Definition of SEA models for structures with restricted accessibility

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ABSTRACT

The definition of an SEA model is usually focused on the determination of the SEA loss factors that are related to the power balance between SEA subsystems. There are several techniques available to perform this task that is usually referred as Experimental SEA. One of the most widely used techniques that has led to the definition of a set of different experimental procedures, is the Power Injection Method. This method is based on the experimental determination of several independent power balances for the system under study in order to calculate the whole set of SEA loss factors. For complex structures, some elements may have reduced accessibility or not accessibility at all. This fact means that it may be impossible to measure their response or to apply an external load in order to obtain one of the independent power balances required. Two methods are presented to analyse such systems whose study through the Power Injection Method is not feasible. Their application is shown through a study case: an L-shaped structure composed of two sandwich panels in a reverberant room. Results show that the methods presented allow determining the system SEA loss factors if one of the subsystems considered cannot be excited.

Keywords: Statistical Energy Analysis, Loss Factor I-INCE Classification of Subjects Number(s): 75.2

1. INTRODUCTION

One of the most used techniques to analyse the behaviour of a system at high frequencies is the Statistical Energy Analysis (SEA) that is focused on analysing the power balance within the system. Therefore, a system is subdivided into a set of subsystems (elements with similar nature) and the power exchange among them is analysed. The resulting set of equations is linear and it is defined by a set of coefficients related to the power loss within each subsystem (due to internal losses) and to the power transfer between the different subsystems (depending on their level of coupling). These coefficients are the internal loss factors (ILF) and coupling loss factors (CLF) respectively.

The definition of these coefficients is possible analytically for a reduced set of simple structural or fluid elements (1). Although there are numerical approaches (2) for other geometries, the determination of these coefficients for an actual system is usually performed experimentally. The first procedure proposed was the Power Injection Method (PIM) (3), based on the in-situ measurements of the power balance under different input powers. This method has also been used as base for different methodologies focused in solving some numerical problems (4, 5,6) associated to the measured energy.

For a SEA model defined by a set of n subsystems, the energy balance is determined by the set of n^2 SEA loss factors: n ILFs plus $n(n-1)$ CLFs. However, the number of equations derived from the power balance of each subsystem provides only n equations. PIM is based on analysing the power balance of each subsystem to n independent input power cases leading to the formulation of n^2 independent equations. The independent power cases are usually defined as successive excitation of each subsystem. This method (as well as all the derived ones based on this set on n^2 equations) requires to be able to measure the response of all the subsystems considered and also to be able to apply external loads in all the subsystems to produce independent input power distributions what imposes an important restriction for those complex and large systems that might involve accessibility issues to some part of the system (7). In order to overcome this problem, alternative formulations have been

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proposed: the Power Coefficient Method (8) or a formulation based on transmissibility (9) that only require to measure the response of the system but not the input power itself, although the latter one requires to know the internal or total loss factors of the system.

Then, if no additional information is available to reduce the number of unknowns, PIM can be used only for systems whose all elements are accessible to apply external loads and measure their response. This can be not the case for very large and complex structures but it can also be a practical issue: as to measure the whole system for n successive load cases might become a problematic issue from the point of view of the experimental set up and cost.

This work presents two methods to study systems with no access to all the elements (either to measure their response or to apply external loads) so that they cannot be studied by means of PIM. Each method, based on a different principle, is briefly presented and their use is illustrated by an application case consisting in an L-shaped structure composed of two sandwich panels.

2. SEA AND PIM FORMULATION

As stated in the Introduction, SEA equations analyse the power balance for each of the subsystems that make up the system, taking into account the input power due to external loads, the power dissipated within the own subsystems due to its vibration and the power transferred to or from other subsystems that it is coupled to. For a given frequency band, centred in ω, this power balance for subsystem i of a system composed of n subsystems is

$$P_i = \omega\eta_{ii}E_i + \sum_{j=1,j\neq i}^{n} \omega\eta_{ij}E_i - \sum_{j=1,j\neq i}^{n} \omega\eta_{ji}E_j \quad (1)$$

where P_i represents the power input due to external loads, η_{ii} is the ILF of the subsystems i, *and* η_{ij} (η_{ji}) is the CLF from subsystem i (j) to subsystem j (i). The set of equations representing the power balance of the n subsystems to a single input power case is expressed in matrix form as

$$\overline{\mathbf{P}} = \omega\mathbf{L}\overline{\mathbf{E}} \quad (2)$$

where $\mathbf{L}$ is a matrix whose coefficients are defined from the SEA loss factors and it is called the SEA Loss Matrix. If n different input power cases are considered then the matrix algebra leads to

$$\mathbf{P} = \omega\mathbf{L}\mathbf{E} \quad (3)$$

where $\mathbf{P}$ is a matrix of n rows (associated to the n subsystems) and n columns (associated to the n independent input powers) and $\mathbf{E}$ is also a matrix of size nxn with the same associations. Equation (3) is the PIM equation that allows obtaining straightforwardly the SEA Loss Matrix $\mathbf{L}$ although, due to the characteristics of the energy matrix $\mathbf{E}$, some numerical issues may have to be overcome (4).

3. METHODS FOR SYSTEMS FOR RESTRICTED ACCESIBILITY

3.1 Systems with restricted accessibility

The methods presented in this section can be applied to large or complex systems in which some elements cannot be accessed to measure their response or applied loads, making not possible to apply the PIM equation (3). This is the case of internal elements in spacecraft structures or for elements of large deployable structures (like intermediate panels in a solar wing in folded configuration). For such systems, the number of elements that can be adequately excited to define independent input powers (n_e) is lower than the number of subsystems ($n_e<n$) and PIM cannot be used to determine the set of loss factors. For these cases, equation (3) does not allow to calculate $\mathbf{L}$ as both matrices $\mathbf{P}$ and $\mathbf{E}$ have n rows but only n_e columns and some of the rows of $\mathbf{P}$ are null. The following methods allow calculating a partial set of the loss factors.

3.2 Error minimisation method

The first method is based on the straightforward minimisation of the error (Euclidean norm) of equation (3) by minimising the functional

$$J_1 = \left\| \mathbf{P} - \omega\mathbf{L}\mathbf{E} \right\| \quad (4)$$

Numerical algorithms found on the literature allow also including characteristics of the solution space (as equivalent ILFs, for example) in the implementation of the method.

3.3 Model updating method

The direct minimisation of the power balance equation error (J_1) can be used to determine the loss factors of small systems that can be solved from scratch. Usual problems involve large and complex systems whose design is based on previous ones, allowing defining a preliminary model based on the system characteristics (physical or mechanical) as well as from previous modelling experience. This knowledge can be used to define a preliminary model for the SEA Loss Matrix $\boldsymbol{L_M}$. This method is based on updating this model through a modal updating technique (9) in order to minimise the functional

$$J_2 = \left\| \mathbf{L} - \mathbf{L_M} \right\| \tag{5}$$

Subject to the power balance measured experimentally by equation (3). Formulation based on Lagrange multiplier allows defining analytically the updated model from the preliminary one and the experimental measurements:

$$\mathbf{L} = \mathbf{L_M} + \frac{1}{\omega}\left(\mathbf{P} - \omega\mathbf{LE}\right)\left(\mathbf{E}^T\mathbf{E}\right)^{-1}\mathbf{E}^T \tag{6}$$

The analytical solution of this method ensures a fast and robust method as will be shown in the application case that follows.

4. APPLICATION CASE

4.1 Case description and PIM application

The methods described in previous section are used to determine the SEA loss factors of a three subsystems SEA model. The structure analysed is an L-shaped structure composed of two sandwich panels surrounded by air that is the third subsystem considered. This structure was analysed within the ESA project “Random Derivation by Vibro-acoustic simulation” (RANDERIV) (11) to determine its vibro-acoustic response under a diffuse acoustic field and its SEA model by means of the Power Injection Method (12).

Figure 1 – L-shaped structure made up of two sandwich panels joined by two points

The SEA model is defined as composed of three subsystems: Subsystem 1 represents the larger panel (1.4 x 0.7 metres) made of up an aluminium honeycomb core of 35 millimetres thickness and two aluminium skins 0.5 millimetres thick. Subsystem 2 represents the smaller panel (0.65 x 0.65 metres) made up also of an aluminium honeycomb core (23 mm thick) and 1 millimetre thickness skins. Both panels are assembled by two L 2024 aluminium joints as shown in Figure 1 and they are modelled as SEA point junctions. Subsystem 3 represents the air of a reverberant room of 200 m^3, where the structure was placed. The dimensions of the reverberant room, at the ITEFI/CSIC facilities, are shown in Figure 2.

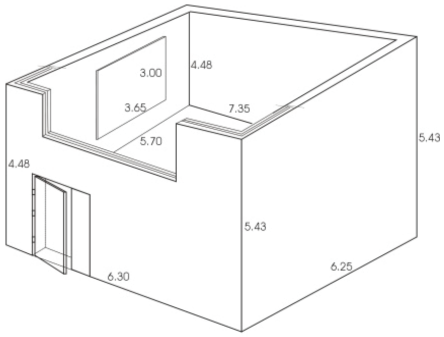

Figure 2 – Dimensions (metres) of ITEFI/CSIC Reverberant chamber

Within the RANDERIV project, the vibro-acoustic response of the structure and the air in the reverberant chamber was determined by a PIM test. The response of each subsystem was measured under successive excitation of each subsystem. For the structural excitation of the panels several excitation points were synthetized to approximate modal statistical independence (3,5). For each of them, the excitation consisted in a force with a white noise spectrum applied with an electromagnetic shaker (LDS450) as shown in Figure 3.

Figure 3 – Structural and acoustic excitation for the PIM test

The response of each subsystem was determined as the mean value of several discrete measurements of acceleration on the panels (for each excitation location) and the pressure measurements on four locations around the structure. The latter were also used to measure the acoustic load when exciting subsystem 3.

From the test measurements, the input power and subsystems energy was determined under each excitation configuration (over one of the subsystems at a time) defining so three independent input powers. Then PIM equation (3) can be solved to determine the SEA loss factors of the model. Results are shown in Figure 4 in logarithmic scale so some bands are not represented as negative values of the loss factors are obtained. This fact is found for bands in the higher end, from 4000 Hz and above. This fact is usually associated to a numerical issue due to the ill conditioning of the energy matrix in these bands. This ill conditioning could be originated by a poor measurement of the panels energy (due to lack of enough measurement points).

The values of the SEA loss factors obtained with PIM are used as reference to assess the methods proposed in previous section for a scenario of restricted accessibility.

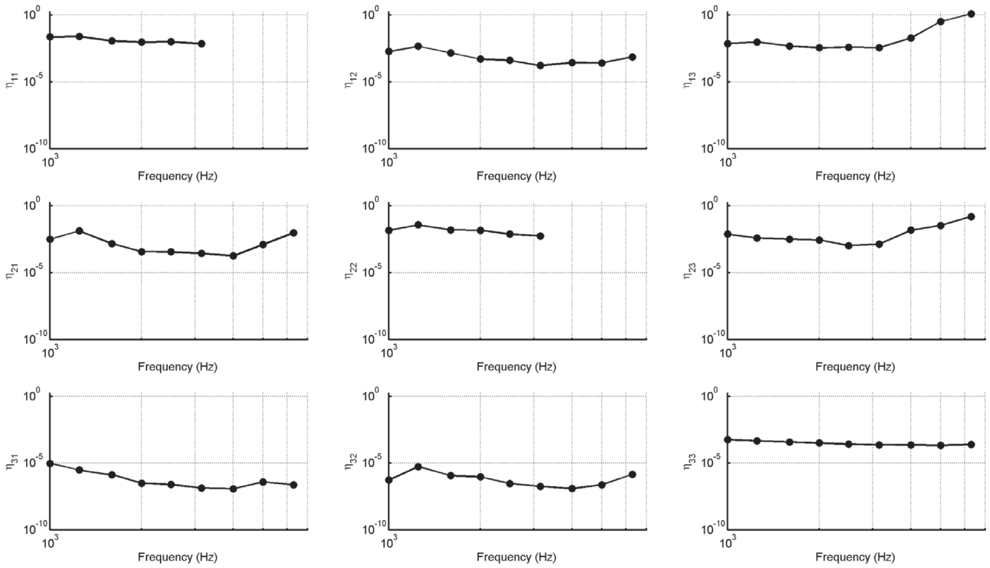

Figure 4 – SEA loss factors determined by means of PIM for successive input power in each subsystem (negative values not displayed)

4.2 Restricted accessibility case

In order to show the results that can be achieved by the methods proposed, it is considered from now on that the excitation to the smaller panel (subsystem 2) is not possible, and therefore only the energy of the whole system is known under structural excitation on the larger panel (subsystem 1) and acoustic load on the surrounding air (subsystem 3). In this scenario, the number of subsystems in the model is $n=3$ but the number of subsystems excited is $n_e=2$. Under this assumption PIM cannot be used to determine the loss factors and the methods described in previous section are used: First, the method based on minimising the error of equation (3). Second, the method based on model updating techniques. For this method, the preliminary model is defined from the result of the first method.

The SEA loss factors obtained by means of the first method proposed (error minimisation) are shown in Figure 5 along with the reference values corresponding to PIM taking into account 3 independent input power cases.

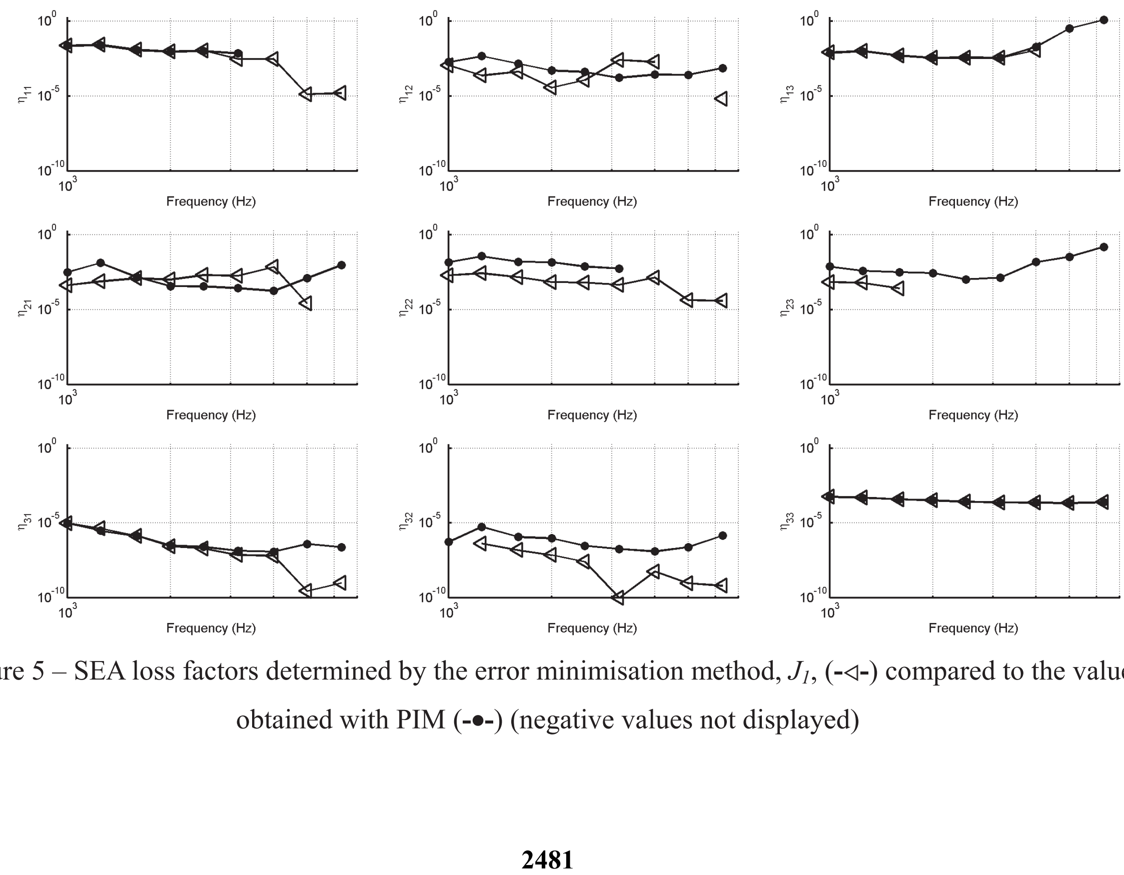

Figure 5 – SEA loss factors determined by the error minimisation method, J_1, (-◁-) compared to the values obtained with PIM (-●-) (negative values not displayed)

Results show that despite taking into account only the power balances in the system for two input power cases (excitation on the larger panel and the acoustic load) the results on the loss factors are about similar to the values obtained with PIM. On the subsystems that are excited (one and three), their ILFs (η_{11} and η_{33}) and the CLFs associated to their interaction (η_{13} and η_{31}) show very good agreement with the PIM values and even good results (positive values) are obtained in the higher end of the frequency spectrum analysed. Despite information on the energy distribution when the second subsystem is driven is unknown, results on the second subsystem (η_{22} and the CLFs related to it) are good at qualitative level as the values are of the same order and the trend is partially kept.

In order to improve the result obtained with this method, the second method is applied updating the results shown in Figure 5 taking into account the power balance measured (equation 6). The results are shown in Figure 6 along with the values corresponding to the application of PIM and the method based on J_1 to ease comparison.

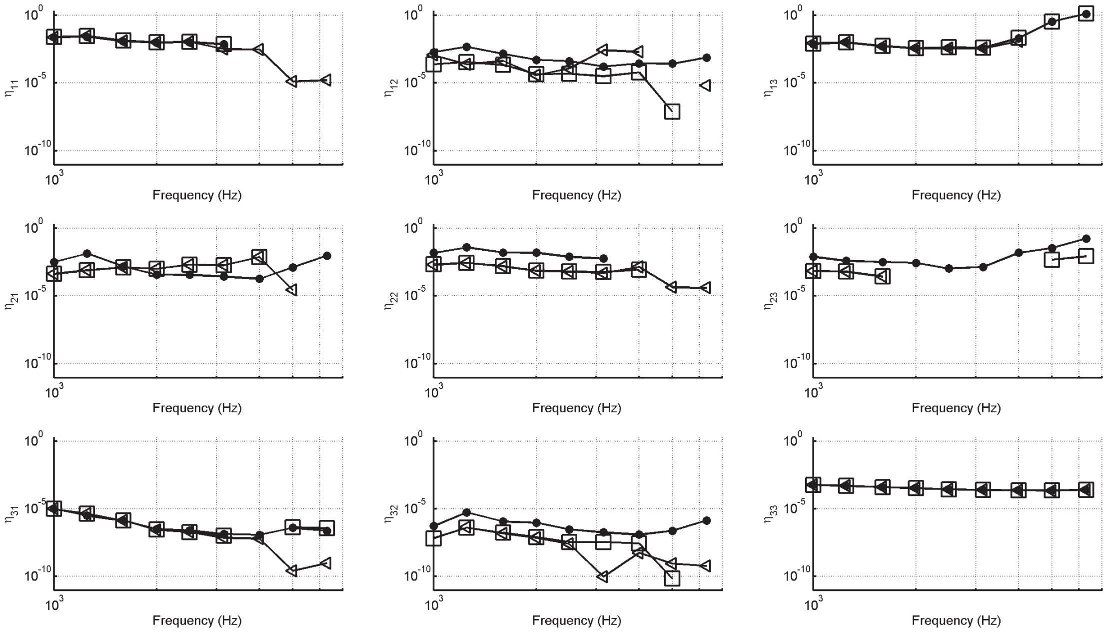

Figure 6 – SEA loss factors determined by updating law (-□-), J_2; values obtained minimising the error (-◁-), J_1 and the values obtained with PIM (-•-) (negative values not displayed)

As Figure 6 shows, the second method, which takes into account twice the experimental results (the former by the definition of the starting model and the latter by imposing the measured power balance through the Lagrange multipliers) improves the results on both the loss factors associated to the subsystems excited and also for the loss factors due to the interaction with the not excited panel (like η_{12} or η_{32}).

On the other hand, as with the direct application of PIM to the system, some of the bands show negative values of the SEA loss factors (on the high frequency end). Again, from the numerical point of view, this fact is related to the numerical conditioning of the measured energy matrix whose inverse is taken into account in the updating law (equation 6). This result, together with the one from the application of PIM shows that the measurement of the subsystems energy is key to define a proper energy matrix that both represents the actual energy in the system and that is well conditioned. To analyse this subject and deep into the performance of both methods, a Monte Carlo study follows.

4.3 Monte Carlo study

Taking the energy measured as the input random variable of the Monte Carlo analysis,that follows a Gaussian distribution with a standard deviation of 5% respect to the values measured experimentally, a sample of 3000 cases is generated. The results obtained by the method based on minimising the error in the system power balance equation leads to the set of results depicted in Figure 7.

From these plots two main conclusions can be stated. First, that an increasing uncertainty is found as the analysis frequency is increased. This result is in line with the increasing condition number of the experimental energy matrix. This trend is clearly observed in the loss factors associated with the first subsystem. The second conclusion is related to the bands for which negative values of the loss factors were found. The Monte Carlo simulation provides a set of positive values for these bands (for example the 5000 Hz and 6000 Hz bands in the case of η_{13}). Then, the use of a Monte Carlo simulation around the experimental results allows estimating the values of the loss factors for these problematic bands. This result also illustrates the high influence of the accuracy of the measurement of the system energy during the PIM tests in order to obtain a coherent energy matrix.

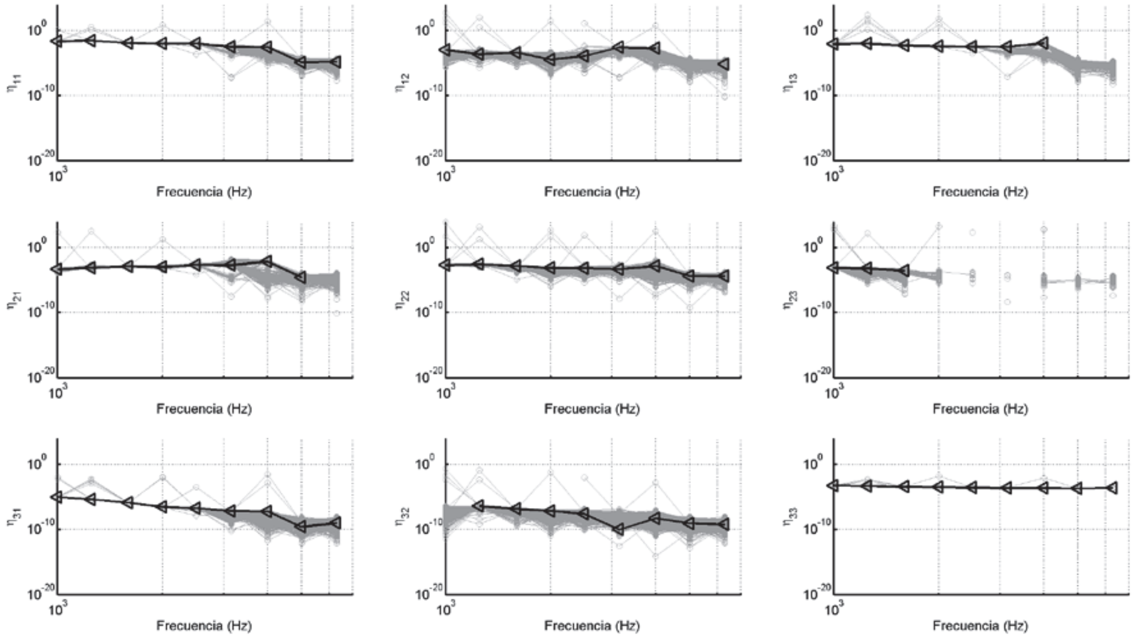

Figure 7 – SEA loss factors determined minimising the error in the power balance equation (J_1) from the nominal energy (-◁-) and the Monte Carlo sample (-○-) (negative values not displayed)

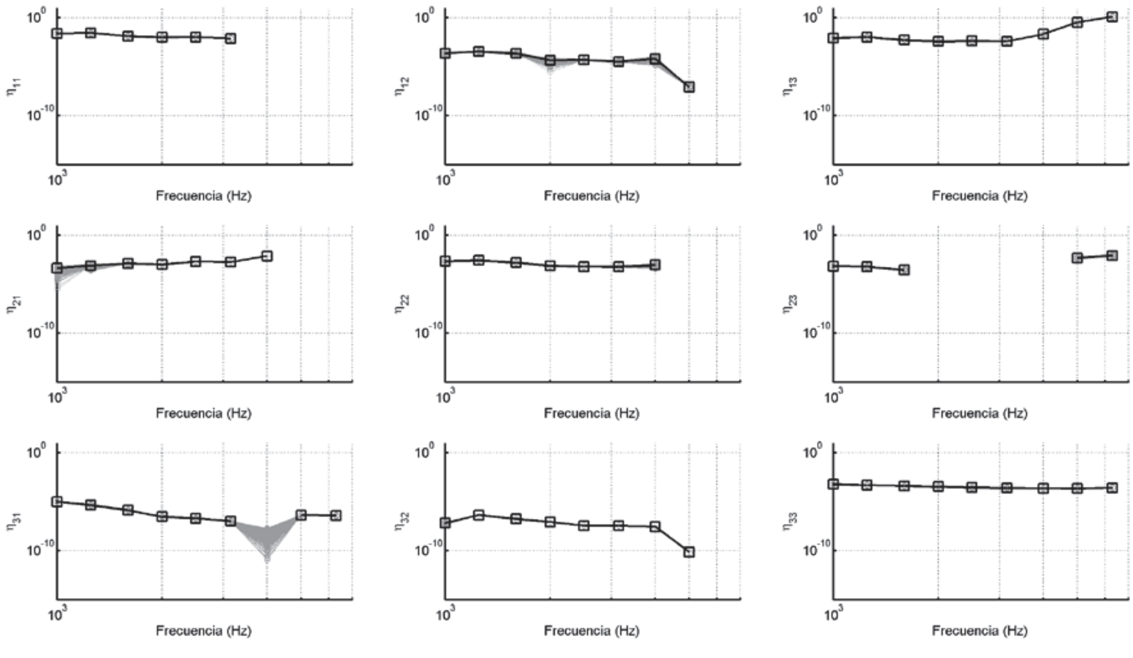

Figure 8 – SEA loss factors determined updating the previous model (J_2) from the nominal energy (-□-) and the Monte Carlo sample (-○-)

The same study by means of a Monte Carlos simulation (identical sample) is presented in Figure 8 for the second method proposed: updating the result of the previous method by the updating law in equation (6). From this equation, the relative uncertainty can be calculated also straightforwardly and as Figure 8 shows it is as small as the initial uncertainty on the measured energy. This leads again to negative values for the same bands found for the nominal results (Figure 6) although some higher uncertainty is found for specific bands and loss factors (for instance η_{31} at 4000 Hz) that shows that for some bands the ill conditioning of the energy matrix still has high influence even for small input uncertainties.

5. CONCLUSIONS

This work presents two methods that can be applied to determine the SEA loss factors of systems with restricted accessibility. If some of the subsystems defined cannot be accessed to apply external loads, PIM, as standard method, cannot be used to calculate the loss factors.

The methods are based on two principles: minimising the error of the power balance equation and modal updating techniques. These methods allow determining the loss factors of the system for systems with no access to some of its elements, although not all of them can be determined with accuracy. Results show that the loss factors related to the subsystems that can be excited (their ILF and the CLF associated to the transfer of energy from this subsystem to the other ones) can be determined adequately. On the other hand the loss factors associated to the subsystems not excited can be only grossly estimated with the first method. The combined use of both methods (taking the results of the first one as the starting model for the second one) improves these results and accurate results can be obtained for the majority of the SEA loss factors.

An uncertainty study shows the dependency of the methods on the condition number of the energy matrix that is obtained from the experiment. It also shows that a Monte Carlos simulation on the measured system energy can be useful to define a set of coherent energy matrices that lead to the determination of a set of positive SEA loss factors. The average of the Monte Carlo simulation can be used then as an estimator of the SEA loss factors in these specific bands.

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REFERENCES

1. Lyon RH, DeJong RG. Theory and Applications of Statistical Energy Analysis. 2nd ed. Butterworth-Heinemann: 1995.
2. Maxit L, Guyader JL. Estimation of SEA coupling loss factors using a dual formulation and FEM modal information, Part I, theory. J Sound Vib 2001;239(5):907-903.
3. Bies D, Hamid S. IN situ determination of loss and coupling loss factor by the power injection method. J Sound Vib 1980;70(2):187-204
4. Lalor N. Practical considerations for the measurement of internal and coupling loss factors on complex structures. ISVR University of Southampton: 1990. 13 p. Report No.: 182
5. Lalor N. Measurement of SEA Loss Factors on a Fully Assembled Structure. ISVR University of Southampton: 1987. 9 p. Report No.: 1506. Hopkins C. Experimental statistical energy analysis of couples plates with conversion at the junction. J Sound Vib 2009;322(1-2):155-166
7. Renji K, Mahalakshmi M. High frequency vibration energy transfer in a system of three plates connected at discrete points using statistical energy analysis. J Sound Vib 2006;296(3):539-553
8. Fahy F. An alternative to the sea coupling loss factor: rationale and method for experimental determination. J Sound Vib 1998;214(2):261-267.
9. Guasch O. A direct transmissibility formulation for experimental statistical energy analysis with no input power measurements. J Sound Vib 2011;330(25):6223-6236.
10. Friswell MI, Mottershead JE. Finite Element Model Updating in Structural Dynamics. Springer Science & Business Media; 1995.
11. ESA/ESTEC. Statement of Work: Random Vibration Environment Derivation by Vibro-Acoustic Simulation, 2006. Contract No. 20779/07/NL/SFe
12. ECE-RDR-TNO-0014, CLF Test Report, Random Vibration Environment Derivation by Vibro-Acoustic Simulation, Contract No. 20779/07/NL/SFe.