Directivity Control of a Large Loudspeaker by Multi-zone Control using a Small Loudspeaker Array

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ABSTRACT

This article addresses the control approach for directional source implemented by two types of loudspeakers, i.e., a main loudspeaker (large size) and control loudspeakers (small size). A directional source using an array system composed of large loudspeakers has two weaknesses: a large physical size, and low Nyquist frequency. In this paper, we propose the control approach to solve such problems. When the main loudspeaker emits a sound source toward two regions: audible and inaudible region, the original sound of the main loudspeaker is generated in the audible region and simultaneously the sound is suppressed in the inaudible region by a control loudspeaker array. Therefore, the directivity of a main loudspeaker using a control loudspeaker array can be controlled based on Beamforming techniques. In this paper, the feasibility study was performed using GENELEC 8240A (a main loudspeaker) and Cambridge audio MINX min11 (control loudspeakers) in a frequency band from 250 Hz to 2 kHz.

Keywords: Directivity control, Loudspeaker array

I-INCE Classification of Subjects Number(s): 74.9

1. INTRODUCTION

Implementation of a directional sound source using multi-loudspeakers has been studied based on Beamforming techniques [1~3] or Kirchhoff-Helmholtz integral equation [4] in a theoretical point of view. In terms of hardware, previous studies based on multiple sources have been implemented by using the same loudspeakers.

However, if the physical size of the loudspeaker used in the directional control is large, it causes two problems in terms of implementation for the directional sound. First, the array system using large loudspeakers makes a huge array, so it is difficult to set up the array system in small space. Second, when the spacing between loudspeakers is distant, Nyquist frequency for controlling the main beam is lower.

Therefore, we address the approach for controlling the directivity of a large loudspeaker using a combination with the different size of loudspeakers. In this paper, the large loudspeaker called as a main loudspeaker is used by GENELEC 8240A, and the small loudspeaker named as a control loudspeakers is used by MINX min11 of Cambridge audio. The feasibility study is performed in an anechoic chamber.

2. PROBLEM DEFINITION

2.1 The concept of directivity control using proposed loudspeakers configuration

Figure 1 shows the concept of directivity control approach using proposed loudspeakers configuration. The configuration is set up with a large loudspeaker and relatively small loudspeakers.
The large loudspeaker is used to generate a sound source which users want, whereas the small loudspeakers are used to control the directivity of the large loudspeaker. Therefore, in this paper, the large loudspeaker is called as main loudspeaker, and the small loudspeakers are named as control loudspeakers.

![Figure 1](image1.png)

**Figure 1** The concept of directivity control approach using two types of loudspeakers.

In Fig. 1, main loudspeaker emits a sound, and simultaneously control loudspeakers make the controlled sound with the same magnitude and anti-phase toward a selected direction (blue dash line). In other words, control loudspeakers manipulate the sound generated by the main loudspeaker for cancellation only toward a selected direction, and the sound in other directions maintains the original sound generated by the main loudspeaker.

Therefore, from a viewpoint on control loudspeakers, the specific sound field is controlled in some direction for cancellation, and the sound field with the relatively low pressure is formed in other directions. For example, the directivity pattern is presented in Fig. 2.

![Figure 2](image2.png)

**Figure 2** The example of the proposed directivity pattern.

Figure 2 shows the example of the proposed directivity pattern. Red line means the directivity pattern of a main loudspeaker, and blue dash line means that of a loudspeaker array for controlling the main loudspeaker. In the targeted direction in Fig. 2, the directivity pattern of the main loudspeaker has the higher pressure than the directivity pattern in the cancellation direction as 10 dB difference. So, the sound from the main loudspeaker is heard better than the sound of a control loudspeaker array because of masking effects. In the cancellation direction in Fig. 2, the magnitudes of both sounds generated by main and control loudspeakers are the same, and the phases of both main and control loudspeakers have the opposite phase to each other. Therefore, in this paper, the targeted direction can be considered as an inaudible region, and the cancellation direction can be
thought of as an audible region in terms of a control loudspeaker array.

2.2 The setup of control points for a feasibility study

Figure 3 illustrates the control points of a loudspeaker array for a feasibility study. The control points in an inaudible region are three points with 30, 45, and 60 degrees, respectively. In other words, the pressure of three field points can be cancellation. The control points in an audible region are seven points with total 60 degrees, which means the right side pressure of the control loudspeaker array is suppressed.

In this paper, for satisfying above conditions, the beamforming approach is applied. The main beam is formed in the inaudible region using three control points, and side lobes are suppressed in the audible region. In the following chapter, we address the formulation in terms of the beamforming approach to control the directivity of the main loudspeaker.

3. CONTROL ALGORITHM

3.1 Linearly Constrained Minimum Variance

Linearly constrained minimum variance (LCMV) algorithm is widely known in the array system. Veen and Buckley [5] introduce to the advantages and disadvantages of LCMV algorithm. The advantages are that the algorithm can be flexible and has general constraints, whereas the disadvantages are computation problems for constrained weight vector. However, since the LCMV algorithm has an objective function of the quadratic form, and single equal constraint (or constraints vector) in an optimization point of view, the LCMV can be expressed in a variety of constraint forms. Therefore, if the reasonable constraints are selected, then we can design the filters and also satisfy the system requirements using the modified LCMV form.

Equation (1) represents an objective function and a constraint in LCMV, and Eq. (2) describes the analytic solution result of the LCMV by using Lagrange multipliers to solve Eq. (1) [5].

$$\begin{align*}
\min_{\mathbf{w}} & \quad \mathbf{w}^H \mathbf{Rw} \\
\text{subject to} & \quad \mathbf{d}^H(\theta, \omega)\mathbf{w} = g^* \\
\mathbf{w} &= g^* \frac{\mathbf{R}^{-1}\mathbf{d}(\theta, \omega)}{\mathbf{d}^H(\theta, \omega)\mathbf{R}^{-1}\mathbf{d}(\theta, \omega)}
\end{align*}$$

where $\mathbf{w}$ is a weighting value in a temporal frequency ($\omega$), $\mathbf{w}^H \mathbf{d}(\theta, \omega)$ is a beamformer response, $g$ is a targeted response (a complex constant), $\mathbf{R}$ is a correlation matrix, and $\mathbf{w}^H \mathbf{Rw}$ is a
beamformer output power. Since the optimization problem in Eq. (1) is solved by using Lagrange multiplier, Eq. (2) is an exact solution, so the targeted response can be well satisfied. However, if the number of constraints increase, the constraints described in Eq. (3) and (4) are changed, and also the computation is more complex.

\[
C^H w = f
\]

\[
\begin{bmatrix}
    d^H (\theta_1, \omega) \\
    d^H (\theta_2, \omega)
\end{bmatrix} w = \begin{bmatrix} g^* \\ 0 \end{bmatrix}
\]

where \( C \) is a \( N \) by \( L \) matrix (constraints matrix), \( f \) is a targeted response vector.

When the constraints assumed to be linearly independent and \( C \) has rank \( L \), the solution response is followings;

\[
w = R^{-1}C \left[ C^H R^{-1}C \right]^{-1} f
\]

In this research, however, spatial correlation matrix \( R \) and constraints matrix \( C \) are not easy to satisfy the full rank of each matrix. It’s because the transfer function between source positions and control positions can be similar.

Therefore, to accurately obtain the Lagrange multiplier, we propose the sequential approach with regularization. This approach is stated in the following session.

### 3.2 Proposed control approach

\[
\begin{align*}
J &= q^H R q + \beta q^H q \\
\text{Subject to} & \quad \begin{cases} 
(G_{r,1} q)^H = P_{d,1,\text{inaudible}}^H \\
(G_{r,2} q)^H = P_{d,2,\text{inaudible}}^H \\
\vdots \\
(G_{r,n} q)^H = P_{d,n,\text{inaudible}}^H
\end{cases} \\
R &= \begin{bmatrix} G_A \mid G_I \end{bmatrix}^H \begin{bmatrix} G_A \mid G_I \end{bmatrix}
\end{align*}
\]

Figure 4 shows the proposed optimization problem that has an objective function of a quadratic form and multi-equal constraints. Such equal constraints indicate the control points of an inaudible region or an audible region. \( k \) and \( m \) are selected by three and seven, respectively. \( R \) is the spatial correlation matrix, \( G_{ik} \) and \( G_{Am} \) are the transfer function between a source strength vector (\( q \)) and control points in the inaudible region or in the audible region, respectively. \( P_{d,k,\text{inaudible}}^H \) is the
targeted responses in the inaudible region, and \( P_{dm, \text{Audible}}^H \) is the desired values in the audible region.

In this paper, \( P_{dm, \text{Inaudible}}^H \) is selected based on the same magnitude and anti-phase of a sound source by main loudspeaker for cancellation. \( P_{dm, \text{Audible}}^H \) is defined by zero value for minimizing the side lobes.

Equation (6) is the analytic solution of the proposed optimization problem in Fig. 4, and Eq. (4) is derived by the same procedure for obtaining Eq. (2).

\[
q = M \left( \sum_{i=1}^{m} \lambda_i G_{di}^H + \frac{1}{\theta} \sum_{i=1}^{k} G_{li}^H \right) \tag{6}
\]

where \( M = (R + \beta I)^{-1} \), \( \beta \) is a regularization parameter.

Equation (6) shows the source strength vector (q) depends on the combination of transfer functions \( G_{di} \) and \( G_{li} \). Therefore, the Lagrange multipliers of \( s_i \) and \( \lambda_m \) are important, and they have to be calculated accurately. It is the reason why we do not consider the approach of Eq. (5), and we propose the approach of Eq. (6). In this paper, although LCMV form is addressed to solve the problem defined in session 2.1, for example, LCMV form of a microphone array is not equal to LCMV form of a Loudspeaker array. In other words, it is difficult to directly apply Eq. (5) into our problem because the matrix based on Green function can be easy to become singular.

Equation (7) represents the matrix for calculating the Lagrange multipliers. This matrix has two weaknesses, and it can be difficult to obtain the exact Lagrange multipliers. First, \( M \) matrix can be singular. So, Tikhonov regularization [6] is applied, and \( \beta \) is a parameter to control the regularization in terms of mathematics. In terms of physics, \( \beta \) has the meaning that input power \( (q^Hq) \) is assured by using different \( \beta \) in each frequency. Second, in Eq. (7), the inverse matrix has to be obtained for exact Lagrange multipliers. Therefore, various regularization techniques can be applied to solve Eq. (7) [6].

\[
\begin{bmatrix}
G_{d1}^H G_{M1}^H & G_{d1}^H G_{M2}^H & \cdots & G_{d1}^H G_{Mk}^H & 0 & 0 & \cdots & 0 \\
G_{d2}^H G_{M1}^H & G_{d2}^H G_{M2}^H & \cdots & G_{d2}^H G_{Mk}^H & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\
G_{dk}^H G_{M1}^H & G_{dk}^H G_{M2}^H & \cdots & G_{dk}^H G_{Mk}^H & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & G_{M1}^H G_{A1}^H & G_{M2}^H G_{A1}^H & \cdots & G_{Mk}^H G_{A1}^H \\
0 & 0 & \cdots & 0 & G_{M1}^H G_{A2}^H & G_{M2}^H G_{A2}^H & \cdots & G_{Mk}^H G_{A2}^H \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\
0 & 0 & \cdots & 0 & G_{M1}^H G_{An}^H & G_{M2}^H G_{An}^H & \cdots & G_{Mk}^H G_{An}^H \\
\end{bmatrix}
\begin{bmatrix}
s_1 \\
s_2 \\
\vdots \\
s_k \\
\lambda_1 \\
\lambda_2 \\
\ddots \\
\lambda_m \\
\end{bmatrix}
= \begin{bmatrix}
P_{d1, \text{Inaudible}} \\
P_{d2, \text{Inaudible}} \\
\vdots \\
P_{dm, \text{Audible}} \\
0 \\
0 \\
\ddots \\
0 \\
\end{bmatrix} \tag{7}
\]

4. FEASIBILITY TEST IN AN ANECHOIC CHAMBER

4.1 Experimental Configurations and procedure for the feasibility test

Figure 5 describes the experimental configuration for a feasibility test in an anechoic chamber. In this paper, GENELEC 8240A is used as a main loudspeaker and MINX min11 of Cambridge Audio is used as control loudspeakers. All control points are followed in Fig. 3, but the test area was measured by the microphones in order to confirm the control results. In the proposed approach to minimize the sound pressure in the inaudible region, the sound can be suppressed only at control points. Thus, we measured sound pressure in the test area for analyzing the control results. The test area was selected by a distance from 1.37m to 2.91m on the origin of a main loudspeaker. The number of microphones is 29, and the spacing between each microphone is 0.07m.

For the feasibility test, we carried out in three steps. First, all transfer function (TF) is measured by the angle step of 5 degrees from 0 degrees to 180 degrees using Swept Sine method. Second, the
proposed control approach in session 3.2 is applied based on TF data at center frequencies of one-third octave band. Thus, the source strength vector ($q$) is calculated per each center frequency. Third, we apply the obtained source strength ($q$) into all TF data in the test area, and then we can acquire controlled sound field. These results are summarized in the following session.

![Figure 5](image_url) The experimental configuration for a feasibility test in an anechoic chamber.

### 4.2 Experimental Results

Figure 6 presents the experimental results regarding to center frequencies of one-third octave band. The experimental results are considered under a selected frequency band because of cut-off frequency of an anechoic chamber, the threshold frequency of the woofer driver in a main loudspeaker, and the Nyquist frequency of the array. The cut-off frequency is 200 Hz, the threshold frequency is 3 kHz, and the Nyquist frequency is around 2 kHz. Therefore, the control results are examined from 200 Hz to 2 kHz.

![Figure 6](image_url) Experimental results of a feasibility test in an anechoic chamber.

(a) Experimental results in 250 Hz
(b) Experimental results in 500 Hz

(c) Experimental results in 1 kHz

(d) Experimental results in 1.25 kHz

(e) Experimental results in 1.6 kHz

Figure 6: Experimental results of a feasibility test in an anechoic chamber.
The left sound field of Fig. 6 displays the uncontrolled results of the main loudspeaker. The center sound field indicates the results of the designed array response, so the main beam is formed in the inaudible region and the side lobes are suppressed in the audible region. The right sound field of Fig. 6 presents the controlled results of a main loudspeaker. In this paper, although control points are few for cancellation, the proposed approach can control the directivity pattern of a large loudspeaker using a small loudspeaker array. From all experimental results, we confirmed the feasibility of the proposed approach.

5. CONCLUSIONS

In this paper, we verified the feasibility regarding to the directivity control of a main loudspeaker using the different loudspeaker with a relatively small size. The proposed approach has advantages of increasing the control frequency and also decreasing the physical size of audio system with a specific directivity pattern in comparison with the array system composed of large loudspeakers.

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REFERENCES