Influence of directivity and spectral shape on the measured sound power level

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ABSTRACT
The sound power is the main quantity to describe the total amount of sound emitted by a source. According to the substitution method, the sound power of an unknown sound source can be determined by replacing it with a reference sound source of known power. At present, this method is lacking a quantitative relation between the characteristics of the sound sources used and the uncertainty budget of the determined sound power level. To change this, models describing the spectral shape and directivity of sound emission of unknown sources were established. The models describe sound pressure levels on enveloping surfaces for the source under test. These could be compared to measurement data for a reference sound source. Using the substitution method, sound power levels for the model data could be calculated. Models were generated using both a pseudo-random number generator as well as finite element software. Sound sources were assumed to be operated in closed rooms. The wall properties were varied to provide smaller or larger amounts of reflections. Using Monte-Carlo simulations the uncertainty that is due to the spectral shape and directivity of the sound emission of sources under test was investigated.

Keywords: Sound power, directivity, spectral shape

INTRODUCTION
The substitution method is a widely used and accepted method for sound power determination. By measuring a sound source of unknown sound power and a sound source with known sound power output, the sound power of the unknown source can be determined. This procedure is described in ISO standards 3741, 3743-1 and 3747. The uncertainties associated with the determined sound power level are described using the term \( \sigma_{RO} \). Specifically, this term describes the uncertainty that is due to the measurement procedure itself. As such, it includes contributions from different sound emission patterns of the two sources. Upper bounds for \( \sigma_{RO} \) are listed in the ISO standards for sources with a "relatively flat" spectrum. However, being a term that lumps together different uncertainty contributions, the influence of individual factors is not quantified. Numerical studies focusing on two specific factors are introduced here. These factors are the directivity of sound emission and the spectral content of the two sources. Through variation of these two source descriptors, their influence on the uncertainty of the determined sound power is described qualitatively. The investigation is focused on the application of the substitution method in approximated hemi-free fields. Previous studies on sound sources with different directivities focused on the comparison of sound power levels determined using different ISO standards [4]. Results of the study reported here were first presented at the German annual acoustics conference (DAGA) [6].

THE SUBSTITUTION METHOD
As described above, the substitution method is a two-step measurement process. First, a reference sound source (RSS), whose sound power output is known, is installed in the measurement room. Its emitted sound pressure levels (\( L_{p,RSS} \)) are recorded on an enveloping surface. Second, the RSS is replaced by the sound source whose sound power is to be determined. The sound pressure levels (\( L_{p,DUT} \)) of this device under test (DUT) are then measured on the same enveloping surface used before (see Fig. 1). The sound power of the DUT can then be determined by applying a correction term to the known sound power of the RSS (Eq. 1)

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\[ L_{W,DUT} = L_{W,RSS} + L_{p,DUT} - L_{p,RSS} \]  \[ \text{[dB]} \]  

**REFERENCE DATA**

Data from a reference sound source were collected in PTB’s hemianechoic room. Measurements of sound pressure levels were performed in one-third octave bands using a hemicircular arc of radius 2 m. 24 microphones were mounted on the arc and it was moved in 1° steps from the floor to an elevation angle of 84° [5]. The 1 kHz third octave band sound power \((L_W)\) and 84x24 sound pressure levels \((L_p)\) were used as reference for the directivity analysis (Fig. 2). For the analysis of the influence of the spectral content of the source reference data were comprised of sound powers and sound pressures for all third-octave bands between 20 Hz and 20 kHz.

**DIRECTIVITY**

Differences in the directivity between reference sound source and sound source under test are most crucial in (hemi-)free environments. This is due to the lack of reflections from the room walls. Reflections diminish directivity differences as they cause an overlap of incident and reflected sound waves in the measurement room. For this reason only a hemianechoic environment was considered in the following numerical simulation.

To develop models for devices under test that differ from the RSS in the directivity of their sound emission, the directivity index \((DI)\) - defined as difference between maximum and mean sound pressure level on the enveloping surface - was used (Eq. 2).

\[ DI = \max(L_{p,i}) - \text{mean}(L_{p,i}) \]  \[ \text{[dB]} \]  

\[ \text{Figure 1: Measurement set-ups for the application of the substitution method in an approximated hemi-free field.} \]

\[ \text{Figure 2. Distribution of sound pressure levels for the 1 kHz one-third octave band of the RSS.} \]
Directivity indexes used were 0, 2, 4, 6, 8 and 10 dB. For each DI $10^3$ models were developed, where each model consisted of 84x24 sound pressure levels to match the data from the reference sound source (Fig. 3). Sound pressures ($p^2$) were distributed uniformly in a first run and in a second run sound pressure ($p$) followed a normal distribution.

To match realistic measurement set-ups subsets with 2 to 200 points were chosen randomly from each 84x24 point modeled DUT. For each subset size $10^3$ distinct variations were produced. This resulted in a Monte Carlo method evaluation with $10^6$ variations per subset size and directivity index (Fig. 4).

It should be noted that the use of a random sampling strategy to obtain the described subsets makes a consideration of the specific directivity shape of the device under test unnecessary. The key descriptors for these subsets are the difference between maximum and minimum sound pressure levels and the standard deviation of the mean of the sampled set. As the sound pressure levels drawn from the created distributions do not correspond to specific measurement locations, this means that they are relevant for all sound sources whose directivity plots show that same difference in maximum and minimum sound pressure level with associated standard deviation.

Sound power levels ($L_{W,i}$) were calculated for each variation using the substitution method (Eq. 1). For each subset size a corresponding number of sound pressure levels were chosen randomly from the data set of the reference sound source. In this way the sampling size for the averaged sound pressure levels $L_{p,DUT}$ and $L_{p,ref}$ were always the same. The sound power levels obtained in this way were compared to the sound power level ($L_{W,84x24points}$) obtained using all 84x24 points for each modeled device under test. The assumption made was that these 84x24 values correspond to a complete sampling of the sound field and thus to the correct value of the sound power for the modeled sound source. The standard deviation (Eq. 3) of the difference between sound power calculated from an under-sampled sound field vs. completely sampled sound field was used as indicator for the uncertainty contribution of the directivity of sound emission.

Figure 3. Samples of modeled devices under test. Sound pressures $p^2$ follow a uniform distribution.

Figure 4: Representation of the Monte Carlo method used. Each rectangle represents a matrix or vector of specified size. For each directivity index 1000 models were developed and from each model 1000 different combinations per subset size generated.
\[ u = \text{std}(L_{W,i} - L_{W,84x24\text{ points}}) \text{ [dB]} \] (3)

Results show that an increase in measurement microphones corresponds to an exponential decrease of the standard deviation of the difference in sound power levels between under-sampled and completely sampled sound fields. The uncertainty associated with the determined sound power is expected to behave in the same way. As expected, smaller directivities in sound emission lead to a smaller uncertainty contribution of the directivity. No significant difference was observed between DUTs with normal or uniform distributions of sound pressures (Fig. 5).

Figure 5: Results on the behavior of the uncertainty due to directivity differences in the sources used in the substitution method.

**SPECTRAL CONTENT**

To investigate the influence of spectral content on uncertainty, A-weighted sound power levels (Eq. 4) were calculated in two different ways. First, the A-weighting was applied after the substitution method was used on every one-third octave band (Eq. 5). This corresponds to the correct procedure for A-weighted sound
power determination and was thus regarded as reference. Second, A-levels were calculated for each one of
the constituents of the substitution method. The substitution method itself was then used on those A-levels (Eq. 6). This method corresponds to a simplified A-level determination. Its validity was evaluated using the difference, $\epsilon$, of the two calculated sound power levels (Eq. 7).

$$L_{W,A} = 10 \log \left[ \sum_{i=1}^{n} 10^{\frac{L_{W,i} + A_i}{10}} \right] \text{[dB]}$$  \hspace{1cm} (4)

$$L_{W,ref} = (L_{W,i,DUT})_A = (L_{W,i,RSS} + L_{p,i,DUT} - L_{p,i,RSS})_A \text{ [dB]}$$  \hspace{1cm} (5)

$$L_{W,DUT} = L_{W,A,DUT} = L_{W,A,RSS} + L_{p,A,DUT} - L_{p,A,RSS} \text{ [dB]}$$  \hspace{1cm} (6)

$$\epsilon = L_{W,DUT} - L_{W,ref} \text{ [dB]}$$  \hspace{1cm} (7)

The reference sound source data used for this analysis were the same as before. As these data sets were collected in a hemianechoic room, they corresponded to that environment. To model more diffuse sound fields, sound pressure levels were modified using a room correction coefficient, $K_2$. This $K_2$ was multiplied by a random number, $N_{[0,1]}$, with mean 0 and standard deviation 1 for each one-third octave band. With $K_2=0$ dB (unchanged RSS data), $K_2=7$ dB and $K_2=12$ dB three different acoustic environments were modeled (Eq. 8).

$$L_{p,RSS,K_2,i} = L_{W,RSS,i} - 10 \log(2\pi r^2) + K_2 N_{[0,1]} \text{ [dB]}$$  \hspace{1cm} (8)

The modeled sound sources under test were of one of three types:

1. A tolerance level between 1 and 20 dB was set around the one-third octave band sound pressure levels of the RSS. Modeled sound pressure levels followed a uniform distribution within this tolerance level. $5 \times 10^5$ models per tolerance level were developed (Fig. 6).

2. Tonal DUTs were modeled by randomly choosing a specific one third octave band so that this single one-third octave band was assigned a sound pressure level between 75 and 120 dB. The base sound pressure level was 70 dB and was assigned to all other one-third octave bands. $5 \times 10^5$ such models were developed (Fig. 7).

3. Broadband noise sound sources were modeled by prescribing the sound pressure level for the first one-third octave band. All other one-third octave band sound pressure levels were then given through sequential addition/subtraction of a constant value in the range of $\pm 5$ dB (Fig. 8).

Figure 6: Examples of modeled DUTs where a tolerance level was defined around each one-third octave band sound pressure level of the RSS.
Results show the value $\epsilon$, describing the difference in A-weighted sound power levels calculated using the two methods detailed above (Eq. 5-7). Results are shown separately for each type of sound source modeled for the three different types of rooms used.

The results show the same behavior for all types of modeled DUTs. $\epsilon$ is not significant in free field conditions ($K_2 = 0$ dB). It does become significant for approximated free fields ($K_2 = 7$ dB) and diffuse fields ($K_2 = 12$ dB), though (Figs. 9-11). This means that for these environments the simplified approach to A-weighted sound power determination is not valid. Correct sound power levels are obtained only by applying the substitution method to each one-third octave band and then calculating an A-weighting afterwards for non-free sound fields (Eq. 5).
CONCLUSION

Two factors contributing to the uncertainty of the substitution method were investigated. The first factor was the directivity of sound emission. Namely, numerical models for sound sources under test were developed whose sound emission displayed a directivity that was different from that of the reference sound source. Results indicate that the uncertainty in determined sound power levels that is due to this difference in directivity can be minimized through the use of more measurement microphones. Specifically, the uncertainty that is due to different directivity patterns decreases exponentially with an increase in measurement microphones used.

The second factor investigated was the influence of a difference in spectral content. Three different environments - free, approximated free and diffuse fields - were contrasted and compared. Devices under test displayed a spectral content that was different from that of the reference sound source. The sound power levels calculated were A-weighted either prior or post to the use of the substitution method. Analyses showed that the two A-levels obtained in this way are only equivalent in free sound fields. For non-free sound fields the difference between the sound power levels is significant. This means that for non-free sound fields A-weighted sound power levels have to be calculated by using the substitution method on each one-third octave band with subsequent A-weighting.
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