

## Study the efficiency of dynamic vibration absorbers in minimizing vibration power flow radiated from a double-deck tunnel

Behshad NOORI<sup>1</sup>; Arnau CLOT<sup>2</sup>; Robert ARCOS<sup>3</sup>; Jordi ROMEU<sup>4</sup>

<sup>1</sup> AV Ingenieros company, Spain

<sup>2,3,4</sup> Universitat Politècnica de Catalunya, Spain

### ABSTRACT

In the present study, dynamic vibration absorbers (DVAs) are introduced as innovative countermeasures in ground-borne vibration control of double-deck tunnel, which is an innovative tunnel design that has been recently implemented in some underground railways and metro lines worldwide. The analytical model based on the receptance method is employed to calculate the mean power flow radiated from a double-deck circular tunnel. The model describes the dynamics of the interior floor using the thin plate theory and considers the Pipe-in-Pipe (PiP) model to describe the coupled tunnel-soil system. DVAs are implemented on the interior floor of the tunnel and their efficiency in minimizing radiated power flow is studied. The effectiveness of DVAs depends on the parameters like their masses, damping ratios, natural frequencies and positions. Therefore, the most favorable design of DVAs requires optimal parameters which are found using Genetic Algorithm (GA). Evaluating the efficiency of the optimal DVAs shows that they are efficient countermeasures to mitigate underground railway-induced vibration.

Keywords: Railway-induced Vibration, Dynamic Vibration Absorber, Double-deck Tunnel

I-INCE Classification of Subjects Numbers: 13.4.1, 46.2

### 1. INTRODUCTION

Nowadays, fast and easy commuting is one of the most important necessities in our life; it can be satisfied by use of underground railways. One of the innovative designs of underground tunnels, implemented in Line 9 of the Barcelona underground railway system, is the double-deck tunnel (Figure 1), in which the tunnel structure is divided into two parts by an intermediate floor supported on the tunnel walls. Not only do these provide timely and costly effective transportation but also cause less air pollution and noise emission. In spite of their healthy, economic benefits, generated vibration propagates through the tunnel and surrounding soil into nearby buildings and can cause annoyance to the building dwellers and structural damage. Therefore, efficient countermeasures, mainly grouped as isolating the source or receiver and interrupting the vibration path, are needed. In order to employ them, accurate vibration prediction models are needed. Those models can be classified in three principal types: numerical, experimental and analytical models.

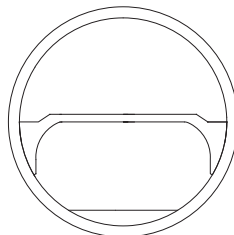


Figure 1 – Cross-section of the Barcelona's underground railway Line 9; double-deck circular tunnel.

<sup>1</sup> behshad.noori@avingenieros.com

<sup>2</sup> arnau.clot@upc.edu

<sup>3</sup> robert.arcos@upc.edu

<sup>4</sup> jordi.romeu@upc.edu

Numerical models are powerful tools to estimate underground railway-induced vibration, however, these demand great computational costs. Empirical prediction models present easier, cheaper and faster ways for making a prediction of vibration, while neither of them is suitable for parametric studies. Analytical models results in better understanding of the dynamic behavior of the system, require far less computational time and also gives the possibility of doing parametric studies. One of the most well-established analytical models is the PiP model. Forrest and Hunt (1) developed the PiP model as a 2.5D semi-analytical model in order to compute soil vibration due to underground railway traffic in a full-space. The soil response obtained with the PiP model is compared with the one obtained using finite element-boundary element (FE-BE) model in the work of Gupta et al. (2); good agreement in results is found, being the PiP model computationally more efficient. The PiP model has been used by Hussein and Hunt (3) to evaluate vibration countermeasure performance. It has been also employed by Hussein et al. to calculate vibration from underground railways in a homogeneous (4) and multi-layered half-space (5).

Coupled plate-cylindrical shell structures have been studied by several researchers because of its interest as airplane fuselage models. Peterson and Boyd (6) present the first analytical model for a shell with a partitioned floor. Using the receptance method, Lee et al. (7) obtained the free vibrations of a simply supported shell-plate structure. The receptance method was also used by Wang et al. (8) and Zhao et al. (9) to study the power flow characteristics and the forced response of the plate-cylindrical shell structure. In the first semi-analytical double-deck model, presented by Clot (10), the PiP model is coupled to an infinite strip plate model. The model was used by Clot et al. (11) to compare the power flow radiated by a double-deck tunnel and a simple tunnel.

In order to control the vibration induced by underground railways, different countermeasures have been proposed. The most used solutions are interrupting the vibration path by using vibration isolation screens (12), open and in-filled trenches (13) or a row of piles (14). Another well-known control devices for cutting the vibration path are DVAs or tuned mass dampers (TMDs). Application of DVAs is an effective way of improving the dynamic performance of the structures like bridges, wide-spanned floors and high-rise buildings.

The concept of a DVA was outlined by Watts in 1883 (15). However, the practical design of vibration absorbers was proposed by Frahm in 1911 (16). During the last century, because of the important role of DVA in vibration attenuation, many studies have been conducted to evaluate the performance of the absorbers as a passive, active and semi-active countermeasures (17). The application of largest TMD in the world (660 tons) in the second tallest skyscraper in the world at Taipei 101 (18), utilization of eight horizontal and fifty vertical DVAs in Millennium bridge in London (19) and the installation of a 140 tones pendulum absorber in Doha sport city tower in Qatar (20) can be named as some of the most prominent and spectacular usage of DVAs. Optimization of the DVA parameters is one of the most important issues in order to lead to the highest levels of DVAs performance in vibration mitigation (21).

According to the vibration isolation performance of DVAs in vibration control, it is expected to be also an efficient abatement solution for railway-induced vibrations. This efficiency has been evaluated in this study. The remainder of the paper is organized as follows. Firstly, a 3D model of a double-deck circular tunnel embedded in a full-space, developed by Clot et al. (10), is described. Afterwards, it is explained how to couple DVAs to the interior floor. Then, a genetic algorithm (GA) is employed in order to find the optimum parameters of the DVAs, which are those that minimize the mean power flow radiated by a double-deck circular tunnel under the action of a harmonic line load. Finally, the efficiency of DVAs is evaluated by defining the insertion loss (IL) in mean power flow due to inserting DVAs.

## 2. MODEL DESCRIPTION

The double-deck tunnel (a tunnel divided by an interior floor) is modelled in this work as an infinitely long circular cylindrical shell of constant thickness  $h_t$  and constant mean radius  $r_t$  divided into two equal parts by an interior floor of constant thickness  $h_p$  and no curvature. The model used for describing the dynamic response of a double-deck tunnel structure embedded in a homogeneous soil is schematized in in Figure 2. Noteworthy, as both subsystems are invariant along the longitudinal direction, the problem can be decomposed into a set of two-dimensional (2D) models which depend on the wavenumbers along the invariant direction.

In the next subsections, the mechanical models for each subsystem and the coupling conditions are explained briefly. The reader is advised to consult with references (1,11) for more details.

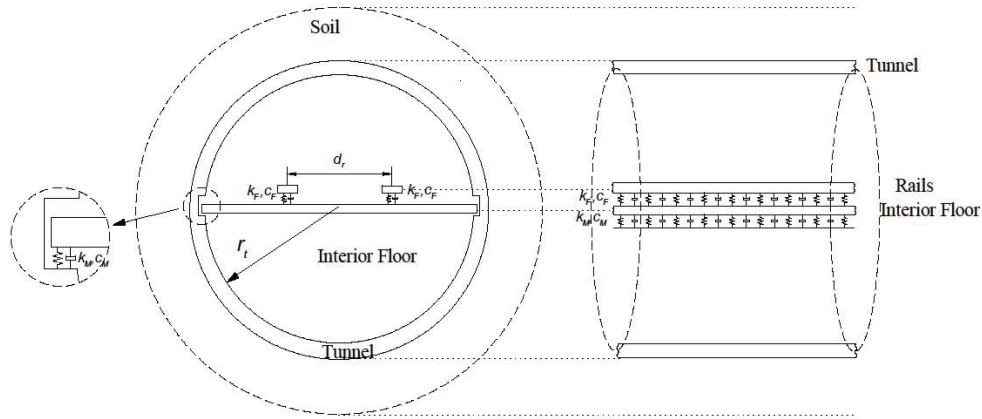


Figure 2 - The proposed double-deck tunnel model.

### 2.1 Interior Floor Model

The interior floor is considered to be a homogeneous isotropic strip plate of constant cross-section with thickness  $h_p$  width  $L_p$  and no pre-stressing effects acting on it. The ratio between the thickness and the width is assumed to be small enough to consider thin plate theory. The deflection of the plate, due to a unitary harmonic line load, is assumed to be harmonic. The general solution of the equation of motion of a thin plate in plane-strain conditions is obtained using the modal participation method.

### 2.2 Tunnel-Soil Model

The PiP model, as the name indicates, can be understood as the coupling of two pipe structures. The inner and the outer pipes, with outer radius being set to infinite, represent the tunnel wall and the surrounding boundless soil with cylindrical cavity, respectively. The former is modelled by using thin shell theory and the soil is modeled as a linear elastic solid. The coupling between both systems is done considering that, at the interface, their displacement fields are equal and that the stresses that one causes to the other are equal and opposite. The PiP model is to obtain the soil response to a load applied at the interior floor of a double-deck tunnel.

### 2.3 Superstructure

The rails are represented as Bernoulli-Euler beams of infinite length. Both rails are coupled to the tunnel's interior floor with direct fixation fasteners. The rail pads are modelled as a continuous layer with a constant stiffness per meter  $k_F$  and a constant viscous damping per meter  $c_F$ . Two vertical harmonic point loads are assumed to be applied at both rails. The rails are centered in the interior floor and separated a distance  $d_r$ .

### 2.4 Coupling of the Systems

The coupling of the interior floor and the tunnel-soil systems is done using the receptance method. The interaction between the tunnel walls and the interior floor of a double-deck tunnel depends on the construction method used to build it. In many cases, the interior floor is a separate precast slab structure supported on the tunnel walls. This double-deck tunnel case is the one assumed in this work and has been modelled using pinned connections at both edges of the interior floor. With this coupling conditions, only vertical harmonic loads in the interior floor and tangential harmonic loads in the tunnel-soil system are required.

## 3. COUPLING DVAs TO THE INTERIOR FLOOR

In this section, it is explained how to coupled DVAs to the interior floor which is coupled to the tunnel embedded in a full-space. Initially, in order to achieve a better understanding of the problem, only one periodic distribution of DVAs along the track direction is considered (see Figure 3).

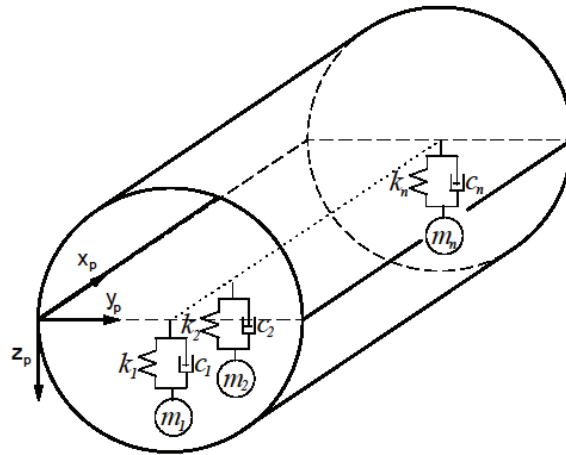


Figure 3 – One longitudinal periodic distribution of DVAs.

The applied forces to the system due to DVAs and external forces on the rails can be written as

$$f(x, t) = p(t)\delta(x) + \sum_{n=1}^{+\infty} \left( k_n + c_n \frac{\partial}{\partial t} \right) (u_n^{\text{DVA}} - u_n) \delta(x - x_n), \quad (1)$$

where the first and second terms in the right-hand side of the equation are the external force and the summation of the DVAs' forces, respectively.  $k_n$  and  $c_n$  are the stiffness and viscous damping of the  $n^{\text{th}}$  DVA, respectively.  $x_n = nL$ ,  $L$  is the distance between two consecutive DVA,  $u_n$  is the displacement of the system at the position and direction of the  $n^{\text{th}}$  DVA, and  $u_n^{\text{DVA}}$  is the displacement of center of mass of the  $n^{\text{th}}$  absorber; the dynamical response of each DVA represented as

$$-\left( k_n + c_n \frac{\partial}{\partial t} \right) (u_n^{\text{DVA}} - u_n) = m_n \ddot{u}_n^{\text{DVA}}. \quad (2)$$

The Fourier transform has been defined as

$$\bar{F}(k_x, \omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, t) e^{-i(\omega t - k_x x)} dx dt, \quad (3)$$

where upper line and capital letters denote that the variables are defined in the wavenumber and frequency domain, respectively. This Fourier transform has been applied on both Eqs. (1) and (2). So, displacement of center of mass of the  $n^{\text{th}}$  absorber in wavenumber-frequency domain can be found as

$$U_n^{\text{DVA}} = \frac{k_n + i\omega c_n}{k_n + i\omega c_n - m_n \omega^2} U_n \quad (4)$$

By replacing the  $u_n^{\text{DVA}}$  in the transformed form of the Eq. (1) with its equivalent from Eq. (4), the applied forces to the system, Eq. (1), can be rewritten in the wavenumber-frequency domain as

$$\bar{F} = P + \sum_{n=1}^{+\infty} k'_n (\bar{U}_n) e^{ik_x x_n}, \quad (5)$$

where

$$k'_n = (k_n + i\omega c_n) \left( \frac{k_n + i\omega c_n}{k_n + i\omega c_n - m_n \omega^2} - 1 \right) \quad (6)$$

As the forces applied to the system are defined in the wavenumber-frequency domain, Eq. (5), by multiplying them to their associated receptances, response at the DVAs' positions can be written as

$$U_b(k_x, \omega) = H(k_x, \omega) + \sum_{n=1}^{+\infty} H_n(k_x, \omega) k'_n U_n(k_x, \omega), \quad b = 1, 2, \dots, N. \quad (7)$$

By approximating the summation with finite number of DVAs, and applying one inverse Fourier transform, the response at the DVAs' positions can be written as

$$U_b(x_b, \omega) = H(x_b, \omega) + \sum_{n=1}^N H_n(x_b - x_n, \omega) k'_n U_n, \quad b = 1, 2, \dots, N, \quad (8)$$

where  $N$  is the number of the DVAs along the longitudinal direction; it will be defined in the

optimization process.  $H$  and  $H_n$  are the response at the DVAs' positions due to two unitary harmonic load on the rails and the  $n^{th}$  DVA, respectively.

This formulation can be extended for the case of more than one DVAs' distribution along the plate

$$U_{sl} = H(x_{sl}, \omega) + \sum_{p=1}^P \sum_{q=1}^Q H_p(x_{sl} - x_{pq}, \omega) k'_{pq} U_{pq}, \quad s = 1, 2, \dots, P; l = 1, 2, \dots, Q. \quad (9)$$

$P$  and  $Q$  are the number of the DVAs between connections at both edges of the interior floor and along the longitudinal direction of the plate; these are called transverse and longitudinal distributions in the following. The position of the DVAs are defined by their subscripts. Eqs. (8) and (9) result in a system of equations with  $N$  and  $P \times Q$  equations, respectively. By solving that system of equations, the response at the DVAs' positions can be determined. Once those are found, the DVAs' forces can be found by multiplying them to their stiffness.

#### 4. POWER FLOW CALCULATION

The mean local power flow  $P$  at a certain point of the soil is defined as

$$P = \frac{1}{T_0} \int_0^{T_0} \{ \text{Re}[v_r(t)] \text{Re}[\tau_{rr}(t)] + \text{Re}[v_\theta(t)] \text{Re}[\tau_{r\theta}(t)] + \text{Re}[v_x(t)] \text{Re}[\tau_{rx}(t)] \}, \quad (10)$$

where  $T_0$  is the integration time,  $\tau_{rr}$ ,  $\tau_{r\theta}$  and  $\tau_{rx}$  are stress and velocities of the considered point are

$$\begin{pmatrix} v_x \\ v_r \\ v_\theta \end{pmatrix} = i\omega \begin{pmatrix} u_x \\ u_r \\ u_\theta \end{pmatrix} e^{i\omega t}. \quad (11)$$

By calculating the displacements and stresses due to unitary harmonic forces at rails and DVA's forces using the methodology in section 2, choosing the integration time to be  $n$  times the harmonic load and integrating Eq. (11), the mean power flow due to each force, radiated through a circular section defined by angles  $\theta_1$  and  $\theta_2$ ,  $x_1$  and  $x_2$  and radius  $r_0$ , can be obtained as

$$P(x_1, \theta_1, x_2, \theta_2) = \int_{x_1}^{x_2} \int_{\theta_1}^{\theta_2} \{ \text{Re}[U_r] \text{Im}[T_{rr}] - \text{Im}[U_r] \text{Re}[T_{rr}] + \text{Re}[U_\theta] \text{Im}[T_{r\theta}] - \text{Im}[U_\theta] \text{Re}[T_{r\theta}] + \text{Re}[U_x] \text{Im}[T_{rx}] - \text{Im}[U_x] \text{Re}[T_{rx}] \} dx d\theta. \quad (12)$$

The mean power flow radiated upwards by the double-deck tunnel in the presence of the DVAs, is the summation of the mean power flow radiated due to forces applied at the rails and DVAs' forces. This global mean power flow is the one that is considered in optimization process that is explained later.

#### 5. OPTIMIZATION OF DVAs

The aim of this section is developing an optimization algorithm with the purpose of finding the parameters of the DVAs that minimize the mean power flow radiated upwards. In order to solve the optimization problem a GA using MATLAB, in which the design variables are the DVAs parameters with lower and upper bounds constraints, will be implemented. Furthermore, parallel processing has been employed to speed up the optimization algorithm.

The fitness function that GA minimize in this work is the computation of the mean power flow radiated upwards due to two unitary harmonic loads applied at the rails. This function is computed as follows:

- 1) Defining the design variables such as the number of the DVAs between connections at both edges of the interior floor and along the longitudinal direction of the plate, their positions, stiffness, damping and masses. These data are managed as an input for the second function. The lower and upper bounds constrains for mass are 500 and 1000 kg, respectively; and for stiffness are 5 and 50 MN m-1. The damping is considered as 1% of the stiffness and it is not defined as an independent design variable. The minimum distance between consecutive DVA is considered as one meter. Noteworthy, the maximum of six DVAs between connections at both edges of the interior and maximum of forty DVAs for each longitudinal distribution are taken into account.
- 2) Calculating the DVAs' forces by use of the methodology explained in section 3, considering

the characteristics of the DVAs chosen in the previous step.

- 3) Calculating the mean power flow of the double-deck tunnel, subjected two unitary harmonic force at rails in the presence of the DVAs in space-frequency domain by using methodology explained in section 4. This mean power flow radiated upwards is the output of the fitness function which is intended to be minimized.

## 6. RESULTS AND DISCUSSION

In this section the efficiency of DVAs in minimizing mean power flow, radiated upwards, is evaluated. The chosen mechanical parameters for the infrastructure, interior floor, tunnel wall and soil can be found in Table 1, 2, 3 and 4, respectively. In all tables,  $E$ ,  $\rho$ ,  $\eta$  and  $\nu$  represent the Young's modulus, density, loss factor and Poisson's ratio, respectively.  $d$  is the distance between two rails.  $I_p$  is second moment of area of the rail cross-section.  $D_p$  and  $D_s$  are the hysteretic damping ratios for P- and S-waves. The values for interior floor and the tunnel wall are the typical ones for reinforced concrete. The frequency range of [1 – 80] Hz is considered as the ground-borne vibration underground railways are perceived in this frequency (5).

Table 1 – Parameters used to model the rail.

Parameters	$I_r$ , m	$E_r$ , GPa	$\rho_r$ , kg m <sup>-3</sup>	$\eta_r$	$d$ , m
Values	0.4	207	7850	0.02	1.8

Table 2 – Parameters used to model the interior floor as a thin plate.

Parameters	$L_p$ , m	$h_p$ , m	$E_p$ , GPa	$\rho_p$ , kg m <sup>-3</sup>	$\eta_p$	$\nu_p$
Values	10	0.4	25	3000	0.02	0.18

Table 3 – Parameters used to model the tunnel wall.

Parameters	$r_t$ , m	$h_t$ , m	$E_t$ , GPa	$\rho_t$ , kg m <sup>-3</sup>	$\eta_p$	$\nu_p$
Values	5.2	0.4	25	3000	0.02	0.18

Table 4 – Parameters used to model the soil as an elastic continuum.

Parameters	$E_s$ , MPa	$\rho_s$ , kg m <sup>-3</sup>	$D_p$	$D_s$	$\nu_p$
Values	100	2000	0.02	0.02	0.3

The fitness function and the design variables need to be defined before calling the GA. The power flow radiated upwards due to two unitary harmonic loads applied on the rails for  $\theta_1 = 0, \theta_2 = \pi, x_1 = -511$  m and  $x_2 = 512$  m is considered as the objective function that is intended to be minimized. Five design variables are considered:

- 1)  $DVA_{\text{position}}$ : The position of transverse distribution of DVA (see Figure 4-a);
- 2)  $N_{DVA}$ : Number of DVAs in each transverse distribution along longitudinal direction;
- 3)  $DVA_{\text{stiffness}}$ : The stiffness of the DVA;
- 4)  $DVA_{\text{mass}}$ : The mass of the DVA;
- 5)  $DVA_{LD}$ : Distance between two consecutive DVAs along longitudinal direction.

The damping of the DVAs is considered 1% of their stiffness. The upper and lower bounds constraints of the DVAs can be found in Table 5. Analyzing of the initial results showed that more than six DVAs in each transverse distribution has a very slight effect in vibration mitigation, so, maximum of six DVAs is considered as the upper bounds for  $N_{DVA}$ . Lower bound of the DVAs' stiffness and upper bound of the DVAs' mass are chosen by taking into account the maximum of 1 mm static deflection due to the mass weight. Noteworthy, all the DVAs in the same longitudinal distribution have the same

properties.

Table 5 – The design variables and their upper and lower bounds constraints in optimization process.

Design variables	DVA <sub>position</sub>	N <sub>DVA</sub>	DVA <sub>stiffness</sub> , MN m <sup>-1</sup>	DVA <sub>mass</sub> , kg	DVA <sub>LD</sub> , m
Lower bounds	1	1	50	500	1
Upper bounds	41	6	500	800	10

In order to decrease the computational costs of the optimization process, the following receptances are calculated before calling GA:

- 1) Receptances needed to determine the DVAs' forces in section 3: the response at all DVAs' possible positions (see Figure 4-a) due to two unitary forces applied on the rails  $H$  and unitary forces applied on each DVAs position  $H_p$ .
- 2) Receptances needed to calculate mean power flow: displacements and stresses at the upper half of 10 m radius circumference (see Figure 4-b) due to unitary forces applied at all possible DVAs' positions and two unitary forces at the rails.

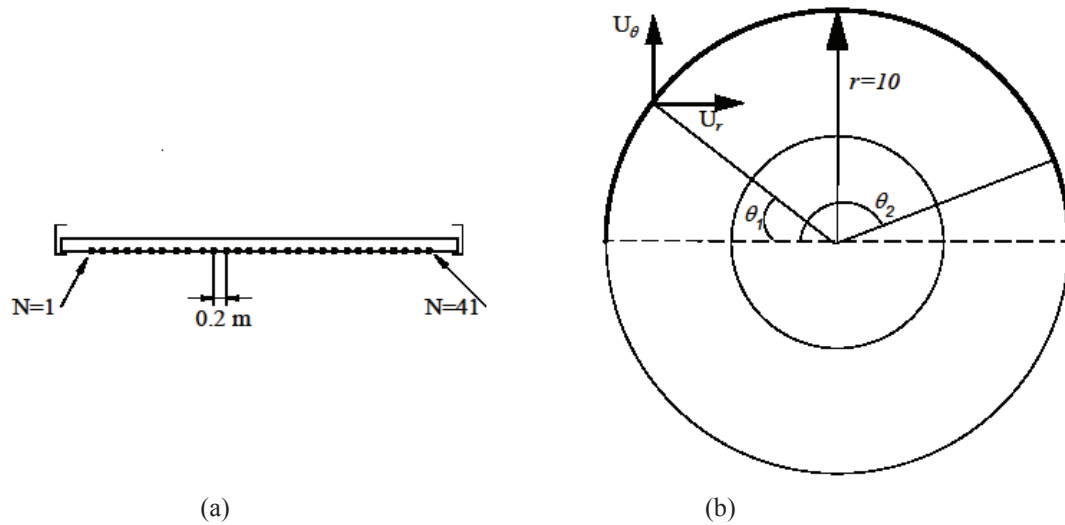


Figure 4 – (a) All DVAs' possible positions; (b) chosen surface for computing radiated power flow.

After calculating all receptances and defining the design variables (Table 5), the GA is called in order to find the optimum parameters of the DVAs. In the optimization process, first, only one distribution of DVAs is considered and optimized, then these parameters are used to find the optimum parameters of the second distribution and so on. The optimum parameters for the first distribution can be found in Table 6. As it was expected the minimum distance between two consecutive DVAs and maximum allowed mass are the optimized  $N_{DVA}$  and  $DVA_{mass}$ .

The mean power flow radiated upwards in a double-deck tunnel, in the presence of the first optimized distribution of DVAs are calculated and it is shown in Figure 5-a. As it can be seen DVAs has an impressive effect at frequency range in which the peaks occurred. The efficiency of DVAs can be evaluated better by defining the insertion loss (IL) as

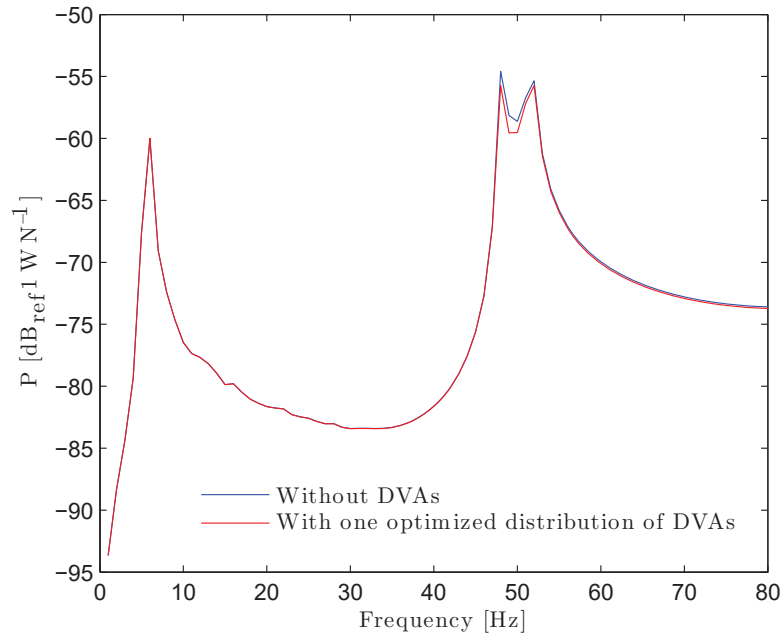
$$IL = 10 \log_{10} \left( \frac{P_{uniso}}{P_{iso}} \right), \tag{13}$$

where  $P_{uniso}$  and  $P_{iso}$  are the mean power flow without and with DVAs. Calculating the IL shows that by using only one distribution with six DVAs, total mean power flow can be reduced by 7.1 dB, which makes the DVAs remarkable in compare to other type of countermeasures. Furthermore, by taking into account the financial aspects, the DVAs becomes more astonishing as implementing the DVAs are much cheaper than other countermeasures that are commonly used like screens.

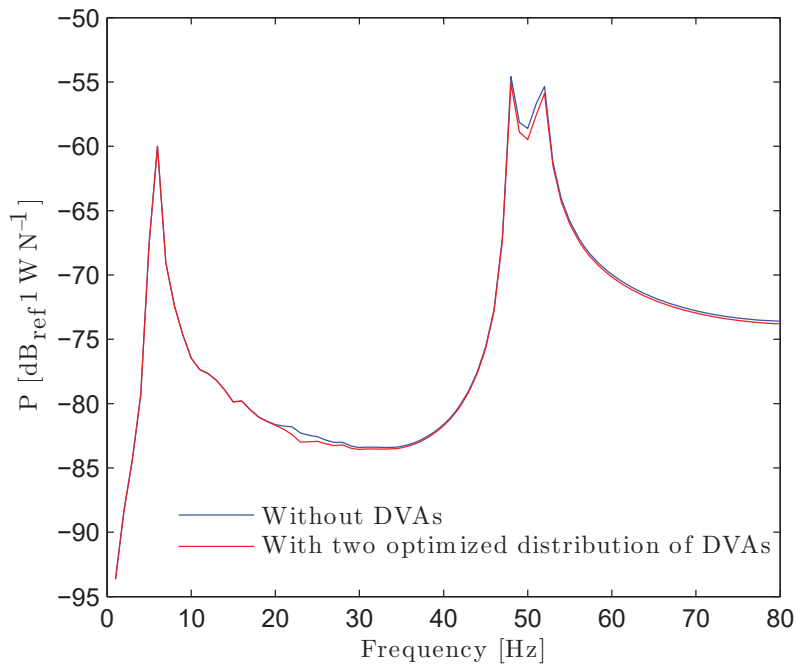
Once the optimum parameters of the first distribution has been achieved, these are used to find the optimum parameters for the second distribution (see Table 6). The mean power flow in the presence of two optimized distribution of DVAs is shown in Figure 5-b. 8.2 dB reduction in total mean power flow can be achieved by using two optimized distribution of DVAs.

Table 6 – Optimum parameters of the first and second distributions of DVAs.

Design variables	DVA position	N <sub>DVA</sub>	DVA stiffness, MN m <sup>-1</sup>	DVA mass, kg	DVA longitudinal distance, m
First optimized DVA	21	6	27.9	800	1
First optimized DVA	39	6	21	800	1



(a)



(b)

Figure 5 – Upwards radiated mean power flow of a double-deck tunnel without and with (a) one optimized distribution, (b) two optimized distribution of DVAs.



## 7. CONCLUSIONS

In the present paper, the efficiency of DVAs as innovative countermeasures in minimizing the railway-induced vibration in a double-deck tunnel embedded in full-space has been studied. The recently analytical model for double-deck has been used to calculate the upwards mean power flow. In this model, the interior floor and the tunnel embedded in full-space has been modeled using thin plate theory and PiP model, respectively. The receptance method has been used in order to couple the interior floor to the tunnel and find the DVAs forces coupled to the interior floor. As efficiency of DVAs related to their parameters, GA has been used to find the optimum parameters of DVAs. According to the results, DVAs can be presented as an effective countermeasure in vibration control of underground railway-induced vibration as a total IL of 7.1 dB in mean power flow has been achieved by using only one distribution with six DVAs. This can be increased to 8.2 dB by inserting two optimized distribution with six DVAs for each distribution.

## ACKNOWLEDGEMENTS

This work has been carried out in the context of the Industrial Doctorates Plan, managed with financial support from AGAUR-Generalitat de Catalunya and AV Ingenieros, developed in a partnership with Universitat Politècnica de Catalunya (UPC). The authors would like to extend their thanks to ISIBUR project: “*Innovative Solutions for the Isolation of Buildings from Underground Railway-induced vibrations*” funded by the Spanish Government - (TRA2014-52718-R).

## REFERENCES

- Forrest JA, Hunt HEM. A three-dimensional tunnel model for calculation of train-induced ground vibration. *J Sound Vib.* 2006;294:678–705.
- Gupta S, Hussein MFM, Degrande G, Hunt HEM, Clouteau D. A comparison of two numerical models for the prediction of vibrations from underground railway traffic. *Soil Dyn Earthq Eng.* 2007 Jul;27(7):608–24.
- Hussein MFM, Hunt HEM. A power flow method for evaluating vibration from underground railways. *J Sound Vib.* 2006;293(3-5):667–79.
- Hussein MFM, Gupta S, Hunt HEM, Degrande G, Talbot JP. An efficient model for calculating vibration from a railway tunnel buried in a half-space. *Proceedings of the Thirteenth International Congress on Sound and Vibration, Vienna.* 2006.
- Hussein MFM, François S, Schevenels M, Hunt HEM, Talbot JP, Degrande G. The fictitious force method for efficient calculation of vibration from a tunnel embedded in a multi-layered half-space. *J Sound Vib.* 2014;333(25):6996–7018.
- Peterson MR, Boyd DE. Free vibrations of circular-cylinders with longitudinal, interior partitions. *J Sound Vib.* 1978;60(1):45–62.
- Lee YS, Choi MH. Free vibrations of circular cylindrical shells with an interior plate using the receptance method. *J Sound Vib.* 2001;248(3):477–97.
- Wang ZH, Xing JT, Price WG. A study of power flow in a coupled plate-cylindrical shell system. *J Sound Vib.* 2004;271(3-5):863–82.
- Zhao Z, Sheng M, Yang Y. Vibration Transmission of a Cylindrical Shell with an Interior Rectangular Plate with the Receptance Method. *Adv Acoust Vib.* 2012;2012:1–9.
- Clot A. A dynamical model of a double-deck circular tunnel embedded in a full-space. PhD thesis, 2014.
- Clot A, Romeu J, Arcos R, Martín SR. A power flow analysis of a double-deck circular tunnel embedded in a full-space. *Soil Dyn Earthq Eng.* 2014;57:1–9.
- François S, Schevenels M, Thyssen B, Borgions J, Degrande G. Design and efficiency of a composite vibration isolating screen in soil. *Soil Dyn Earthq Eng.* 2012;39:113–27.
- Andersen L, Nielsen SRK. Reduction of ground vibration by means of barriers or soil improvement along a railway track. *Soil Dyn Earthq Eng.* 2005 ;25(7-10):701–16.
- Kattis SE, Polyzos D, Beskos DE. Modelling of pile wave barriers by effective trenches and their screening effectiveness. *Soil Dyn Earthq Eng.* 1999;18(1):1–10.
- Watts P. On a method of reducing the rolling of ships at sea. *Trans Inst Nav Archit.* 1883;24:165–90.
- Frahm H. Device for damping vibration of bodies. US Pat No 989958. USA: U.S. Patent; 1911;
- Sun JQ, Jolly MR, Norris MA. Passive, Adaptive and Active Tuned Vibration Absorbers—A Survey. *J Mech Des.* 1995;117(B):234. 3
- Kourakis I. Structural systems and tuned mass dampers of super-tall buildings : case study of Taipei 101.

- Massachusetts Institute of Technology; 2007.
19. Newland DE. Vibration of the London Millennium Bridge: cause and cure. *Int J Acoust Vib.* 2003;8(1):9–14.
  20. Nawrotzki P. Tuned-mass systems for the dynamic upgrade. *Elev east asia-pacific Conf Struct Eng Constr.* 2008;1–9.
  21. Noori B, Farshidianfar A. Optimum design of dynamic vibration absorbers for a beam, based on H infinity and H two optimization. *Arch Appl Mech.* 2013;83(12):1773–87.