Analysis and parametric study of Fork muffler with and without H-connection

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ABSTRACT

Due to unavailability of enough space to mount a single large volume muffler under the vehicle, often the required volume of the muffler system is achieved by dividing the muffler system into two thinner mufflers (located on the two sides of the vehicle) of desirable length and cross-sectional area, which may be identical. This type of dual muffler is known as Fork muffler. Such a muffler falls under the category of the single-inlet multiple-outlet type. The performance of a fork muffler may be favorably altered by inter-connecting the two mufflers (H-connection), making use of the principle of selective phase cancellation. Although there is only one additional tube in the inter-connected fork muffler (fork muffler with H-connection), yet it substantially changes the acoustic impedance felt at the source. In order to understand the effect of the H-connection, we need to carry out analysis of the whole exhaust system.

This paper deals with the analysis of the fork muffler with and without H-connection followed by a parametric study in order to evolve some design guidelines. The analysis is carried out using the transfer matrix approach, the results of which have been validated by means of a 3-D finite element analysis.

Keywords: Fork Muffler, H-connection

1. INTRODUCTION

For paucity of space under the vehicle, a given muffler system may be divided into two identical mufflers of the same (desirable) length, but with half the cross-sectional area of the respective exhaust pipe, tail pipe, intermediate pipe and the muffler shell (Fig. 1). Fork mufflers (or dual mufflers) are very commonly used for high volume engines (engines with high mass flow rate, for example V6 engines), as splitting the muffler into two thinner mufflers eases the design process. Fork mufflers are also preferred to improve subjective noise; i.e., in addition to reduction of its overall sound pressure level (SPL), the noise spectral components at the low as well as high frequencies are optimized to improve the tone of the vehicle exhaust noise as perceived by the customers. The performance of the fork muffler may be favorably altered by inter-connecting the two mufflers, as phase cancellation may be enhanced by the connecting pipe due to the Herschel–Quincke Tube phenomenon (1-3).

Understanding the role of each component in a complex exhaust system is of key value to Faurecia as it helps in developing a more efficient exhaust system especially in the emerging markets. The analytical models and implementation of such analytical models in our linear models enhances the simulation capabilities of Faurecia and therefore brings benefit in the development process. This paper deals with the analysis of the fork-muffler with and without H-connection, followed by parametric studies in order to evolve some design guidelines.

2. ONE-DIMENSIONAL ANALYSIS OF FORK MUFFLER

Figure 1 shows the schematic diagram of a single muffler configuration and the corresponding fork muffler configuration with the same overall muffler volume and same acoustic performance as well as back pressure (3).

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The muffler proper used here is the three-pass double-reversal muffler with tubular bridges (4). The configuration of the three-pass double-reversal muffler with tubular bridges is shown in Fig. 2. Its one-dimensional analysis has been duly validated against 3-D FEM in reference (4) and is shown in Fig. 3.

(a) – Schematic diagram of the single muffler configuration

(b) – Schematic diagram of the equivalent Fork muffler configuration

Figure 1 - Schematic diagram of the single muffler configuration and its equivalent Fork muffler configuration

Figure 2 - Schematic diagram of the three pass double-reversal muffler with tubular bridges with mean flow distribution (4).
Figure 3 - Validation of the 1-D analysis by 3-D FEA of the muffler of Fig. 2 (Mach number, \( M = 0.2 \)), adopted from reference (4)

Using the transfer matrix obtained for the three-pass double-reversal muffler with tubular bridges (4), the governing equations for the fork muffler (3) are given below, where ‘\( p \)’ represents acoustic pressure and ‘\( v \)’ represents mass velocity:

\[
P_a = p_1 = p_4
\]
\[
v_u = v_1 + v_3
\]
\[
\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} J_{1,2} \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix}
\]
\[
\begin{bmatrix} p_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} J_{3,a} \end{bmatrix} \begin{bmatrix} p_4 \\ v_4 \end{bmatrix}
\]

Here, \([J_{i,i}]\) is the transfer matrix for the pipe element connecting points ‘i’ and ‘j’.

Using the transfer matrix obtained for the three-pass double-reversal muffler with tubular bridges (4), the transfer matrix for each muffler proper can be written as:

\[
\begin{bmatrix} p_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} T_{11,a} & T_{12,a} \\ T_{21,a} & T_{22,a} \end{bmatrix} \begin{bmatrix} p_{d,a} \\ v_{d,a} \end{bmatrix}
\]
\[
\begin{bmatrix} p_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} T_{11,b} & T_{12,b} \\ T_{21,b} & T_{22,b} \end{bmatrix} \begin{bmatrix} p_{d,b} \\ v_{d,b} \end{bmatrix}
\]

Now, at the radiation end (d, a) and (d, b), we have
\[ p_{d,a} = v_{d,a} Z_{0,a}(M) \]  \hspace{1cm} (12)

\[ p_{d,b} = v_{d,b} Z_{0,b}(M) \]  \hspace{1cm} (13)

where, \( Z_{0,a} \) and \( Z_{0,b} \) are the impedances (5) at the radiation end (d, a) and (d, b), respectively and ‘M’ is the mean flow Mach number.

There are thirteen equations (1 to 13) and fourteen unknown variables. So we can obtain the values of thirteen variables in terms of the fourteenth variable, say ‘\( v_u \)’. For ease of calculation, ‘\( v_u \)’ is taken as unity.

Now, ‘\( p_u \)’ and ‘\( v_u \)’ can be written in terms of complex amplitudes of progressive waves A and B as follows:

\[ p_u = A_u + B_u \]  \hspace{1cm} (14)

\[ v_u = \frac{A_u - B_u}{Y_u} \]  \hspace{1cm} (15)

where ‘\( Y_u \)’ is the characteristic impedance at the upstream point ‘u’.

Thus, from the above two equations, \( A_u \) is calculated. Similarly, at the downstream end complex amplitudes \( (A_{d,a} \) and \( A_{d,b} \)) are calculated. Thus, the incident power and transmitted power are given as (3):

\[ W_i = \frac{1}{2 \rho_0} |A_u|^2 \left( 1 + M_{u,a} \right) / Y_u \]  \hspace{1cm} (16)

\[ W_{t,a} = \frac{1}{2 \rho_0} |A_{d,a}|^2 \left( 1 + M_{d,a} \right) / Y_{d,a} \]  \hspace{1cm} (17)

\[ W_{t,b} = \frac{1}{2 \rho_0} |A_{d,b}|^2 \left( 1 + M_{d,b} \right) / Y_{d,b} \]  \hspace{1cm} (18)

where, \( Y_{d,a} \) and \( Y_{d,b} \) are the characteristic impedances at the radiation end (d, a) and (d, b), respectively.

Finally, transmission loss is calculated as follows (3):

\[ TL = 10 \log \frac{W_i}{W_{t,a} + W_{t,b}} \]  \hspace{1cm} (19)

The TL calculated for the fork muffler (Fig. 1(b)) and equivalent single muffler (Fig. 1(a)) configuration are plotted in Fig. 4. From this graph, we see that the TL curve for the fork muffler and the equivalent single muffler configuration are very nearly identical, which was expected as the area expansion ratio is same for both the mufflers.

For calculation of insertion loss, the characteristic impedance in the power expression is replaced by their corresponding convective radiation impedances given as below (5, 3):

\[ R_{c,d,a} = \text{real} \left( \left( Z_{0,a} + MY_{d,a} \right) / \left( 1 + M \left( Z_{0,a} / Y_{d,a} \right) \right) \right) \]  \hspace{1cm} (20)

\[ R_{c,d,b} = \text{real} \left( \left( Z_{0,b} + MY_{d,b} \right) / \left( 1 + M \left( Z_{0,b} / Y_{d,b} \right) \right) \right) \]  \hspace{1cm} (21)

\[ R_{c,u} = \text{real} \left( \left( Z_u + MY_u \right) / \left( 1 + M \left( Z_u / Y_u \right) \right) \right) \]  \hspace{1cm} (22)

where, notation \( R_{c,d,a} \) represents the real part of the convective impedance at the radiation end (d, a) and ‘M’ is the mean flow Mach number,

\[ M = M_{d,a} = M_{d,b} \]  \hspace{1cm} (23-24)

Sound power at the radiation end (d, a) is given by (3):
\[ W_{d,a} = \frac{|v_{c,d,a}|^2 R_{c,d,a}}{2\rho_0} \]  

(25)

where, \( v_{c,d,a} \) represents the convective acoustic velocity at the radiation end (d, a), and is given as below:

\[ v_{c,d,a} = v_{d,a} + p_{d,a} M_{d,a} / Y_{d,a} \]  

(26)

\[ = v_{d,a} \left( 1 + Z_{a,a} M_{d,a} / Y_{d,a} \right) \]  

(27)

Figure 4 - Computed TL for Fork muffler and equivalent single muffler configuration

Similarly, the sound power \( W_{d,b} \) at the radiation end (d, b) is calculated and thus total sound power level at the radiation end is given as:

\[ L_{w,d} = 10 \log \left( \frac{W_{d,a} + W_{d,b}}{W_{ref}} \right) \]  

(28)

Similarly, the sound power level ‘\( L_{w,i} \)’ at the upstream point is calculated and the insertion loss is then given as:

\[ IL = L_{w,i} - L_{w,d} \]  

(29)

The IL of the fork muffler is plotted in Fig. 5, where the low frequency dip represents the interaction of the muffler compliance with the tail pipe inertance (3).

3. ANALYSIS OF FORK MUFFLER WITH H-CONNECTION

Figure 6 shows the schematic diagram of the Fork Muffler with a connecting pipe, the so called H-connection.
The length ‘$l_c$’ denotes the length of the connecting pipe. Here also, the muffler proper used is the three-pass double-reversal muffler with tubular bridges (4) shown in Fig 2.

Using the transfer matrix obtained for the three-pass double-reversal muffler with tubular bridges (4), the governing equations for Fork muffler with H-connection are given below, where ‘$p$’ represents acoustic pressure and ‘$v$’ represents mass velocity:

\[
\begin{align*}
  p_u &= p_1 = p_3 \\
  v_u &= v_1 + v_3
\end{align*}
\]  

The transfer matrices for different pipe elements can be written as:

\[
\begin{align*}
  \begin{bmatrix}
    p_1 \\
    v_1
  \end{bmatrix} &= \begin{bmatrix}
    J_{1,2}
  \end{bmatrix} \begin{bmatrix}
    p_2 \\
    v_2
  \end{bmatrix}
\end{align*}
\]  

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**Figure 5** - Computed IL for Fork muffler without H-connection

**Figure 6** - Schematic diagram of the Fork-muffler with H-connection
Also we have,
\[ p_2 = p_3 = p_9 \]  
\[ v_2 = v_3 + v_9 \]  
\[ p_6 = p_7 = p_{10} \]  
\[ v_7 = v_6 + v_{10} \]

Using the transfer matrix obtained for the three-pass double-reversal muffler with tubular bridges (4), the transfer matrix for each muffler proper can be written as:
\[
\begin{bmatrix}
    p_4 \\
    v_4
\end{bmatrix} =
\begin{bmatrix}
    T_{11,a} & T_{12,a} \\
    T_{21,a} & T_{22,a}
\end{bmatrix}
\begin{bmatrix}
    p_{d,a} \\
    v_{d,a}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    p_8 \\
    v_8
\end{bmatrix} =
\begin{bmatrix}
    T_{11,b} & T_{12,b} \\
    T_{21,b} & T_{22,b}
\end{bmatrix}
\begin{bmatrix}
    p_{d,b} \\
    v_{d,b}
\end{bmatrix}
\]

Now, at the radiation end of the two branches (d, a) and (d, b), we have
\[ P_{d,a} = v_{d,a}Z_{0,a}(M) \]  
\[ P_{d,b} = v_{d,b}Z_{0,b}(M) \]

where, \( Z_{0,a} \) and \( Z_{0,b} \) are the impedances (5, 3) at the radiation end (d, a) and (d, b), respectively, and ‘M’ represents the mean flow Mach number.

There are 25 equations (30 to 54) and 26 unknown variables. So we can obtain the values of 25 variables in terms of the 26th variable, say ‘\( v_u \)’. For ease of calculation, ‘\( v_u \)’ is taken as unity.

Now, ‘\( p_u \)’ and ‘\( v_u \)’ can be written in terms of complex amplitudes of the progressive waves A and B as follows:
\[ p_u = A_u + B_u \]  
\[ v_u = \frac{A_u - B_u}{Y_u} \]
where ‘\(Y_u\)’ is the characteristic impedance at the upstream point ‘u’.

Thus, from these two equations, \(A_u\) is calculated. Similarly, at the downstream end, complex amplitudes \((A_{d,a}\) and \(A_{d,b}\)) are calculated. Thus, the incident power and the transmitted power are given as (3):

\[
W_i = \frac{1}{2\rho_0} |A_u|^2 \left(1 + M_{u,a}\right) / Y_u
\]

\[
W_{t,a} = \frac{1}{2\rho_0} |A_{d,a}|^2 \left(1 + M_{d,a}\right) / Y_{d,a}
\]

\[
W_{t,b} = \frac{1}{2\rho_0} |A_{d,b}|^2 \left(1 + M_{d,b}\right) / Y_{d,b}
\]

where, \(Y_{d,a}\) and \(Y_{d,b}\) are the characteristic impedances at the radiation end (d, a) and (d, b), respectively.

Finally, transmission loss is calculated as follows:

\[
TL = 10\log\left(\frac{W_i}{W_{t,a} + W_{t,b}}\right)
\]

For calculation of the insertion loss, the characteristic impedance in power expression is replaced by their corresponding radiation impedances (5, 3) given below:

\[
R_{c,d,a} = \text{real}\left(\frac{(Z_{0,a} + MY_{d,a})}{\left(1 + M\left(Z_{0,a} / Y_{d,a}\right)\right)}\right)
\]

\[
R_{c,d,b} = \text{real}\left(\frac{(Z_{0,b} + MY_{d,b})}{\left(1 + M\left(Z_{0,b} / Y_{d,b}\right)\right)}\right)
\]

\[
R_{c,u} = \text{real}\left(\frac{(Z_u + MY_{u})}{\left(1 + M\left(Z_u / Y_u\right)\right)}\right)
\]

where, notation \(R_{c,d,a}\) represents the real part of the convective impedance at the radiation end (d, a) and ‘\(M\)’ is the mean flow Mach number,

\[
M = M_{d,a} = M_{d,b}
\]

Sound power at the radiation end (d, a) is given by (3):

\[
W_{d,a} = \frac{|v_{c,d,a}|^2 R_{c,d,a}}{2\rho_0}
\]

where, \(v_{c,d,a}\) represents the convective acoustic velocity at the radiation end (d, a), and is given as:

\[
v_{c,d,a} = v_{d,a} + p_{d,a} M_{d,a} / Y_{d,a}
\]

\[
v_{c,d,a} = v_{d,a} \left(1 + Z_{0,a} M_{d,a} / Y_{d,a}\right)
\]

Similarly, the sound power \(W_{d,b}\) at the radiation end (d, b) is calculated and the total sound power level at the radiation end is given as:

\[
L_{w,d} = 10\log\left(\frac{W_{d,a} + W_{d,b}}{W_{ref}}\right)
\]

Similarly, the sound power level at the upstream end ‘\(L_{w,i}\)’ is calculated and then insertion loss is defined as:

\[
IL = L_{w,d} - L_{w,i}
\]

The computed TL and IL spectra are plotted and compared with those of the fork muffler in Figs. 7 and 8. The default dimensions (refer Fig. 6) are \(l_1=l_3=700\) mm, \(l_2=l_4=300\) mm, \(l_c=1000\) mm and diameter of
connecting pipe is 40 mm. Length of the each tail pipe is 140 mm. Sound speed = 618.8 m/sec corresponding to the temperature of 680°C. The mean Mach number of the flow is 0.2 which has been chosen to be consistent with the Mach number used in three pass double-reversal muffler with tubular bridges (4).

A closer look at the H-connection in Fig. 7 shows that the upper half of the branch connecting tube would act as a branch resonator of length $l_c/2$ for muffler branch ‘a’, the lower half would play the same role for muffler branch ‘b’. This explains the three additional peaks as zeros of the branch impedances $-jY_c \cot (kl_c/2)$ or $\cos (kl_c/2)$. Thus,

$$\frac{kl_c}{2} = (2n-1) \frac{\pi}{2}$$

This implies that the $n^{th}$ natural frequency of the branch resonator is given by:

$$f_n = \frac{(2n-1) c_0}{2l_c}$$

$$= 314 \text{ Hz}, \ 942 \text{ Hz} \text{ and } 1570 \text{ Hz}.$$ (73)

These peaks may be seen to appear in the IL curve as well (Fig. 8). Thus length $l_c$ can be decided depending upon the firing frequency of the engine and thus resonant frequencies due to H-pipe can be tuned accordingly.

4. PARAMETRIC STUDY OF FORK MUFFLER WITH H-CONNECTION

4.1 Effect of the position of the connecting pipe

Maintaining the symmetry, the position of the connecting pipe ($l_a$ and hence $l_b$) is varied, and its length $l_c$ changes accordingly. Its effect on IL is plotted in Fig. 9. From this graph we can see a shift of the peaks to the right as $l_c$ decreases. Larger $l_c$ may be desirable so as to cancel the firing frequency noise.
Figure 8 - Computed IL for fork muffler with and without H-connection

Figure 9 - Effect of the position of the connecting (symmetrical) pipe on the insertion loss
By slanting the connecting pipe as shown in Fig. 10, the lengths \( l_{a1}, l_{a2}, l_{b1}, l_{b2} \) and \( l_c \) get altered. Its effect on IL is plotted in Fig. 11. By keeping the position more unsymmetrical i.e. rotating the connecting pipe more (increasing the deviation from perpendicularity to the axis of the muffler), the troughs are lifted a little, while the peaks are lowered. There is a general moderating effect like the effect of damping on a dynamical filter.

**4.2 Effect of the diameter of the connecting pipe**

The effect of the diameter of the connecting pipe has been found to be marginal (6) and therefore is not shown here.

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**Figure 10 - Slanting the connecting pipe of Fork muffler with H-connection**

**Figure 11 - Effect of the position of the connecting (unsymmetrical) pipe on the insertion loss**
4.3 Effect of the length of the connecting pipe
Here, the length of the H-pipe is varied without changes in its position. It is perpendicular to the axis as shown in Fig. 6. The effect of the length of the H-pipe is shown in Fig. 12. The peak may be seen to shift to lower frequencies as the length is increased.

![Graph showing effect of length on insertion loss](image)

Figure 12 - Effect of the length of the connecting pipe on the insertion loss of the Fork muffler with H-connection

5. CONCLUSION
The fork or twin muffler with H-connection has been analyzed, and the effect of the length, location and diameter of the connecting pipe on the insertion loss curve is investigated. In general, the H-connection helps inasmuch as it produced three additional quarter-wave-resonator peaks in the TL curve, the frequency of which may be tuned to the firing frequency and its odd multiples at the desired engine speed. However, in the IL curve, the advantage of the H-connection is not unconditional. The peak is accompanied by a trough in the insertion loss but the width of the peak spans over a wider range of frequency as compared to the width of the trough. Also, the unsymmetrical position of the H-connection lift the troughs and subdue the peaks. This is like the effect of damping on a dynamical filter. The increase in the length and diameter of the H-pipe shifts the resonance to the lower frequencies, bringing the frequencies closer to the firing frequency range.

REFERENCES