results can also be verified with the measured data in terms of the line plot instead of a contour plot by comparing the sound pressure level at each microphone's positions as shown in Figure 6. It is observed that the results are in good agreement with less than 10% error.

Figure 7– Measured and reconstructed sound pressure at 149 Hz frequency. Left: Measured, Middle: Reconstructed, Right: Comparison

Figure 7 shows the comparison of measured and reconstructed sound pressure in the form of contour plot and line plot at a frequency of 149 Hz. It is observed from the plot that there is a discrepancy between the measured and reconstructed results. The noise radiation pattern varies as it keeps moving away from the source surface to the holographic plane since the evanescent waves presented in the near-field decay exponentially. And also the noise radiated at 149 Hz is due to the coupling effect. So, coupling effect is stronger on the source surface than the measurement plane. Therefore, the measured pressure may not have the information of the evanescent waves, and it leads to the errors in the reconstruction results.

5. CONCLUSIONS

In the present paper, NAH technique based on equivalent source method is applied to reconstruct sound pressure distribution on a flexible surface in structural-acoustic coupled systems. A box with one wall flexible surface is considered for the study. Tikhonov regularization with L-curve parameter selection method is used to solve the ill-posed problem. The effect of noise (which is induced in the measured pressure) on the reconstruction accuracy is studied in terms of signal-to-noise ratio (SNR). From the reconstructed results, it is observed that the sound source parameters can be accurately reconstructed at an uncoupled frequency with less than 20% error at 50 dB SNR value. However, at the coupled frequencies even by using 50 dB SNR value, the reconstruction accuracy is poor. There is a scope to study the effect of the number of virtual sources, spacing between the virtual sources and retreat distance involved in the ESM technique to improve reconstruction error at coupled frequencies.

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