Small Bore Dynamics and some Fatigue Related issues

Noise and Vibration Consultant[1]

[1] Glasgow, United Kingdom.

ABSTRACT

Small Bore Connection fatigue is a problem of considerable interest in the Oil and Gas and Nuclear Industries. Almost 50% of hazards from hydrocarbon release in petrochemical plants occur as results of fatigue failure of small bore connection at welds or in the vicinity.

In the industry, the term Small Bore Connection (SBC) is used to refer to a branch-pipe that is less or equal to 2” in diameter or to any small bore attachment. From a dynamic point of view however, it is more appropriate to differentiate between a Small Bore Attachment (SBA) and a Small Bore Branch (SBB). This investigation is concerned with Small Bore Attachment.

In 2005, Mousseau (of **C**ommissariat à l'**E**nergie **A**tomique, Paris - CEA) developed a dynamic model for Small Bore Branch (SBB). Failure of Small Bore Branches in the nuclear industry constitutes a major hazard and can lead to small Loss-of-Coolant Accident (SLOCA).

In the Oil and Gas Industry, we are not aware of any development of the subject. EI Guidelines, AVIFF of January 2008, for example do not refer to any analysis when dealing with small bore likelihood of failure (LOF) analysis.

The analysis in here shows how it can be extended to include the effect of **flange rotary inertia**, **connection-header** interaction and more importantly, **header shell deformation**. A list of findings, which help at an early stage of design against vibration induced fatigue failure, is presented.

The model developed allows insight to the parameters affecting SBA vibration and shows the resemblance between SBA dynamics and that of a high rise building on elastic foundation.

Keywords: Small Bore Connection, Fatigue, Dynamics.

1. DYNAMICS OF SMALL BORE ATTACHMENT

Consider $\mathbf{x_H}$ to be the movement of the header and $\mathbf{x_{B0}}$ the movement of the attachment *relative* to the header. As a result of the offset of branch inertia, bending moment at the connection results in the header rotating an angle $\boldsymbol{\theta}$. Relative to the header, the small bore attachment movement therefore has two components. One due to stem flexibility, while the second due to connection flexibility. This analysis applies to in-plane and out-plane vibration. For in-plane vibration however, the effect of branch vibration on axial pipe movement can be neglected.

The equation of motion of mass of the connection is given by:

$$m\, a_t + k_B\, x_B = 0 \qquad (1)$$

where

a_t is the *total* acceleration of the connection mass, given by

$$\begin{aligned} a_t &= a_B + a_H \\ &= a_{B0} + L\,\theta + a_H \end{aligned} \qquad (2)$$

where,

m	is the mass of SBA
k_B	is the stiffness of SBA, assuming infinitely rigid connection
L	is the length between the centers of mass of SBA and header.
a_{B0}	is the SBA acceleration relative to the header, for infinitely rigid header

Neglecting inertia, the equilibrium of the SBA alone gives,

[1] sfahmy@talk21.com

$$k_{B0}\, x_{B0}\, L \quad = \quad k_\theta \;\; \theta \qquad (3)$$

where

k_θ is the connection rotational stiffness (Nm/rad)

Substituting equations (2) & (3) into (1) thus,

$$a_B \quad + \quad \omega_n^{\,2} \quad x_B \quad = \quad - a_H \qquad (4)$$

where

$$\omega_n^2 \quad = \quad (k_B / m) \qquad (5)$$

$$k_B \quad = \quad k_{B0} / (1 + k_{B0}\, L^2 / k_\theta) \qquad (6)$$

and a_H *is the header acceleration*

The dynamic force at the connection to the header due to branch line vibration is given by, $\mathbf{k_B}\ \mathbf{x_B}$ which from equation (1), is also equal to $\mathbf{m\ a_t}$

For harmonic header of radian frequency ω, equation (4) becomes:

$$(- \omega^2 + 2\, i\, \zeta\, \omega\, \omega_n + \omega^2_n)\, \boldsymbol{x_B}\, (i\omega) = - \boldsymbol{a_H}\, (i\omega) \qquad (7)$$

where a damping term, ($2\, i\, \zeta\, \omega\, \omega_n$) is introduced for convenience.

To enable a solution, it is necessary to define the header excitation. For the purpose of this investigation, it is assumed that the header vibration is stationary random.

It is assumed that the excitation spectrum is broad-band in the vicinity of the resonance frequency of the SBA and that the branch fitting is lightly damped such that it peaks steeply at resonance. In this case, the root mean square response of the branch fitting $<\mathbf{x_B}>$, can be obtained in terms of the power spectral density of the header acceleration, G_H (f) as follows, (2)

$$k^2_B \quad < x^2_B > \quad = \quad \pi^2\, m^2 \quad f_n\, G_H\, (f) \,/\, \zeta \qquad (8)$$

where $f_n \quad = \quad \omega_n / 2\pi$

$$G_H(f) \quad = \quad <a_H^2>/\Delta f_H \qquad (9)$$

$$\Delta f_H \quad = \quad \text{Excitation bandwidth} \qquad (10)$$

The mean square force at the connection, $<\mathbf{F_d^2}>$ is given by $<\mathbf{F_d^2}> = \mathbf{k^2_B} <\mathbf{x^2_B}>$. Peak values of response can be estimated if required (2). In the present calculation, a pseudo peak value of 1.414 times the rms value of $\mathbf{F} = <\mathbf{F}^2>^{1/2}$ is assumed.

The small bore attachment vibrates then at its "natural frequency" with a random but slowly varying amplitude (2).

The forgoing analysis quantifies the effect of the different parameters on the dynamic force and gives an insight to the problem. For instance, the analysis shows that the dynamic force per unit header acceleration is proportional to,

- the equipment mass, **m**
- the square root of the SBA natural frequency, $\mathbf{f_n}$
  and inversely proportional to
- the square root of the damping ratio, $\boldsymbol{\zeta}$

Counter intuition, the stiffening of the fitting for example by adding bracing leads to an increase to the force transmitted to the header rather than a decrease as commonly believed. In practice however, this the introduction of bracing would 'split' the path of force leading to a reduction in the force transmitted from the attachment stem to the header.

The forgoing analysis addresses the dynamic force acting on the header rather than stress. The stress per unit force (MPa/N) will depend on geometrical parameters of the attachment-header combination such as header wall thickness, branch fitting to header diameter, weld contour etc. and can be obtained from Codes or from FEA (3). In comparing two SBA-Header connections, the one with higher natural frequency or higher mass should not therefore result in higher dynamic stress. This is important to note as some vibration risk analysis methodologies (VRA) depends solely on determining the risk from a measure of the attachment natural frequency.

The stress at the connection S is given by,

$$<S> = S_0 < \mathbf{F_d} > \quad (11)$$

where S_0 is the stress per unit force (MPa/N) applied at SBA

The stress S_0 can be obtained by applying a unit force at center of flange and calculating the static stress at the connection crotch using FEA (3). Alternatively, it can simply be obtained from Codes (e.g. PDA 5500). The dynamic stress is then obtained by simply multiplying the stress value obtained by the dynamic amplification factor Γ,

$$\Gamma = a_H [(\pi/2)\ \eta\ m^2 f_n / (\Delta f_H\ \zeta)]^{½} \quad (12)$$

where $G_H(f)$ in equation (10) is equated to $1/\ \Delta f_H$ for unit header acceleration.

The factor η is introduced to allow for the finite bandwidth of excitation and is a function of system parameters and excitation bandwidth (2).

The choice of forcing function does not limit the application of the findings to any other type of excitation. For example, for an harmonic header excitation, assuming resonance condition, the SBA acceleration is given by,

$$a_B = a_H / 2\ \xi \quad (13)$$

It can be seen that the derived equation is very similar to that of a high rise building on elastic foundation, a problem of major interest in civil engineering.

The analysis can be extended to include the effect of:
- Flange rotary inertia for SBA.
- Attachment-header dynamic interaction.
- Shell deformation.

When flange rotary inertia is included, a two degrees of freedom analysis however is required and two modes are calculated (4). This should be applied for SBA with large mass (e.g. diaphragm valve), and is important to consider for accurate results (4).

SBA – Header interaction helps in interpreting measurements and explaining the sudden drop in header vibration level as it approaches the SBA connection.

The inclusion of Shell deformation on the other hand is a problem of considerable interest due the increase in the use of high tensile thin piping (5).

2. DISCUSSION AND CONCLUSIONS

Formulation of Small Bore Attachment dynamics is given from first principles. It facilitates calculating the stress and fatigue life from vibration sources at design stage. It is also useful in vibration risk screening and in providing comparison with measurements. The resemblance of high rise building on elastic foundation is highlighted. Three issues, namely: flange rotary inertia, connection-header interaction, and header shell wall deformation, are also discussed.

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