



Acoustic Fatigue Analysis of Weld on a Pressure Relief Line

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ABSTRACT

This work presents an acoustic fatigue analysis of a welded connection that has initially failed the EI Guidelines, Avoidance of Vibration Induced Fatigue Failure in Process Pipework, 2008 and demonstrates the level of conservatism imbedded in the method therein.

A detailed finite element model of the connection was developed and a random acoustic field equivalent to the acoustic power level calculated from Carucci & Mueller criteria is applied. This is achieved by generating and applying a pressure field on the walls and summing up the stresses resulting from excitation. The summation took into consideration the random nature of the noise field by randomly selecting the phase angles of each harmonic up to and including the cut-off frequency for each acoustic mode. The stresses are obtained by summing up all elemental stresses using the SRSS rule.

The acoustic field generated is assumed to peak at 500Hz, corresponding to Strouhal number = 0.2 and a 5% attenuation is applied to the noise field, resulting in limiting excitation bandwidth to 2.5 kHz. No sensitivity study was carried out to evaluate the sensitivity of the results to cut-off frequency, but a conservative value of 2% hysteretic structural damping is assumed.

The analysis indicates that the acoustic induced stresses are well below the allowable stress at the weld and that fatigue damage per safety relief valve operation can be mitigated over and above the life time of the connection.

The analysis quantifies the conservatism embedded in the Carucci & Mueller empirical data and demonstrates that *risk assessment tools* should never be applied to effect design changes without further investigation.

1. INTRODUCTION

The purpose of this analysis is to conduct an acoustic fatigue analysis of an equal tee connection on a relief line that failed the acoustic fatigue criterion set up in the EI-Guidelines (1). Acoustic fatigue criterion of the EI-Guidelines is based on Carucci & Mueller data (2).

A detailed finite element model of the T-connection is developed and a random acoustic field, depicting the acoustic power level (see appendix A) is applied.

A detailed stress and fatigue analyses were then carried out. A Computational Fluid Dynamics model was also developed to calculate the acoustic pressure field to verify the noise levels predicted by the Carucci & Mueller method. This analysis however was not conclusive.

2. METHODOLOGY

Thin-walled pipes exhibit shell deformation pattern that makes them vulnerable to high frequency excitation. This, in addition to other factors, makes them more susceptible to fatigue damage.

The response to acoustic excitation can be amplified in two different mechanisms (3). Firstly, when one or more excitation frequency coincides with one of the structural natural frequencies. Secondly, when acoustic and structural wave lengths coincide. The first effect is known as *resonance* while the second is referred to as *coincidence*. When the two effects combine, large structural response can occur.

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In a pressure relief event, high level of turbulence is usually encountered which results in pipework acoustic excitation. Derivation of the intensity of the turbulence and associated acoustic field generated from flow parameters is not attempted. Instead an acoustic field that matches that calculated by Carucci & Mueller WPL is generated and applied internally to the pipe wall.

Over and above plane wave frequency = $C/2d_i$, where C is the speed of sound and d_i is the internal pipe diameter, a three-dimensional wave analysis involving high order modes is required – see Appendix A for details.

In a straight pipe with no imperfection or intersections, a plane wave can only excite the pipe breathing mode. However, due to imperfection and the presence of discontinuity at any connection, a plane wave can excite higher order structural modes. This effect is automatically taken care of in modelling and need not be concerned about. Frequencies in the range 250 -2.50 kHz are found more likely to contribute to structural excitation. This could be due to that resonance and coincidence effects occurring simultaneously is more likely over and above mid-range frequency of 250 Hz (due to the increase in the structure modal density).

The acoustic field generated (aerodynamic noise) is characterised by a broadband random field peaking at one or more frequency. In this analysis, it is assumed that the acoustic field peaks at frequency, corresponding to excitation at the pressure relief valve. This is given by the Strouhal frequency, $f_s = 0.2 V/D_i$ where d is the flow Vena Contract diameter at the valve throat (4). Based on the process condition, the Strouhal frequency is calculated to be around 500 Hz. The acoustic field is assumed to attenuate at 5 times this frequency.

The acoustic excitation field therefore is calculated up to 2.5 kHz. Structural natural frequencies were calculated up to 3 kHz. To reflect the random nature of acoustic field, acoustic excitation harmonics were randomly selected. The resulting stresses were also SRSS summed.

There are two energy dissipating mechanisms. The first is in the acoustic field, assumed 5% as highlighted earlier, while the other is in the structure, conservatively assumed equal 2%.

3. ENERGY BALANCE

It is constructive to understand the exchange of energies between the acoustic field and pipe wall excitation. Energy balance requires that the combined acoustic energy, vibration energy in the pipe and energy radiated from pipe wall remains constant, i.e.

$$E_t = E_{\text{pipe}} + E_{\text{ro}} \tag{3.1}$$

The acoustic energy inside the pipe is equal to the acoustic energy generated by turbulence E_g , plus the energy trapped by reflection at the pipe wall, E_r (5),

$$E_t = E_g + E_{r_i}, \tag{3.2}$$

The reflected energy from pipe wall can further be divided into *blocked* and *radiated* energies. These details however are not necessary as it is the pipe vibration energy, E_{pipe} , which is of concern.

The energy per mode (per unit length), assuming equi-partitioning of energy, can be found from the following expression (5),

$$E_{\text{pipe}} = m \langle v \rangle^2 / \pi \tag{3.3}$$

while the stress can be evaluated from (6),

$$\sigma = E \varepsilon / (1-2\nu) \tag{3.4}$$

$$\langle v \rangle = c \varepsilon \tag{3.5}$$

where m is the pipe mass per unit length, v is pipewall vibration velocity, ε is the micro strain (mm/mm E-06), ν is Poisson's ratio and c is a constant equal to $2.95 \text{ E}+06 \text{ mm/s}$. $\langle \rangle$, denotes the rms value of variable.

Equation (3.4) therefore allows calculating the stress σ from a knowledge of the vibration field at the surface v , while equation (3.3) allows calculating the energy per unit length per mode. Equation (3.4) was used to check computer modelling results.

4. COMPUTER MODELLING

4.1 Finite Element Model

A finite Element model is generated on ANSYS Release 15.0 using 20 nodes isoperimetric element and tetrahedral, pyramid and prism shaped elements, each with three degrees of freedom per node. Two elements per pipe wall thickness were applied. The model depicts an equal forged Tee connection of an outer diameter = 906.9 mm and thickness 9.52 mm. Welds are assumed be dressed and heat treated. As a result, no stress concentration or residual stresses were assumed. The effect of these two assumptions has not been investigated. To limit the model size, the model was terminated at 5D length from center of the connection. Thrust pressure was applied at the edge to depict tensile field due to internal pressure. Appropriate boundary conditions are applied and care is taken to minimize their effect. External loadings (i.e. weight and thermal loadings) are not considered in PD5500 approach (7) which requires only dynamic stress range to be applied.

4.2 Calculation of Maximum Element Size

Maximum element size is based on allowing $10 \lambda_{\max}$ per element where λ_{\max} is the maximum wave length in the analysis. The speed of sound in the shell at 2.5 kHz is calculated at 526 m/s (8).

4.3 Acoustic Pressure Loading

The acoustic pressure loading is derived in Appendix A. A typical acoustic pressure loading plot at excitation frequency 500 Hz is shown in Figure 9.3 and a tabulated pressure data at same frequency is given in Appendix B.

5. RESULTS

5.1 Modal Analysis

A free vibration analysis of the connection was carried out using ANSYS Release 15.0 and modes up to 3000 Hz were calculated. 743 modes were found in this frequency range.

Frequencies and mode shapes obtained from Modal Analysis were used applying modal summation method.

5.2 Dynamic Stress Analysis

The three principal stress amplitudes (0-Pk) for frequency range from 0-2.5 kHz were calculated. As 'thrust' lip forces were applied to the model to simulate traction of cutout pipework, stress resulting from these forces were subtracted from the axial stress according to PD-5500 (7).

6. DISCUSSION AND CONCLUSIONS

The analysis carried out indicates that the rmss values of the three principal stress are: 9.69, 7.09 & 11.50 MPa (0-PK) respectively and that the maximum fatigue damage at weld per one-minute operation, assuming a class D butt weld and applying Miner's rule (7), are: 1.2E-4, 4.9E-5 and 2.2E-4 respectively. This is equivalent to 4545 minutes (or 75.8 hours) venting operations.

The analysis therefore concludes that welds at the tee connection are capable of withstanding fatigue damage.

This reflect the conservatism imbedded in the Carucci & Mueller criteria which, for this particular case, predicts an imminent failure at one relief operation. In this particular case, it is concluded that Carucci & Mueller criteria is conservative by a factor of 2.00.

It is worth keeping in mind, as mentioned earlier, that the acoustic field generated is assumed to peak at 500Hz, corresponding to Strouhal number = 0.2 and decay at *five* times this frequency resulting in 2.5 kHz excitation bandwidth. This requires a 5% attenuation to be applied. No sensitivity study is carried out to assess the effect of attenuation on fatigue life.

7. GRAPHS & PLOTS

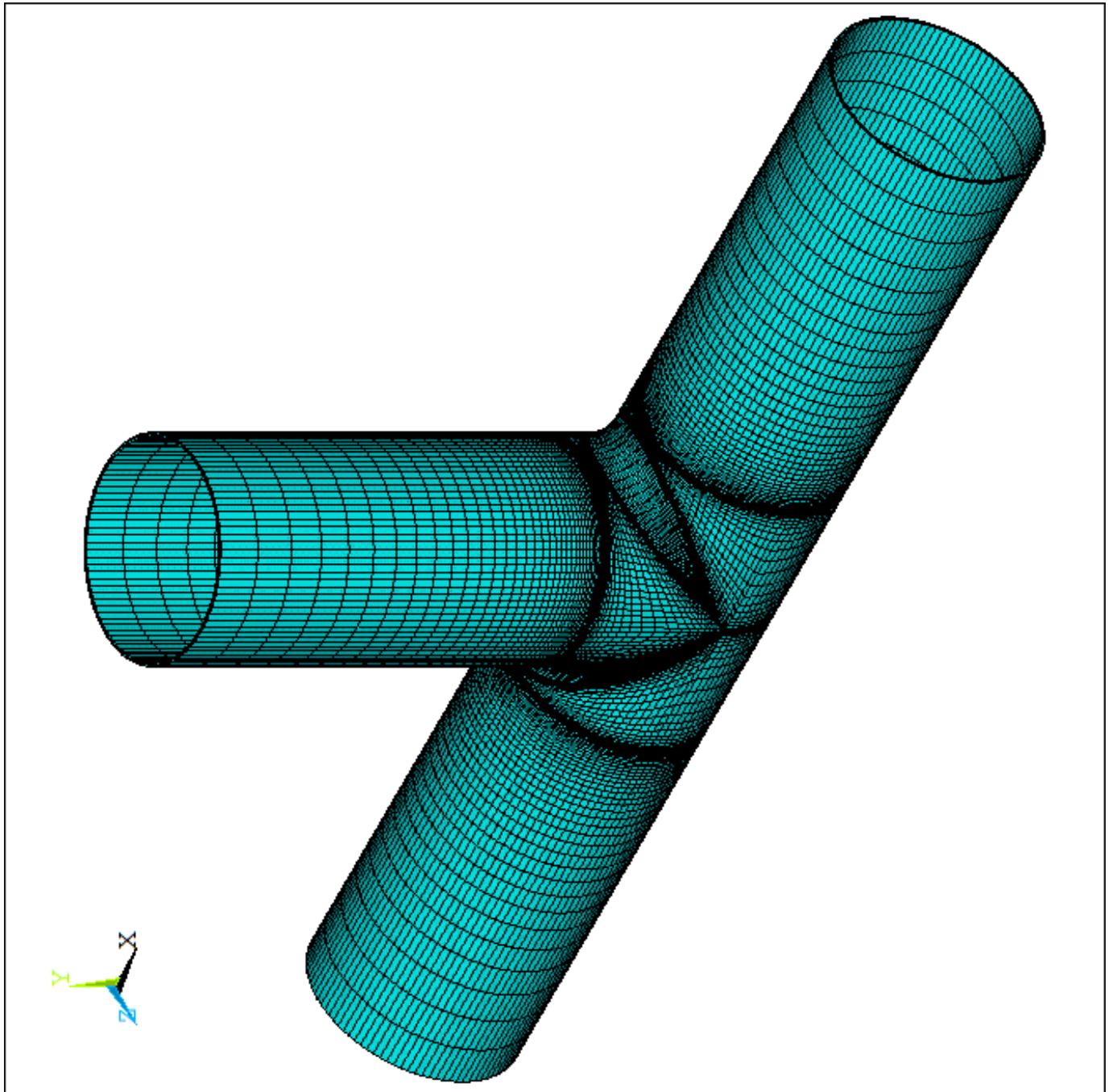


Figure 7.1 - Element Mesh of Tee Connection

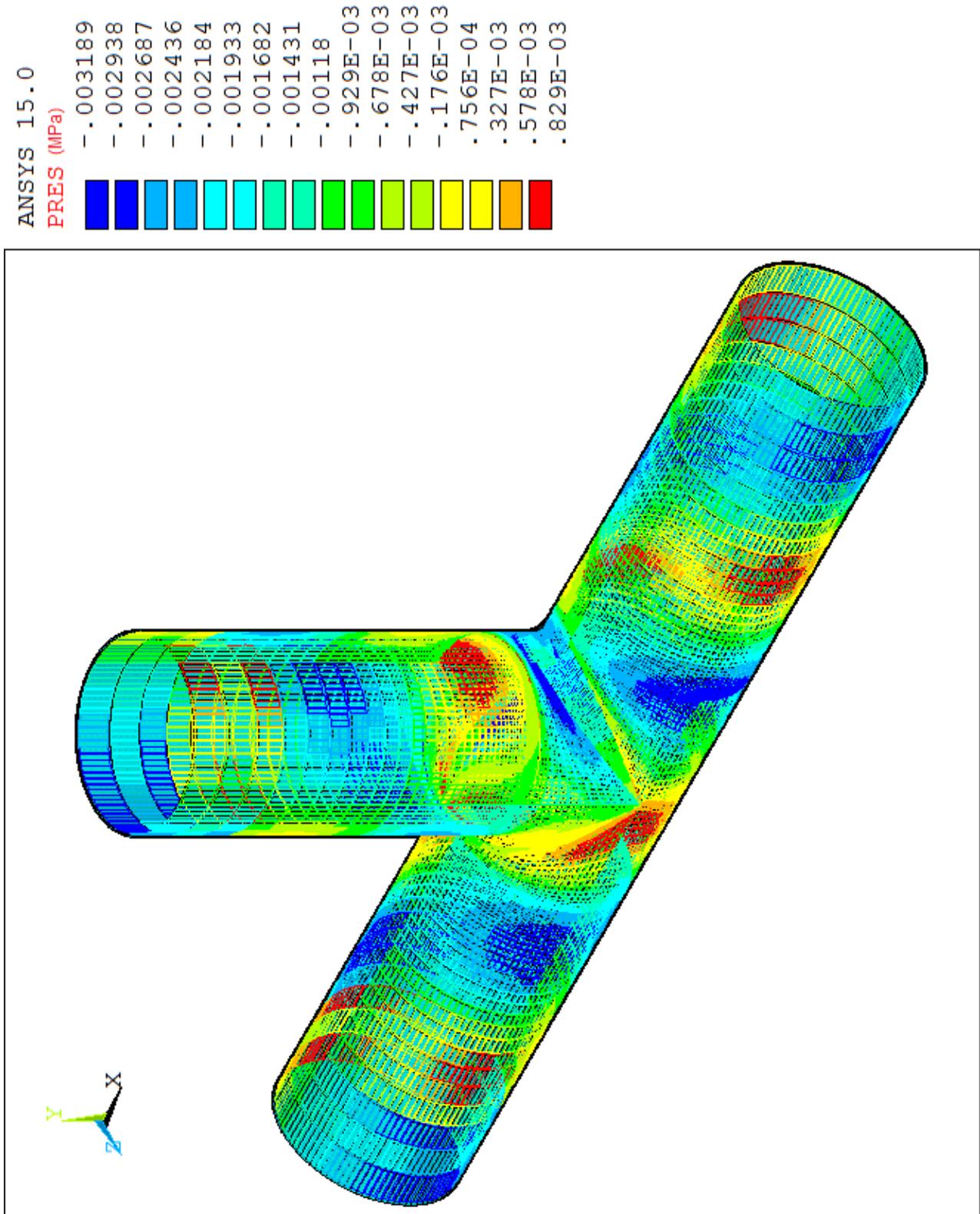


Figure 7.2 Typical Acoustic Pressure Loading Plot for Excitation Frequency at 500 Hz

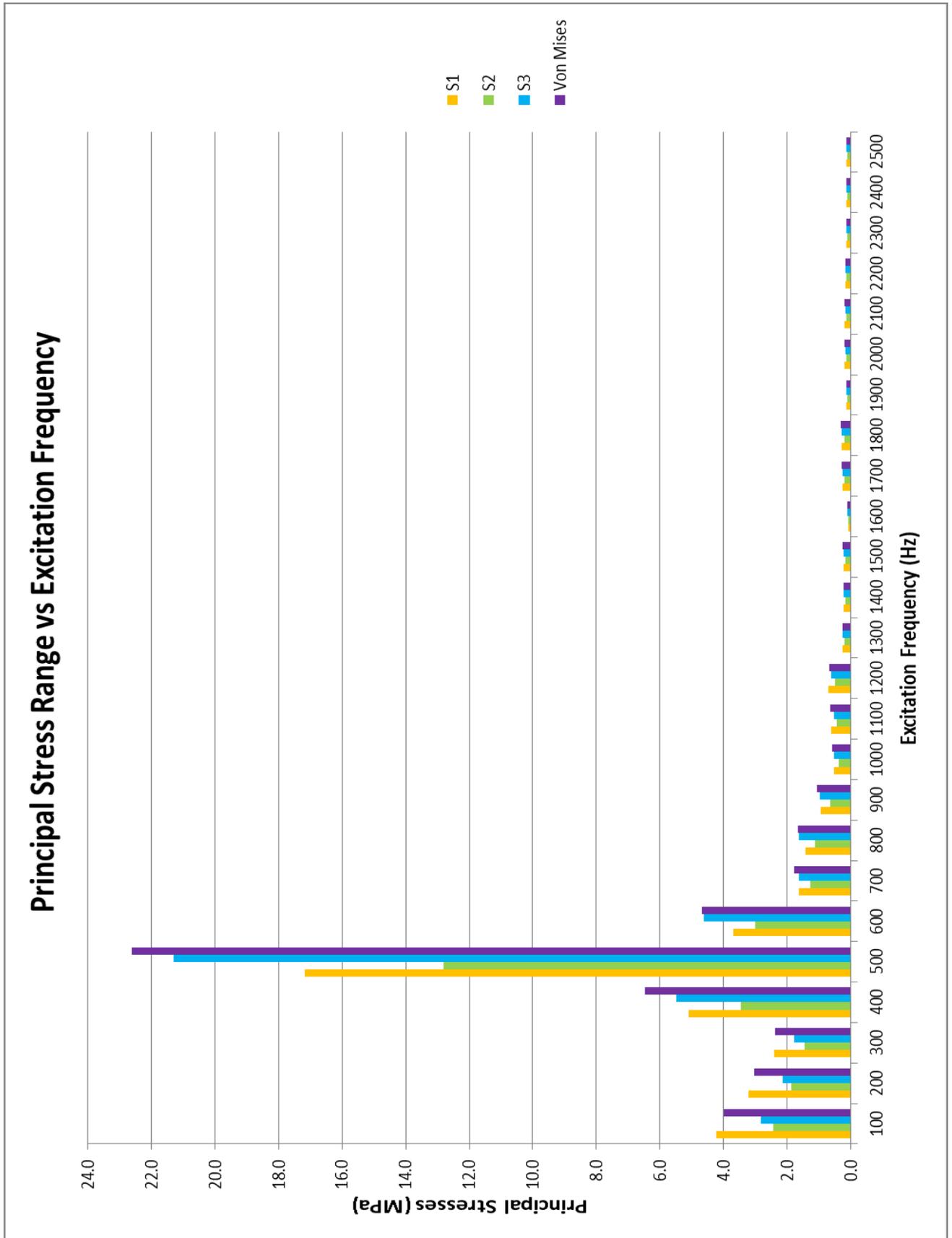


Figure 7.3 - Principal Stress Range Spectrum

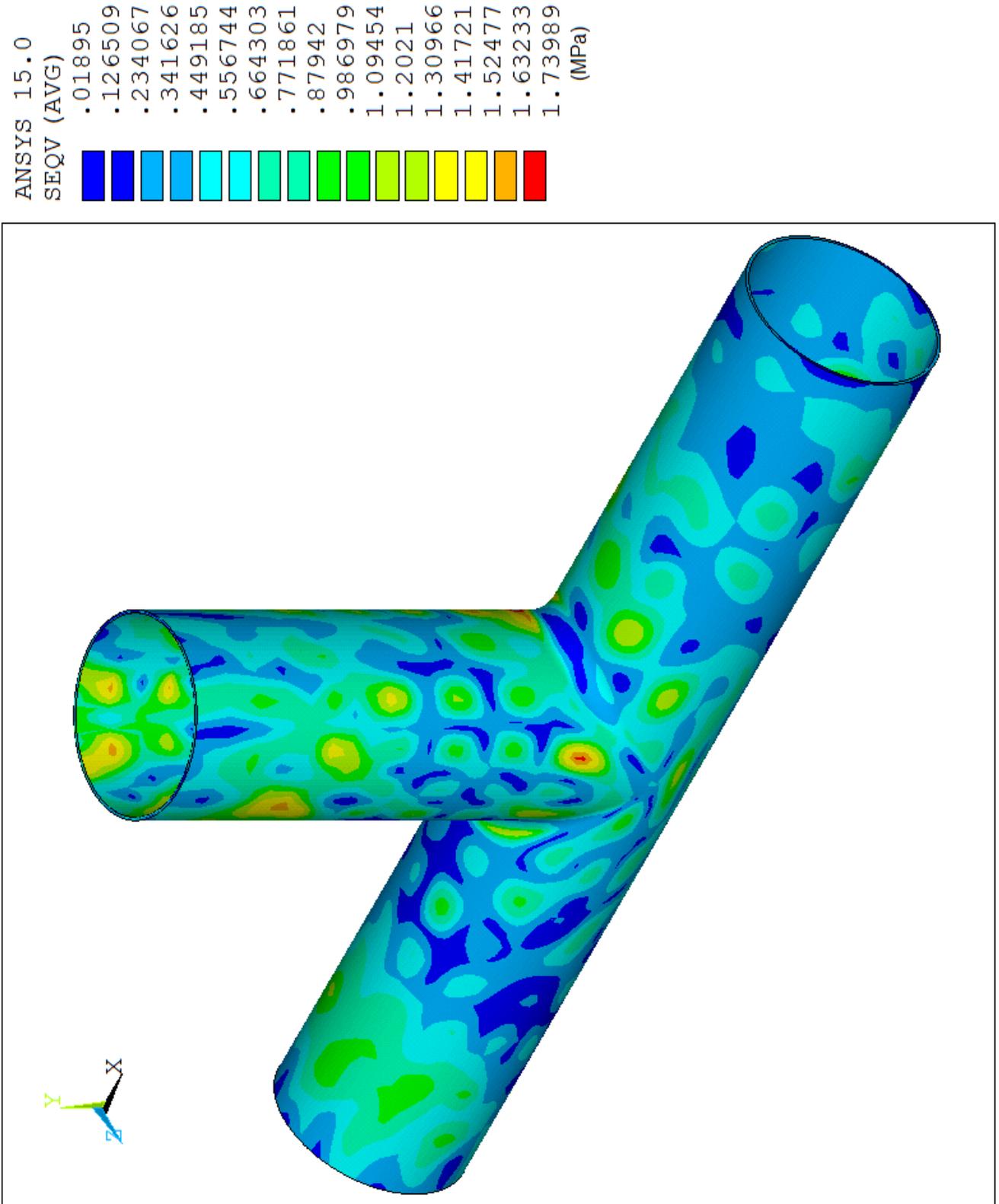


Figure 7.4- Typical Von Stress Contour Plot for Excitation Frequency at 500 Hz

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APPENDIX A

GENERATION OF ACOUSTIC LOADING

1.0 THEORY

Neglecting convection, the acoustic pressure in acoustically hard walled duct can be described by (3),

$$p(x,r,\theta,t) = f(x).f(r).f(\theta). \exp(i\omega_j t) \tag{1}$$

Where,

$$f(x) = A_1 \exp(i k_x x) + B_1 \exp(-i k_x x) \tag{2}$$

$$f(r) = A_2 J_m(k_r r) \tag{3}$$

$$f(\theta) = A_3 \exp(i m \theta) + B_3 \exp(-i m \theta) \tag{4}$$

$$k^2 = (\omega/a)^2 = k_x^2 + k_r^2 \tag{5}$$

$$J'_m(k_r R) = 0 \tag{6}$$

ω_j is the angular frequency and j is a random none integer variable

J_m is the Bessel function of integer order $m = 0,1,2,3 \dots$

J'_m is the first derivative of the Bessel function of order m

R is the pipe radius

It should be noted that although the variables in equation (1) are separable, the three functions f(x), f(r) and f(θ) are interrelated.

Equation (6) determines the eigenvalues of elemental waves. Each wave propagates at a distinct frequency (eigenvalue). The higher the frequency, the higher these modes can propagate through the pipe. These modes are bell shaped along the radius and harmonic across it. The waves are seen to spiral along the pipe axis.

To calculate the pressure at any point, the summation of the elemental solutions given by equation (1) is to be carried out over all the values of m for all possible solutions (determined from the solution of the eigenvalue equation (6)).

2.0 SIMPLIFICATIONS

The following simplifying assumptions are made:

2.1 It is assumed that the presence of flow does not affect the acoustic pressure loading on the pipe. This assumption will be checked when obtaining published papers, but intuitively flow is expected to have secondary effect on pressure loading.

2.2 It will be assumed that the two families of circumferential waves $A_3 \exp(i m \theta)$ and $B_3 \exp(-i m \theta)$ in equation (4) have equal chance of propagating. In a random pressure field, such as that created by turbulence, this is an acceptable assumption simplifying equation (4) to,

$$f(\theta) = A_m \cos (m \theta) \quad m = 0,1,2,3 \tag{7}$$

2.3 It will be assumed that the reflection due to the side branch can be represented by the reflection of a zero order wave. This assumption simplifies equation (2) to,

$$f(x) = A \{ \exp(i k_x x) + \alpha \exp(-i k_x x) \} \tag{8}$$

where α is given by, [2]

$$\alpha = B_1 / A_1 = - \rho a / 2S / (\rho a / 2S + Z_b) \tag{9}$$

S being the branch cross section area and Z_b the acoustic impedance of the branch.

Substituting equations (7) & (9) into equations (1) - (4), the following equation is obtained:

$$p(x,r,\theta,t) = F(f_j) \{ \exp(i k_x x) + \alpha \exp(-i k_x x) \}. \cos (m \theta). J_m(k_r r). \exp(i \omega_j t) \tag{10}$$

with $\omega_j = 2 \pi f_j$

3.0 DETERMINATION OF THE FUNCTION F(F_j)

The noise field inside the pipe is assumed to be a pink noise, peaking at the Strouhal frequency, f₀ (3)

$$f_0 = 0.2 M V / D \quad (11)$$

where,

M is the molecular mass kg/kmol
 V is the velocity of gas in the Vena Contracta
 D is the valve diameter

Over and above f₀, the pressure field is assumed to decay by viscous action and thermal diffusion. The rate of decay is obtained empirically. The following function is found to satisfy these criteria,

$$\Gamma(f / f_0) = 1 / [\{1 - (f / f_0)^2\} + 2 i \zeta (f / f_0)] \quad (12)$$

The function F(f) therefore can be written as,

$$F(f_j) = F_{j0}(f) \cdot \Gamma(f / f_0) \quad (13)$$

From 0.0 to 0.1/ $\zeta = 2.5$

$$\zeta = 5\% \quad (14)$$

4.0 DETERMINATION OF THE SOUND PRESSURE LEVEL

The Sound Power Level, PWL is defined as

$$PWL = 10 \text{ Log } (I/I_0) \quad (15)$$

where

$$I = p^2/Z \quad \text{Watt/m}^2 \quad (16)$$

$$I_0 = 10^{-12} \quad \text{Watt/m}^2 \quad (17)$$

$$Z = \text{the characteristic impedance of air at standard conditions} \quad (18)$$

Thus the measured sound power is

$$I = I_0 X 10^{PWL/10} \quad \text{watt/m}^2 \quad (19)$$

From equation (22), the acoustic pressure therefore is given by,

$$P_{PWL} = (Z \cdot I)^{1/2} = (Z \cdot I_0 X 10^{PWL/10})^{1/2} \quad \text{Pa} \quad (20)$$

The sound pressure is determined from summing over all elementary solutions given by equation (3). It is understood that the summation is over m values from 0 to M, where M is dictated by propagation condition (equation (5), i.e. k_x > 0.0).

Therefore, at any given frequency, f_j one can write,

$$P_{PWL} = F_0(f_j) \Gamma(f / f_0) \sum \{ (\exp(i k_x x) + \alpha \exp(-i k_x x)) \cdot \cos(m \theta) \cdot J_m(k_r r) \cdot \exp(i \omega_j t) \} \quad (21)$$

At any given frequency f_i, the summation includes all waves from m=0 to M.

5.0 ORGANISATION OF CALCULATIONS

- 5.1 Assume f_j , x and θ .
- 5.2 Calculate $\omega_j = 2 \pi f_j$
- 5.3 Calculate the wave number $k_j = \omega/a$.
- 5.4 Calculate the eigenvalues, $k_j R$ from equation (6)
- 5.5 Calculate all corresponding real values of k_{xj} from equation (5)
- 5.6 Calculate $F_0(f_j, \theta)$ from equation (22)

The two terms $\cos(m\theta) J_m(k_r R)$ in equation 22 should be replaced by,

$$I(m, k_r) = \int_0^{2\pi} \int_0^R \cos(m\theta) \cdot J_m(k_r r) 2\pi r dr d\theta / \pi R^2$$

Equation (21) becomes,

$$F_0(f_j) = P_{PWL} / [\Gamma(f_j / f_0) \cdot \sum \{(\exp(i k_x x) + \alpha \exp(-i k_x x))\} \cdot I] \tag{22}$$

- 5.7 Repeat calculations (6.1- 6.6) for $f_j + \Delta f$
- 5.8 Plot $F_0(f_j)$ versus frequency f_j .

The function $F_0(f_j)$ is calculated in the frequency range, 0-3.0 kHz, only non-integer values of j were assumed to avoid periodicity and guarantee randomness. Matlab package was used.

It is to be noted that, The Function $F_0(f_j, \theta)$ is a complex function. Only the modules were taken. It is worth also noting that $\sum \sum \sum f(x).f(y).f(z)$ is generally not equal to $\sum f(x) \cdot \sum f(y) \cdot \sum f(z)$ and treble summation was applied. The three functions seem to be independent, but in reality are coupled through their arguments manifesting propagation zones.