# Uncertainty of standardized aircraft noise descriptors evaluated from measurements

Alois HEISS[1]; Berthold VOGELSANG[2]

[1] Independent Researcher, former State Ministry for Environment of Bavaria/Germany

[2] Ministry for Environment, Energy and Climate Protection of Lower Saxony/ Germany

**ABSTRACT**

The standards ISO 20906 (2009) and DIN 45643 (2010) define a set of descriptors for aircraft noise impact which are to be evaluated on basis of measured sound pressure levels. As boundary conditions fluctuate, in general randomly, the magnitudes of those descriptors also vary correspondingly. Hence they reveal an inevitable uncertainty whose common basic measure is the variance. The specific variance algorithms are outlined and - in a further step - the confidence intervals. To evaluate the uncertainty of the averages the variance of a sum is performed explicitly, not as usual the sum of the single variances assuming a priori mutual statistical independence. This straightforward leads to the covariance matrix whose n•(n-1)/2 distinct elements quite comfortable can be calculated numerically by computer. By this way an eventually existing autocorrelation can be uncovered for sake of a realistic presentation of the uncertainty. A further aspect to be accounted of in the correct description of variance when measuring over finite time intervals is the variation of the event number. This aspect is outlined for time series of the single events. Examples for the evaluation of short- and long-term measurements in the vicinity of an airport in operation are presented.

Keywords: Aircraft Noise, Measurement Uncertainty, Correlation Analysis
I-INCE Classification of Subjects Number(s): 13.1.1; 77

## 1. INTRODUCTION

It is a fact of experience that the levels and the sequence in time of aircraft noise events fluctuate like other ambient noise in the environment. To a certain extent these fluctuations are not predictable due to their random nature. Concerning outdoor sound impact this can - also if the airplanes are standardized in their construction - be caused by

Different pilots,
Varying capacity utilization,
Spreading of the vertical track profile,
Spreading of the horizontal track within the corridor, primarily due to varying situations of wind,
Different conditions of sound propagation,
Random superposition of ambient noise, i. e. residual noise with respect to aircraft noise and
Any further emerging and not permanent recordable or perceptible boundary conditions.

Systematic deviations or lack of the data collected by measurement against the „true“ values, are not considered here. However, the standards ISO 20906: 2009 (1) and DIN 45643 (2) explicitly demand to take the systematic deviations into account within aircraft noise assessment, at least by an appropriate statement within the measurement report.

Topic of this study here is the sound impact with its variety of random fluctuations really going on at the measurement site. In consequence of the fluctuations differing values for the single aircraft noise events, the primary data type, are to be accepted. If there occur extensive quantities of data they constitute distributions, as is well known. In this case obviously no single measured value can be the „true“ result. This would not be representative for the whole situation to be described. But the situation characterized by the sound impact to be investigated can be described by one or more representative descriptors. A descriptor is an adequate algorithm taking into account **all** the sample elements with equal weighting, for example the equivalent continuous sound pressure level (SPL). Exactly this procedure is given by the standards (1) and (2). They define a series of descriptors which are presented in Table 1:

[1] Heiss-Alois@t-online.de
[2] Berthold.Vogelsang@mu.niedersachsen.de

Table 1 – Aircraft noise key descriptors in the standards ISO 20906 and DIN 45643

| Type | Notation | Defined in | |
|---|---|---|---|
| Equivalent continuous sound pressure level (SPL) | $L_{p,A,eq}$ | ISO | DIN |
| Sound exposure level (and ist mean) | $L_{p,A,E}$ | ISO | DIN |
| AS-weighted maximum SPL (and its mean) | $L_{p,AS,max}$ | ISO | DIN |
| N per cent exceedance level | $L_{p,AS,N}$ | ISO | DIN |
| Number above threshold | NAT | ----- | DIN |
| Time above threshold | TAT | ----- | DIN |

As will be shown below, for all kinds of descriptors in Table 1, the corresponding uncertainty of their numerical result, can be calculated by a related algorithm. And by arrangement of the single event-results in a sequence up or down, an event number descriptor like NAT is also a characteristic of the distribution. Usually a descriptor is composed by a random sample of measured primary values. In consequence the descriptor itself also can only be a random variable which itself is also distributed and hence also can only be an estimation due to the always limited sample size in measurement practice. It is to be distinguished between the terms descriptor, i. e. algorithm and its value, i. e. number.

## 2. RELEVANT ASPECTS OF THE UNCERTAINTY OF SOUNDDESCRIPTORS

### 2.1 Statistical basic Descriptors: Variance and Confidence Interval

The main statistical descriptor which is introduced and applied for the quantification of the uncertainty of randomly variating magnitudes is the variance. It is denoted by $\boldsymbol{\sigma^2}$ and defined as the averaged square of the deviation from the expectation value of the total of possible outcomes (total sample with principally infinite size). The expectation value is the mean of a random variable taken from the total sample. If, as usual, there is no confirmed value of the variance available then the variance is to be estimated from a sample of finite size. This estimation is the square of the standard deviation, denoted by **s**. For sound pressure levels and their addition as mean squares of sound pressure (Antilog-values) s is defined for example in the standard VDI 3723-1 (3).

In (3) and in the textbooks it is practice to calculate the variance of a mean from the variance of the n sample elements divided by n-1, the degree of freedom, to get an expectation true estimation. By this procedure the sum of the variances of the single sample elements is performed instead of the explicitly elaborated variance of the *sum* of the sample elements. Additionally it is the rule to assume without test on contradiction that the sample elements are mutually independent, i. e. covariance (autocorrelation) is zero or at least not significant. This kind of uncertainty here is tried to be avoided by explicitly evaluating the variance of the *sum* of the sample elements. This is a general approach according to reality. With this procedure existing autocorrelations can be detected, estimated and thus taken into account within the evaluation of the uncertainty of the descriptor of type "mean value". Furthermore within the following considerations no assumptions will be made on the type of sound pressure level distributions created by aircraft noise. It will be operated only by use of variances. The confidence interval principally is calculated as the product of the standard deviation times a coverage factor k (see Eqs. 14a,b and 17a,b etc. below).

### 2.2 About the Uncertainty Treatment of Aircraft Sound Descriptors in Standards and Literature

In the ISO 20906 (1) the measurement uncertainty is mentioned by a short statement in section 6 as follows: "The uncertainty of results obtained from measurements according to this International Standard shall be evaluated, preferably in compliance with ISO/IEC Guide 98-3 (4). If reported, the expanded uncertainty together with the corresponding coverage factor for a stated coverage probability of 95 % as defined in ISO/IEC Guide 98-3 shall be given". In the Annex B "Uncertainty of reported data" contributions to the measurement uncertainty are listed as a mixture of systematic and random deviations with a total variance $u^2 \approx 0{,}75\ (\mathrm{dB})^2$ of the measurement system. Although in ISO 20906 the decisive elements of the uncertainty are presented an evaluation of the descriptor uncertainty concerning air traffic noise up to the final numerical result on site, is not given. To fill this gap as far as random deviations are considered this paper here may be a contribution. The German DIN 45643 (2) contains in its Annex a part B presenting the same content as Annex B of the ISO 20906.

The method offered by ISO/IEC GUIDE 98-3 primarily is based on a function modeling the interesting descriptor which is dependent on (known) input parameters determining its uncertainty. From this function the first differential is taken as an approximation by linearization, mutual independence of the first derivatives assumed and the sum of the single variances $(u_i)^2$ performed, what results in the combined uncertainty. Precondition of this procedure is, that only small deviations are to be processed, i. e. in the level scale $u_i < 2$ dB. This for example applies if measurements of sound power levels of machinery are to be performed. But this does not work correct in every case if outdoor sound immissions are to be measured, especially concerning aviation noise. Restrictions of this kind do not apply on the considerations and procedures presented below.

The advantage of the German standard VDI 3723-1 (3) is that it explicitly offers a procedure how to evaluate straightforward the uncertainty in the dB-scale with the measured sound values as input. This in consequence represents already the final uncertainty because the distribution of the measurement data on site encompasses the additive variations of emission and sound propagation – like an analog computer. Aditionally the VDI 3723-1 still yields correct results if the standard deviation of levels is ≥ 2 dB and the main part of the probability density of the levels is not too far on the left side. However, in the VDI 3723-1 is assumed that no (significant) correlation exists between the single measurement values of the sample. Also this restriction does not apply on the following considerations and procedures.

In (5) besides other topics the kinds of uncertainties in the measured aircraft noise exposure are formally indicated by Thomann. There the uncertainty components of unattended aircraft noise measurements primarily are presented as a superposition of equipment, missing flights and residual noise. A guidance for quantitative evaluation of the interesting final descriptors by a numerical procedure is not given. Further considerations are made by Thomann on the uncertainty of aircraft noise measurements in (6). In (7) Thomann and coauthors have listed seven kinds of uncertainties of measurements. The uncertainties of type $u_{set}$, $u_{dist}$, $u_{add}$ and $u_{acc}$ are covered by the method presented here in the following chapters. For all these quantities estimations of ≤ 1,0 dB each are given in (7). The deviations $u_{inst}$, $u_{env}$ and $u_{set}$ can be regarded as systematic errors. In (8) Acensio and coauthors analyze identification and detection. It is a substantial completion of the detailed treatment of the whole topic. In (9) Rosin gives a list of uncertainties u of the measurement system which yields a combined standard uncertainty of about 0,67 dB(A).

### 2.3 On the Treatment of the Residual Sound

In the ISO 20906 (1) Annex B an algorithm for a level correction due to the residual sound is given, including some notes to this matter. Concerning the uncertainty there merely is referred to (4). A clearly definition and quantitative determination of the uncertainties within the separation of the residual sound according to the procedures of (4) is presented in (10), there in a general sense, and in an exact performance in (11). However, here in this paper the separation between aircraft noise and residual sound is not explicitly treated.

A prerequisite of DIN 45643 (2) is that the AS-weighted maximal sound pressure levels caused by single aircraft noise events are at least 15 dB above the $L_{p,\mathrm{AS},95}$ of the ambient residual sound. With regard to the close vicinity of the aircraft sound measurement stations such a prerequisite usually can be met. Thus the corresponding level correction due to the residual sound is not more than about 0,14 dB. For this reason it may be neglected.

## 3 DESCRIPTORS FOR AVERAGING AIRCRAFT NOISE AND THEIR UNCERTAINTY

### 3.1 Equivalent Continuous Sound Pressure Level, A-weighted, $L_{p,A,eq}$

According to (2) this energy equivalent averaging descriptor is defined by

$$L_{p,\mathrm{A,eq},T} = 10\,\lg\left(\frac{t_0}{T}\sum_{i=1}^{\mathrm{N}} 10^{0,1 L_{p,\mathrm{A,E},i}/\mathrm{dB}}\right) \quad dB \quad . \tag{1}$$

The sample elements in Eq. (1) are the sound exposure levels $L_{p,\mathrm{A,E},i}$ of the N single flight pass by events measured by a sound monitoring station during the time interval T. The sound exposure levels $L_{p,\mathrm{A,E},i}$ are defined by

$$L_{p,\mathrm{A,E},i} = 10\,\lg\left(\frac{1}{t_0}\int \frac{p_{\mathrm{A},i}(t)^2}{p_0^{\,2}}\,\mathrm{d}t\right)\mathrm{d}B\,\cdot \tag{2}$$

The rating time $t_0$ is set by $t_0$ = 1 s. The distributed measured values of $L_{p,\mathrm{A,E},i}$ and, in addition, their usually also distributed sequence in time are the relevant elements of the statistics which is inherent here.

### 3.2 The Variance of a Sum and Autocorrelation

The procedure for the determination of the random spread and in consequence of the uncertainty of a mean from a sample with distributed elements is to evaluate the variance, as generally introduced in the statistical textbooks. This means as already mentioned above that the expectation value of the squared differences between the single values and their average is calculated – with the usual not analyzed assumption that there is no (significant) autocorrelation which could change the variance. Due to the meanwhile common high computing power it is reasonable to determine the variance of a mean as that what it is by the strong definition, i. e. explicitly by

$$Var(\sum_{i=1}^{n} x_i / n) := <(\sum_{i=1}^{n} x_i - <\sum_{i=1}^{n} x_i>)^2> / n^2 . \tag{3}$$

Here the sample element $x_i$ stands in abbreviation for $10^{L_{p,A,E,i}}$. In Eq. In eq. (3) the notation <•••>, instead of an upper horizontal line, indicates averaging, strictly speaking over all values possible. But those are not accessible. Thus the mean can only taken from the sample available, as an estimation. It is convenient to transform Eq. (3) into Eq. (4) and temporary without the factor $1/n^2$:

$$Var(\sum_{i=1}^{n} x_i) = <\sum_{i=1}^{n}(x_i - <x>)\cdot\sum_{k=1}^{n}(x_k - <x>)> . \tag{4}$$

With the abbreviations $x_i - <x> := \Delta_i$ and $x_k - <x> := \Delta_k$ and with the abbreviated notation Y for $Var(\Sigma)$ we get

$$Y := Var(\sum_{i=1}^{n} x_i) = \sum_{i,k=1}^{n} <\Delta_i \cdot \Delta_k> = n\sigma^2 + \sum_{i\neq k=1}^{n} <\Delta_i \cdot \Delta_k> . \tag{5}$$

The product $<\Delta_i \Delta_k>$ now formally can be understood as a matrix element, with i as row- und k as column index in the Eqs. (4) and (5). The track of the matrix, i. e. the sum of element products with equal indices, is $n \sigma^2$, where $\sigma^2$ is the common variance, to be found in the textbooks of statistics. The sum on the right side of Eq. (5) explicitly represents the covariance. It is convenient to add up the matrix elements in Eq. (5) in the diagonal direction because then the n-2 results can be expressed in terms of the multiple step correlation coefficient. The

diagonal addition results in

$$Y = n\sigma^2 + 2\sum_{l=1}^{n-1}\sum_{i=1}^{n-l} <\Delta_i \cdot \Delta_{i+l}> . \tag{6}$$

Furthermore it is by definition

$$<\Delta_i \cdot \Delta_{i+l}> = \sigma^2 \cdot r(l) . \tag{7}$$

(see for example (12), p. 221.), where r(l) denotes the correlation coefficient of step l (index l running from unity to n-1). The correlation coefficient r(l) is scaled on $\sigma^2$. By this convention we get the general final expression

$$Y = \sigma^2 \cdot \left[ n + 2\cdot\sum_{l=1}^{n-1}(n-l)r(l) \right] . \tag{8}$$

In the following considerations, especially in the presented examples Eq. (8) will be the tool for the evaluation of the uncertainty of the aircraft noise related equivalent continuous SPL and of the energy equivalent mean of the maximum sound pressure level.

### 3.3 Number Variance

If the single events have a variating sequence in time an additional contribution to the descriptor variance respectively uncertainty is created if the descriptor makes an averaging over a finite time interval. This is due to the variation of the number of events within separated time windows with equal widths. For this reason the actual variance of the $L_{eq}$ usually is not only determined by the spread and correlation of the single event levels but also by the distribution of the sequence time intervals. If all the single event levels would have equal magnitude, then for the $L_{eq}$ still would remain a variance due to variating event numbers in case of repetition at constant conditions. This number variance already was investigated by Heiss (13). It is applied in the following chapter. It is determined by the relation

$$Var N_T = <N_T> \cdot \sigma_{\Theta}^2 / <\Theta>^2 . \tag{9}$$

In Eq. (9) $<N_T>$ denotes the averaged number of events which occur during the time interval T and Θ stands for the time interval between immediate successive events. If the sequence has strong periodicity then $N_T = 1$, because fluctuations of the events within the time window of length T can occur only in the first and/or last period which are still part of T. From this and according to the well known rule $Var\ x = <x^2> - <x>^2$ follows $Var\ N_T = 1/12 = const.$, what can be neglected in practice.

By the mentioned definition of variance can be proved that the number variance does not depend on a permanently present minimum number of the events. Such a minimum event number for example can be established by the flight schedule of an airport. On the other hand the whole sound energy doses of the events well effect the uncertainty of the mean.

The number variance is also determined by the so called Characterization Time, defined in (3). This for example for aircraft noise can be all the nights during the 6 most frequented months or all days within a month etc. From view of practice it can be advantageous to combine the data sets collected by the measurement into groups. Every group is characterized by its choice out of equally spaced time intervals into which the total measurement time has been divided. These time intervals can be hours, days, nights, weeks and months. Then the summed up amounts of the single event sound energy values within every time interval can be considered and treated as „the new sample“ to be further processed for evaluation of the uncertainty (see example 2 below). This procedure yields the variance without the explicit use of Eq. (9) respectively the term given by Eq. (12) below. On the other hand Eq. (12) gives the option to present explicitly the contribution of the number variance to the uncertainty budget of the $L_{eq}$ type descriptor.

### 3.4 Variance of the Mean at Aircraft Noise and Confidence Interval in the dB-Scale

When for application on aircraft noise in Eq. (4) the variable of kind $x_i$ (and $x_k$ respectively!) is replaced by $10^{0,1L}{}_{p,A,E,i}$ and, for convenience the notation n by N, then one arrives at the general notation of the $10^{0,1L}{}_{p,A,E,i}$-variance including also possibly existing terms of autocorrelation

$$Var\left\{10^{0,1L_{eq}}\right\}=\frac{1}{T^2}\left[\sigma^2\cdot\left[N+2\cdot\sum_{l=1}^{N-1}(N-l)r(l)\right]+10^{0,2<L_{p,A,E}>}\cdot Var\{N\}\right], \tag{10}$$

where

$$\sigma^2 := Var\left\{10^{0,1L_{p,\mathrm{A,E},i}/\mathrm{dB}}\right\}. \tag{11}$$

T denotes the total duration of the measurement and N the number of events, i. e. recorded pass by flights within T. The term $\sigma^2\cdot[\bullet\bullet\bullet]$ is the square of standard deviation extended by the term due to auto-correlation. The last term in Eq. (10) represents the number variance. It can be evaluated either by Eq. (9) from the distribution of the sequence times or better practicable from n number groups out of equally spaced time intervals covered by the complete interval T, as already mentioned above. Then the further use of the basic definition of variance yields

$$Var\{N\}=n\cdot s_m^2 = N\cdot s_m^2 / <m> \tag{12}$$

because by definition the number n of groups times the average <m> of the numbers m of events per group equals N. Division of Eq. (10) by the antilog square of the equivalent continuous SPL yields $v^2_{Leq}$, the square of the complete variational coefficient as follows:

$$10^{-0,2L_{eq}}\cdot Var\left\{10^{0,1L_{eq}}\right\} = (1/N)\sigma^2 10^{-0,2L_{eq}} + 2\sigma^2\sum_{l=1}^{N-1}(N-l)r(l)\cdot 10^{-0,2L_{eq}}/N^2 + \frac{s_m^2}{\overline{m}}/N \; := \; v_{L_{eq}}^2 . \tag{13}$$

The arrangement of terms at the right side of Eq. (13) represents an uncertainty budget, but different to the definition given in (4). The first term represents the conventional variance coefficient. This is extended by the autocorrelation term and completed by the contribution due to the number variance. In (3) only the first term is considered. The variance coefficient $v^2_{Leq}$ presented by Eq. (13) determines the uncertainty in the decibel scale, caused by random variations of the immission by distinct sound events. From this and with an appropriate coverage factor k the confidence limits in dB for the equivalent continuous SPL are:

$$L_u = L_{eq} + 10\lg(1+k\cdot v_{L_{eq}}) \tag{14 a}$$

and

$$L_l = L_{eq} + 10\lg(1-k\cdot v_{L_{eq}}). \tag{14 b}$$

“u” stands for “upper” and “l” for “lower”. In all the following examples the coverage factor is chosen 1,3 according to a two-sided confidence interval with a confidence level of 80% consistent with the convention laid down in (3).

All examples use selected sound measurement results from the Hanover Airport/Germany.

Table 1 – Example 1 for the evaluation of the uncertainty

Type of descriptor: $\mathbf{L_{p,A,E,eq}}$

| **Conditions:** | |
|---|---|
| Measurement site | At the side of the runway, in about 220 m Distance |
| Measurement time | Daytime |
| Mode of operation | Only landings |
| Sample size | 9 |
| **Results:** | |
| $L_{p,A,E,eq}$ | 88,7 dB(A) |
| Confidence Interval | |
| On basis of σ alone | ± 1,2 dB(A) |
| On basis of $Y^{1/2}$ | ± 2,1 dB(A) |

Table 2 – Example 2 for the evaluation of the uncertainty

Type of descriptor: $\mathbf{L_{p,A,,eq}}$

| **Conditions:** | |
|---|---|
| Measurement site | At the side of the runway, in about 220 m Distance |
| Measurement time | Night |
| Mode of operation | Starts and – in majority - landings |
| Sample size | 4, weekly cumulated |
| **Results:** | |
| $L_{p,A,,eq}$ | 55,4 dB(A), for the groups: 108,4 dB(A) |
| Confidence Interval | |
| On basis of σ alone | ± 2,1 dB(A) |
| On basis of $Y^{1/2}$ | ± 3,4 dB(A) |
| Note | Here over the month stationarity did not exist. In consequenc the uncertainty is relatively high in comparison with normal, i. e. stationary conditions in airport operation. |

# 4 PERCENTILE LEVELS $L_{p,AS,1}$ and $L_{p,AS,95}$

In the VDI 3723-1 (3), section 5.2 a complete set of descriptors of this kind based on measurement including the procedure to evaluate their confidence intervals by the tables 4 to 6. There the evaluation of the confidence intervals is based in principle on the binomial distribution.

# 5 DESCRIPTORS OF SINGLE EVENT STATISTICS

## 5.1 Equivalent mean of the Maximum Sound Pressure Level, $<L_{p,AS,max}>$

This kind of average is defined by

$$< L_{p,\mathrm{AS,max}} > = 10 \lg\left(\frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}} 10^{L_{p,\mathrm{AS,max},i}/10}\right)\mathrm{dB} \quad . \tag{15}$$

In Eq. (15) N denotes the number of identified aircraft noise events which occurred within the time interval T. The sample elements evidently are the observed flight pass by maximum levels $L_{pASmax,i}$. The tool for the evaluation of the variance and the corresponding confidence limits for this averaging descriptor is already provided by the analogous treatment of the equivalent continuous SPL $L_{p,A,eq}$ in section 3.4. The replacement of $L_{eq}$ and $L_{p,A,E,i}$ by the energy equivalent mean $<L_{p,AS,max}>$ of the maximum SPL and $L_{p,AS,max,i}$ respectively, from which in the following Eq. (16) the conventional variance $\sigma^2$ is calculated, gives

$$10^{-0,2<L_{p,\mathrm{AS,max,Fl},T}>}\cdot Var\left\{10^{0,1<L_{p,\mathrm{AS,max}}>}\right\} = (1/N)\sigma^2 10^{-0,2<L_{p,\mathrm{AS,max}}>} + 2\sigma^2\sum_{l=1}^{n-1}(N-l)r(l)10^{-0,2<L_{p,\mathrm{AS,max}}>}/N^2 := v^2_{<L_{p,AS,\max}>} \tag{16}$$

and the confidence limits

$$L_u = < L_{p,\mathrm{AS,max}} > +10\lg(\ 1 + k \cdot v_{<Lp,AS,\max>})\quad \mathrm{dB} \tag{17 a}$$

$$L_l = < L_{p,\mathrm{AS,max}} > +10\lg(\ 1 - k \cdot v_{<Lp,AS,\max>})\quad \mathrm{dB}\ . \tag{17 b}$$

In Eq. (16) the contribution by a number variance does not exist because here is no rating of the average with regard to a given time interval.

Table 3 – Example 3 for the evaluation of the uncertainty

Type of descriptor: $\mathbf{L_{p,AS,max,eq}}$

| **Conditions:** | |
|---|---|
| Measurement site | M09, Heitlingen, Garbsen, in about 3,5 km distance from the western end of the northern take off flight path. Flights in mean distances aside about 400 and 1400 m |
| Measurement time | All nights within 6 months |
| Mode of operation | Starts and landings |
| Sample size | 2111 |
| **Results:** | |
| $L_{p,AS,max,eq}$ | 81,0 dB(A) |
| Confidence Interval | |
| On basis of σ alone | ± 0,156 dB(A) |
| On basis of $Y^{1/2}$ | + 3,67 dB(A)            - ???? dB |
| Note | Here a permanent and nearly constant correlation is present, with a throughout positive correlation coefficient of about 0,65. Due to high correlation the lower confidence level cannot be calcu-lated by the variance algorithm. But it can estimated to be higher than the L95 of the Lmax-distribution, i. e. about 73 dB(A). Due to high correlation: Computation of number variance here omitted. |

The following figures present the cumulative distributions of $L_{max}$ and its antilog values corresponding to example 3:

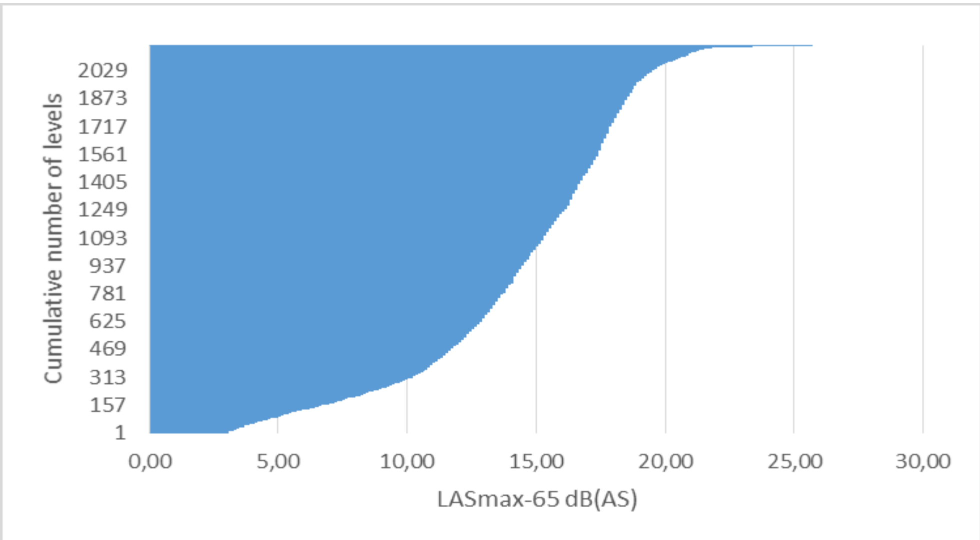


Figure 1 – The cumulative distribution of $L_{max}$ in example 3

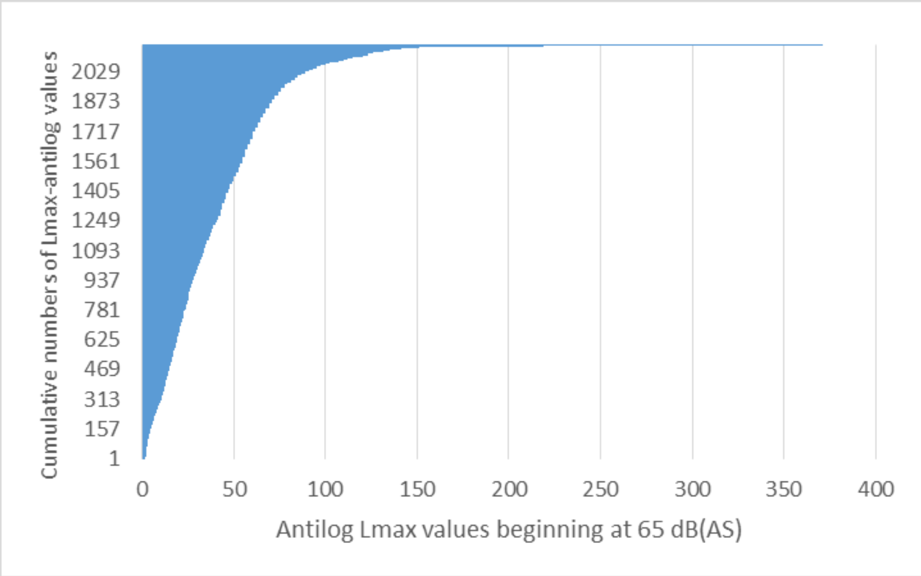


Figure 2 – The cumulative distribution of $10^{0,1\mathrm{Lmax/dB}}$ (antilog) in example 3

### 5.2 N Percent Exceedance Level, $L_{p,AS,max,N}$

According to the standard (2) the N% exceedance level is defined as the level which is exceeded by N % of all occurring aircraft sound maxima within the rating time. It can be determined from the cumulative distribution $S(L_{p,AS,max})$ by

$$S(L_{p,\mathrm{AS,max}}) = \frac{1}{\mathrm{N}} \sum_i \mathrm{n}(L_{p,\mathrm{AS,max},i}) \qquad \text{for} \qquad L_{p,\mathrm{AS,max},i} > L_{p,\mathrm{AS,max}} \quad . \tag{18}$$

Where S is set in advance and meets the constraint 0 < S < 1. In Eq. (18) denote

- $\mathrm{n}(L_{p,\mathrm{AS,max},i})$ the number of events which are fallen into the level class i and
- N the total of (aviation) events occurring at the monitor station and recorded respectively.

The level $L_{p,\mathrm{AS,max}}$ is the outcome, here abbreviated by $L_S$ at the position where the cumulative distribution amounts N %. S and N % as well are subject to specific uncertainty. This descriptor $L_S$ is associated to NAT, the number above threshold due to the relations

$$NAT(L_S) = S(L_S) \cdot N \tag{19 a}$$

and

$$N\%(L_S) = 100 \cdot S(L_S) = 100 NAT(L_S) / N \,. \tag{19 )}$$

This means that N % and – as an integer – NAT are mutually proportional. This allows to transform the variance and the confidence limits of $L_S$ into those of NAT and vice versa. The variance of NAT as an integer easily can be presented and accordingly the corresponding confidence interval. For this reason the variance of N % will be treated in the following section.

### 5.3 Number Above Threshold, $NAT_{LS}$

According to the standard (2) NAT is defined as the number of the aircraft sound events whose AS-weighted maximum SPL exceeds a preset threshold level $L_S$ during the measurement time interval T:

$$\mathrm{NAT}_{LS} = \sum_{i=1}^{\mathrm{N}} w\left(L_{p,\mathrm{AS,max},i}, L_\mathrm{S}\right) \quad , \tag{20 a}$$

where

$$w\left(L_{p,\mathrm{AS,max},i}, L_\mathrm{S}\right) = \left\{ \begin{matrix} 1 \text{ for } L_{p,\mathrm{AS,max},i} > L_\mathrm{S} \\ 0 \text{ for } L_{p,\mathrm{AS,max},i} \le L_\mathrm{S} \end{matrix} \right\} . \tag{20 b}$$

The very definition of NAT is given by Eq. (20 b), i. e. NAT simply is determined by counting the single events whose levels occurred above the threshold (level). For this procedure is convenient to arrange these data in a first step in a descending order. Then, on the SPL scale the threshold be within the sample. The probability that within all possible samples the recorded values are positioned above the threshold be S, with 0 < S < 1. Then, in case of repeating the same sampling the outcome can be another number of values between zero and sampling size above the threshold. This means that NAT obeys a binomial distribution. Details about the binomial distribution are found in the statistical standard literature, for example in (12), p. 111 ff. The expectation value and the estimator value for the NAT is S*N. Also for NAT as a distributed random variable a variance exists (s. for example (12), p. 144). It is determined by

$$Var(NAT) \;=\; N \cdot S(1-S) \;\equiv\; (NAT)\cdot(1-NAT/N) \;. \tag{21}$$

From Eq. (21) is obvious that the NAT variance is determined only by itself and the total sample size N of the relevant single aircraft noise events. If the threshold is positioned very close to the margin of the sample then the variance practically coincides with the variable NAT itself, a relation also well known from Poisson Statistics. Var(NAT) represents a number variance but is distinct from that which is presented above in section 3.3.

The evalulation of the estimator and the confidence limits of the NAT using accordingly the preceding relations is demonstrated in detail in (3), section 5.1.2, tables 7a to 7c. For example: If there are 20 single outcomes from one night and the NAT is preset to be ≤ 6, then the confidence limits (rounded down and up respectively) are positioned at the 9th and the 3rd value counted from the top. The corresponding N % values simply follow by applying Eq. (19 b).

Table 4 – Example 4 for the evaluation of the uncertainty

Type of descriptor: **NAT6**

| **Conditions:** | |
|---|---|
| Measurement time | One night |
| Mode of operation | Starts and – in majority - landings |
| Sample size | 9 |
| | |
| **Result:** | |
| NAT3 | 76,2 dB(A) |
| Confidence Interval | + 2,9 dB(A) - 4,4 dB(A) |
| Note | This result shows that by such a small sample of a single night no uncertainty can be accomplished which is sufficiently low. The confidence Interval encompasses nearly the whole range of all available measured events. |

Table 5 – Example 5 for the evaluation of the uncertainty

Type of descriptor: **NAT3,** representative for the average single night

| **Conditions:** | |
|---|---|
| Measurement time | All nights within 3 months |
| Mode of operation | Starts and landings |
| Sample size | 1057 |
| **Result:** | |
| NAT3 | 80,4 dB(A) |
| Confidence Interval | ± 0,122 dB(A) |

## 5.4 Duration of Level Exceedance, $TAT_{LS}$

The total of the temporary time intervals, denoted by $t_{LS.i}$ during which the instant AS-weigthed level caused by sound from aviation exceeds a threshold $L_S$ within the measurement time interval T is denoted with $TAT_{LS}$ according to Eq. (22) and (2).

$$\mathrm{TAT}_{LS} = \sum_{i=1}^{N} t_{L_{S,i}} \;. \tag{22}$$

The link to the system parameters T and S easily can be seen by Eq. (23):

$$\mathrm{TAT}_{LS} = T \cdot S \tag{23}$$

At the end of the measurement it can be realized which percentile $L_{S\%}$ of the continuous time dependent instant level coincides with the preset threshold $L_S$. By this procedure the interesting value for S to determine $TAT_{LS}$ according to Eq. (23) is found.

## 6 CONCLUSION

For all descriptors based on the measurement of aircraft sound immission appropriate algorithms can be established to evaluate the uncertainties due to the manifold random variations of the sound impact. Of course, some aspects of second order could be considered in addition, for example to which extent the calculation of the autocorrelation itself is subject to its own uncertainty due to the finite sample size in practice and what the physical and/or operational causes for the observed correlations are, etc. On the other hand by the analysis above a tool and an option is available to perform explicitly quality control of aircraft sound measurements. By this tool at least default values of the uncertainty can be established to demonstrate the occurring orders of magnitude and for predictive estimations. It is an opportunity primarily for data processing in case of a big amount of collected data, for example by unattended stations for the monitoring of aircraft sound in the vicinity of airports.

Further benefits can be

- to make statistically significant comparisons, for example between successive time periods with different status of noise reduction etc.
- in general to plan and perform an optimized measurement project based on a given quality requirement and by use of appropriate comparable data from former measurements. This seems primarily to be an option if temporary measurements are to be performed with use of mobile monitoring stations.

## REFERENCES

1. ISO 20906: 2009. (2009) Unattended monitoring of aircraft sound in the vicinity of airports.
2. DIN 45643:2010-09 (2010) Measurement and assessment of aircraft sound (In German Language: *Messung und Beurteilung von Fluggeräuschen*).
3. VDI 3723-1: 1993, Application of statistical methods for the description of variating ambient noise levels (In German Language: *Anwendung statistischer Methoden bei der Kennzeichnung schwankender Geräuschimmissionen*).
4. ISO/IEC Guide 98-3:2008, Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM: 1995).
5. Thomann G. Measurement and Calculation Uncertainty of Aircraft Noise Exposure. Proc. INTER-NOISE13; 15-18 September 2013; Innsbruck, Austria. Paper 0044.
6. Thomann G. Mess- und Berechnungsunsicherheit von Fluglärmbelastungen und ihre Konsequenzen (*Uncertainty of calculation and measurement of aircraft noise impact and its consquences*). Thesis No. 17433 ETH Zürich, Swiss. 2007.
7. Thomann G, Bütikofer R,Krebs W. Uncertainty of measured and calculated aircraft noise. Proc. Managing Uncertainty in the Noise Measurement and Prediction. 27-29 June 2005; Le Mans, France.
8. Acensio C, Ruiz M,Pavon I, Recuero M. Uncertainty derived from the discrimination of events in aircraft monitoring. Proc. INTER-NOISE13; 15-18 September 2013; Innsbruck, Austria. Paper 0473.
9. Rosin C. Uncertainty of aircraft noise measurements. Evaluation for an aircraft noise monitoring network. Proc. INTER-NOISE13; 15-18 September 2013; Innsbruck, Austria. Paper 1007.
10. Heiss, Alois, Klaus-Georg Krapf. Quantification of uncertainty by real time confidence limits in separation of sound immission levels. Noise Control Eng. J. 2007; 55(1). p. 149-158.
11. Heiss A. On the Uncertainty according to ISO/IEC 98-3 of the Equivalent Continuous Sound Pressure Level in Presence of Background Noise. Proc. AIA-DAGA 2013, Conf. on Acoustics; 18-21 March 2013; Meran, Italy. p. 500-503.
12. Hogg RV, Tanis E. Probability and Statistical Inference, Macmillan, 3rd Edition 1989 p. 221.
13. Heiss A. Varianz des zeitlichen Geräuschmittelwertes stochastischer Folgen kurzer Schallereignisse (In German Language) (*Variance of the equivalent continuous sound pressure level from random sequences of short sound events)*. Advances of Acoustics, DAGA'90, Part A, p. 385-388, Vienna 1990.