Dynamic parameters estimation based on longitudinal vibration using an inverse method in FEM

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ABSTRACT

Viscoelastic materials have been widely used in the noise and vibration control since they can provide high damping capability over wide temperature and frequency ranges. Designing or selection of damping structure requires the knowledge of dynamic performance of viscoelastic materials. Resonant bar technique is a classical method which has been used for the parameters measurement of the viscoelastic materials. In this paper, an inverse method for dynamic parameters estimation of viscoelastic thin bar is presented, in which the measured response of the bar longitudinal vibration is gradually approximated by the forward calculated response from finite element computation. Firstly, the loss factor is estimated from the displacement magnitude ratio between the driven end and the free end of the bar. Then the complex-modulus is estimated from the resonant frequency. The accuracy of experimental results validates the proposed inverse method.

Keywords: Viscoelastic material, Dynamic parameters, Inverse FEM
I-INCE Classification of Subjects Number(s): 72.7

1. INTRODUCTION

The viscoelastic material, a kind of material that both has the characteristics of viscous liquid and elastic solid, can store energy and dissipate energy as well. When the viscoelastic material produces dynamic stress and strain, a part of the energy is stored just like potential energy, while the other part of the energy is converted into heat energy and dissipated. The convert and dissipation of the energy is shown as mechanical damping which has the function of reducing vibration and noise. Although the application of viscoelastic material in vibration and noise reducing technology has only decades of development history, it grows quickly due to its excellent effect. The knowledge of the dynamic elastic properties of such materials is essential to predict their performance and to make effective use of this kind of material. While the dynamic elastic properties of such materials is a function of temperature and hydrostatic pressure, the first issue is how to obtain the dynamic parameters at different frequencies under different hydrostatic pressures and at different temperatures.

Several methods have been used to measure the dynamic parameters of viscoelastic materials. Such as forced oscillatory measurements, resonance measurements, dynamic mechanical analyses, etc. Resonance method (1), the most widely used method, which obtains the Young's modulus and the loss factor by testing the longitudinal vibration of the viscoelastic material bar, was introduced by Norris et al. (2) in the 1970s.Madigoski et al. (3)and Guo et al. (4) expanded the frequency range of the measurements using the time-temperature superposition principle. Measurements as a function of both temperature and hydrostatic pressure were reported by Willis et al.(5).Garrett(6) proposed a similar method which tests the Young's modulus and shear modulus of the materials by exciting the torsional wave, longitudinal wave and flexural wave. In recent years, F.M.Guillot et al.(7,8) tested the vibration characteristics of the material utilizing laser Doppler vibrometers in a pressure vessel, the elastic modulus of the material is obtained according to resonance measurements and wave-speed measurements. Using this approach, Young's modulus data can be obtained at frequencies typically

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ranging from 100 Hz to 5kHz.But this method is essentially a single-frequency method and the accuracy of the measurement at high frequency needs to be improved.

With the development of computer technology, finite element method in many aspects, such as engineering technology has been widely used. Willis et al. (9,10) proposed an experimental/numerical technique based on FEM to determine simultaneously the bulk and shear dynamic moduli of viscoelastic material of arbitrary shape. But the inversion code does not converge well under some conditions.

This paper expounds the principle of resonance method testing technology and studies the relationship between the dynamic responses and the dynamic parameters based on FEM, on the foundation of which, a new method of inverting the dynamic parameters of viscoelastic materials is presented, in which the loss factor is firstly estimated from the displacement magnitude ratio between the driven end and the free end of the bar, then the complex-modulus is estimated from the resonant frequency. The method has advantages of rapid convergent speed, stable and accurate inverted results. The research in this article makes the study of the wave propagation in a bar using FEM more intuitive, and establishes the foundation for the application of the finite element method in the inverse of dynamic parameters in a contiguous range of frequencies.

2. RESONANCE MEASUREMENT

2.1 Theoretical basis

Let us consider a homogeneous bar of density ρ and length L, with a constant cross section and no mass attached to its free end (Figure 1). One end of the bar is driven with a harmonic displacement $u_0(t)=U_0e^{j\omega t}$ resulting axial displacement(relative to the driven end) at a distance x from the driven end is $u(x,t)=U(x)e^{j\omega t}$. Assuming a uniform, uniaxial stress distribution inside the bar, and neglecting the effects of lateral inertia, the equation of motion can be written as

$$\frac{F_x}{S}=-E\frac{\partial\left(u+u_0\right)}{\partial x} \tag{1}$$

where F_x is the uniaxial force in the bar in the longitudinal (x) direction, E is the complex Young's modulus.

If F_x represents the internal force at x, then $F_x+(\partial F_x/\partial x)dx$ represents the force at $x+dx$, and the net force to the down is

$$dF_x=F_x-F_{x+d_x}=-\frac{\partial F_x}{\partial x}dx \tag{2}$$

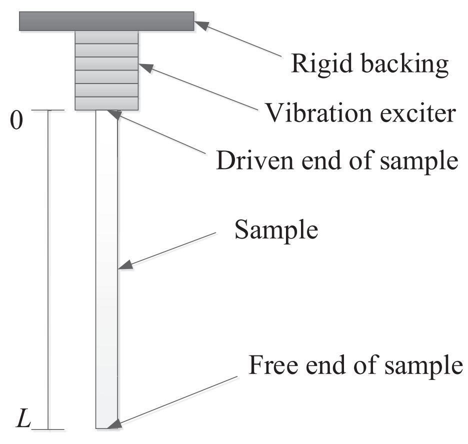


Figure 1 – Resonance measurements diagram

Substituting Eq. (1) into Eq. (2) and according to Newton's second law, one obtains

$$ES\frac{\partial^2(u+u_0)}{\partial x^2}dx=\rho Sdx\frac{\partial^2(u+u_0)}{\partial t^2} \tag{3}$$

which, in turn, leads to the following ordinary differential equation

$$\frac{d^2U}{dx^2}+k^2U=-k^2U_0 \tag{4}$$

where k is the complex wave number, defined as $k=\omega(\rho/E)=\omega/c_b$, and $c_b=(E/\rho)^{1/2}$ is the complex bar wave speed.

The boundary conditions for the problem are zero relative displacement at x=0, i.e., $u(0,t)=0$, and zero stress at $x=L$, i.e., $dU/dx_{x=L}=0$, which lead to the following solution for Eq.(4)

$$U\left(x\right)=U(x)=U_0[\cos(kx)+\tan(kL)\sin(kx)-1] \tag{5}$$

Now that the equation of motion has been solved, let us study the resonance of the sample. According to the equation (5), the complex ratio of the free-end displacement to the driven end displacement Q^* can be expressed as

$$Q^*=\frac{U\left(L\right)+U_0}{U_0}=\frac{1}{\cos\left(kL\right)} \tag{6}$$

For viscoelastic materials, using complex notation, Young's modulus can be written as

$$E^*=E'+iE''=E[\cos(\delta)+i\sin(\delta)] \tag{7}$$

where E' is the storage modulus, E'' is the viscous modulus, E is the magnitude, and $\tan(\delta)$ is the loss factor. Then the Eq. (7) can be expressed as

$$\begin{aligned}\frac{1}{Q^*}&=\cos(kL)=\cos[\omega L(\frac{\rho}{E[\cos(\delta)+i\sin(\delta)]})^{1/2}]\\&=\mathrm{Re}+i\,\mathrm{Im}\end{aligned} \tag{8}$$

$$\mathrm{Re}=\cos\left[\omega L\left(\frac{\rho}{E}\right)^{1/2}\cos\left(\frac{\delta}{2}\right)\right]\cosh\left[\omega L\left(\frac{\rho}{E}\right)^{1/2}\sin\left(\frac{\delta}{2}\right)\right] \tag{9}$$

$$\mathrm{Im}=\sin\left[\omega L\left(\frac{\rho}{E}\right)^{1/2}\cos\left(\frac{\delta}{2}\right)\right]\sinh\left[\omega L\left(\frac{\rho}{E}\right)^{1/2}\sin\left(\frac{\delta}{2}\right)\right] \tag{10}$$

The complex ratio is written as $Q^*=Qe^{i\theta}$, then at resonance, when $\theta=(2n-3)(\pi/2)$, n = 1,2,3,⋯is the resonance number, one can easily obtains

$$\delta=2\tan^{-1}\left[\frac{\sinh^{-1}\left(1/Q\right)}{\left(2n-1\right)\frac{\pi}{2}}\right] \tag{11}$$

$$E=\rho\left[\frac{4Lf_{res}}{\left(2n-1\right)}\cos\left(\frac{\delta}{2}\right)\right]^2 \tag{12}$$

One should again emphasize is that Eqs. (11) and (12) are valid at resonance only, that f_{res} is the resonance frequency, and that Q is the amplitude of the displacement ratio. The storage and viscous moduli can be computed by the above two equations.

Figure 2 – Resonance measurements system

2.2 Measurement procedure

In this study, a certain resin material whose density is 1150kg/m^3 is used, which is cut into a bar with a 9.9- by 8.2-mm cross-section area and a 96-mm length. The experimental apparatus is shown in Fig 2. The sample is glued to a vibration exciter which is rigidly connected to a rigid backing, then the vibration exciter can generate a signal and make the viscoelastic sample bar vibrate longitudinally. The resulting longitudinal vibrations of the free end are measured by a laser Doppler vibrometer (PSV-400), then vibration signals are transferred to a notebook computer for data processing. According to the complex ratio of the free-end displacement to the driven end displacement at corresponding resonant frequency, the storage and viscous moduli can be computed using resonance method (As seen in Table 1).

Table 1 – Physical parameters of liquids

Order	Q	Resonance, Hz	Loss factor	Storage modulus, Pa
1	27.28	4444	0.046	3.34e+009
2	8.454	5820	0.05	6.37e+008

3. FINITE ELEMENT SIMULATION ANALYSIS

3.1 Simulation based on resonance method

According to the measurement system, the finite element model of a long bar is established as seen in Fig.3, which has 1470 nodes and 960 hexahedron elements. One end of the bar is fixed and the other end is free. The fixed end of the bar is excited with a broadband signal. After inputting the storage modulus, the viscous modulus and the density of the bar, the displacement response of the free end can be calculated by Msc.Nastran, a finite element calculation software.

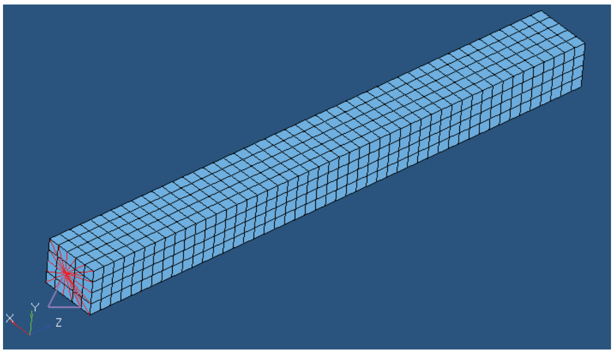


Figure 3 –The finite element model of bar

Input the parameters (storage and viscous modulus at the first order resonance frequency) we measured using resonance method into the finite element system, one can obtain the first-order resonance frequency is 4372Hz and the corresponding ratio of the free-end displacement to the driven end displacement is 27.6978(As seen in Fig. 4). And the relative error between the simulation data and experimental data is only 1.62% for the resonance frequency and 0.43% for the displacement ratio.

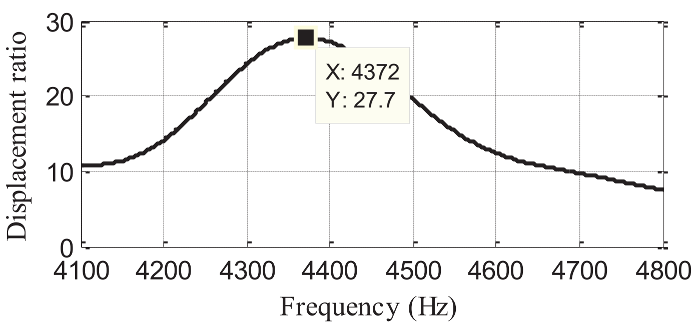


Figure 4 –Simulation of the first mode

For the second mode, repeat the above steps, we can see that the relative errors between the simulation data and experimental data for the resonance frequency and the displacement ratio are only 1.75% and 0.34% respectively.

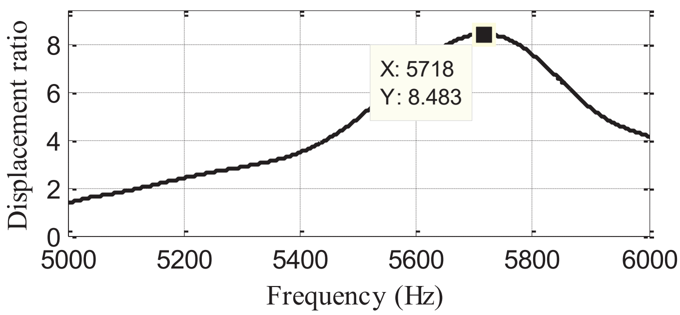


Figure 5 –Simulation of the second mode

According to the analysis above we know that the calculated resonance frequency and displacement ratio by FEM can agree well with the parameters tested by the resonance method, which show that the simulation by FEM is feasible and we can use this finite element model to inverse the dynamic parameters of viscoelastic materials.

3.2 Relationship between dynamic response and the dynamic parameters

To inverse the dynamic parameters of viscoelastic materials based on FEM, we need to analysis the relationship between dynamic response and the dynamic parameters.

According to Eq. (11), one can obtain the relationship between the displacement ratio Q and loss factor

$$Q = \frac{1}{\sinh\left[(2n-1)\frac{\pi}{2}\tan\left(\frac{\delta}{2}\right)\right]} \tag{13}$$

The relationship between resonance frequency and complex Young's modulus can be written as follow by combining the Eq. (12) and Eq. (13)

$$f_{res} = \left(\frac{E\cos(\delta)}{\rho}\right)^{\frac{1}{2}}\left(\frac{1}{\cos(\delta)}\right)^{\frac{1}{2}}\frac{(2n-1)}{4L\cos(\frac{\delta}{2})} \tag{14}$$

3.2.1 Relationship between dynamic response and storage modulus

Eqs. (13) and (14) show that the dynamic responses at different resonance frequencies has similar variation trend with dynamic parameters, then we analysis this phenomenon at the second order resonance frequency, for example.

With the storage modulus range from 600 to 675MPa and the loss factor remains unchanged, we can calculate the resonance frequency and displacement ratio by FEM, as seen in Table-2, which shows that when the loss factor remains unchanged, the second resonance frequencies get higher with the increase of the storage modulus, while the displacement ratio is almost constant.

Table 2 –Keep the loss factor as a constant

Storage modulus, MPa	Loss factor	Resonance frequency, Hz	*Q*
600	0.05	5546	8.4867
625	0.05	5660	8.4868
637	0.05	5714	8.4868
650	0.05	5772	8.4868
675	0.05	5882	8.4868

According to Eq. (14) we know that the resonance frequency is only determined by the storage modulus when keeping the loss factor as a constant. While Eq. (13) indicates that the displacement ratio is only determined by the loss factor. We plot the results from the Table 2, as seen in Fig. 6. The discrete points represent the simulation results, while the solid line represents the theoretical value according to Eq. (14). Figure 6 indicates that simulating results are consistent with theoretical analyses.

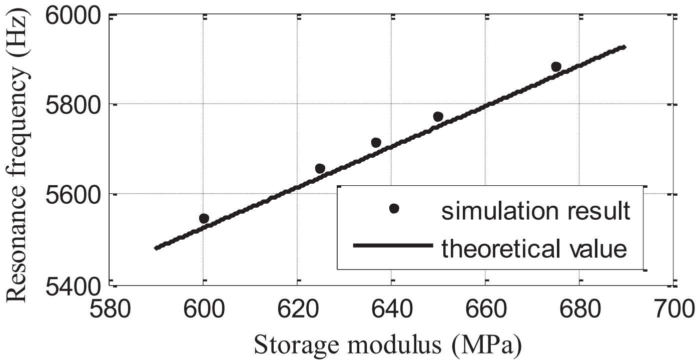


Figure 6 –The relation between the second resonant frequency and storage modulus

By the analysis above, it is concluded that when the loss factor remains unchanged, the displacement ratio Q is almost constant and the resonance frequencies get higher with the increase of the storage modulus. One can obtain $f_{ref} = k\ (E\cos(\delta))^{1/2}$, k is determined by resonance order, loss factor and density of the material.

3.2.2 Relationship between dynamic response and loss factor

With the loss factor range from 600 to 675MPa and the storage modulus remain unchanged, the calculated resonance frequency and displacement ratio by FEM are shown in Table-3, from which we can see that, when the storage modulus remain unchanged and loss factor is small, there is an inverse relationship between the displacement ratio and loss factor while the second resonance frequency is constant. But the simulation results become inaccurate if the loss factor is greater than 0.1.

Table 3 –Keep the storage modulus as a constant

Storage modulus, MPa	Loss factor	Resonance frequency, Hz	Q
637	0.01	5712	42.4911
637	0.03	5712	14.1573
637	0.05	5714	8.4868
637	0.07	5716	6.0539
637	0.1	5718	4.2260
637	0.3	5732	1.3591
637	0.5	5536	0.7926

We plot the results of loss factor and the resonance frequency from the Table 3, as seen in Fig. 7. The discrete points represent the simulation result, while the solid line represent the theoretical value according to Eq. (14). Figure 7(a) shows that simulating results are very different with theoretical analyses when the loss factor is greater than 0.1, which is because that resonance of the sample bar is difficult to be produced, and the test results of the resonance method is not accurate. Then we know that the loss factor needs to be small enough for the inverse method by FEM for dynamic parameters estimation based on longitudinal vibration.

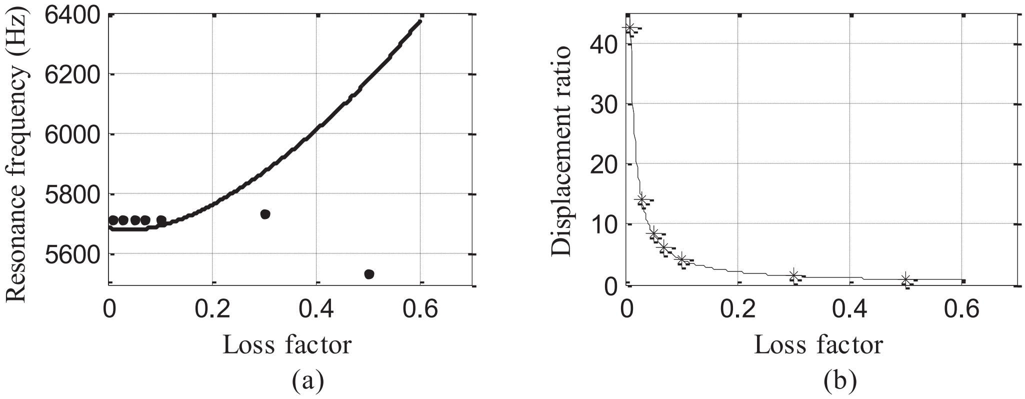


Figure 7 –The relation between the second resonant frequency and storage modulus (a)
The relation between the second resonant frequency and storage modulus (b)

We plot the results of loss factor and displacement ratio from the Table 3, as seen in Fig. 7(b). The discrete points represent the simulation result, while the solid line represent the theoretical value according to Eq. (13). Figure 7(b) shows that simulating results are very consistent with theoretical analyses, and the displacement ratio is in the inverse proportional relationship to the loss factor.

According to what mentioned above, we know that when the storage modulus remain unchanged and the loss factor is very small, the resonance frequency is constant and the displacement ratio Q is in the inverse proportional relationship to the loss factor. While the inverse method by FEM is not accurate when the loss factor is too great.

4. INVERSE METHOD FOR DYNAMIC PARAMETERS ESTIMATION

4.1 Inverse method

The third section shows that the simulation of the longitudinal resonance method based on FEM is feasible, and the relationships between various parameters are given. And we conclude that Q for the corresponding resonance frequency is only determined by the loss factor, while the resonance frequency is determined by both the storage modulus and the loss factor. Thus, for inverse the dynamic parameters, we can determine the loss factor based on the displacement Q. After determining the loss factor, the storage modulus can be determined by the resonance frequency further. The concrete process of the inverse method is shown in Figure 8.

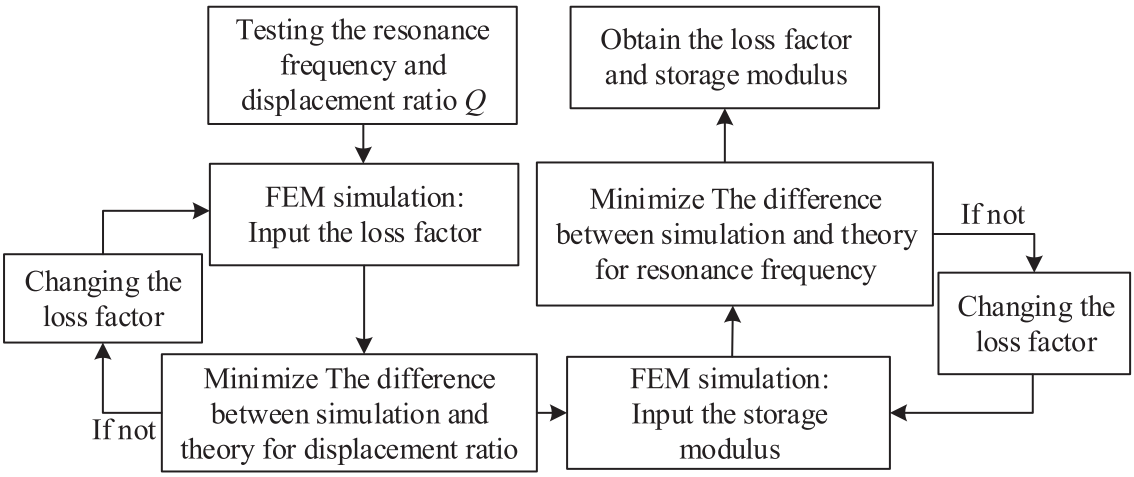


Fig.8 Inverse process

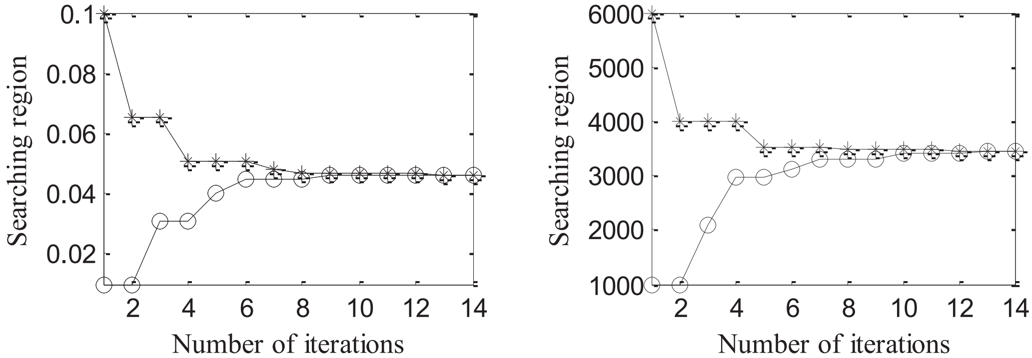
We know that the inverse method includes two steps and only one dynamic parameter needs to be adjusted. So we can use one dimension search method to inverse the dynamic parameters. One dimension search method include: fraction method, 0.618 method, quadratic interpolation method, etc. In this paper, 0.618 method, a usually used method, is used to inverse dynamic parameters.

4.2 The inverse method results

4.2.1 The inversion for the first mode

By resonance measurement, we obtain that the first order resonance frequency is 4444Hz, the amplitude of the ratio of the free-end displacement to the driven end displacement is 27.58, the calculated Young's modulus and loss factor are 3340MPa and 0.046 respectively.

Loss factor as independent variable which range from 0.01 to 0.1, changing the values of the loss factor to make the simulation displacement ratio keep very close to the experimental displacement ratio (with difference no more than 0.02), then we obtain the loss factor. The inverse process is shown in Figure 9(a). Using 0.618 method, through 14 iterations we obtain that the loss factor is 0.0462 when the difference of the simulation displacement ratio and the experiment displacement ratio is 0.0016. And the relative error of the loss factor is only 0.43%.



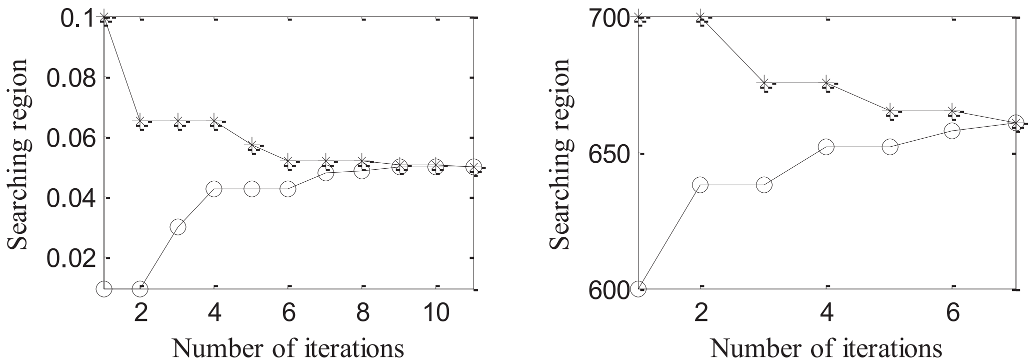
(a) (b)

Fig.9 The inversion of loss factor for the first mode (a)
The inversion of storage modulus for the first mode (b)

For storage modulus, making the searching region spacing 1000MPa to 6000MPa and making the simulated resonance frequency matches the measured one (with difference no more than 4Hz), we obtain the storage modulus through 14 iterations (as seen in Figure 9(b)). The storage modulus is 3445.7MPa and its relative error is 3.16%.

4.2.2 The inversion for the second mode

The experimental value of the second resonance frequency is 5820.And the corresponding displacement ratio is 8.454. Then, by the resonance method we obtain the Young's modulus is 637MPa and the loss factor is 0.05.



(a) (b)

Fig.10 The inversion of loss factor for the second mode (a)
The inversion of storage modulus for the second mode (b)

Loss factor as independent variable which range from 0.01 to 0.1, changing the values of the loss factor to make the simulation displacement ratio keep very close to the experimental displacement ratio (with difference no more than 0.002), then we obtain the loss factor through 11 iterations. The inverse process is shown in Figure 10(a). The loss factor obtained by inverse method is 0.0502 and the relative error is 0.4%.

For storage modulus, making the searching region spacing 600MPa to 700MPa and making the simulated resonance frequency matches the measured one (with difference no more than 1Hz), we obtain the storage modulus through 7 iterations (as seen in Figure 10(b)). The storage modulus is 660.8MPa and its relative error is 3.74%.

4.2.3 Analysis of the inverse method

Through the results of the inverse method for the first and the second mode, we can obtain that the relative error of the loss factor is no more than 0.5% and the relative error of the storage modulus is no more than 4%. Which shows that the results of the inverse method based on resonance measurement is accuracy. And the inverse process shows that the 0.618 method we used has the advantages of fast convergence speed and fewer iterations.

5. CONCLUSIONS

Using FEM, we simulated the process of the resonance method and the results is accurate and reliable. At the resonance frequency, the ratio of the free-end displacement to the driven end displacement is only determined by the loss factor and is in the inverse proportional relationship to the loss factor. While the resonance frequency is determined by the storage modulus and is increasing with the storage modulus increasing. Which should be noted is that the inverse method by FEM is accurate enough only with the loss factor is not too great.

This article proposes an inverse method by FEM based on resonance measurement. By 0.618 method which has the advantages of small relative error, fast convergence speed and fewer iterations, we inverse the dynamic parameters for different resonance frequency and establishes the foundation for inversing the dynamic parameters over a wide frequency range.

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