Estimation of Fatigue Damage from Vibration Measurements

Noise and Vibration Consultant[1]

[1]Glasgow, United Kingdom

ABSTRACT

A method for calculating the fatigue damage in pipework and steel structure from vibration measurements is proposed. The method is based on decomposing the structure into its normal modes, evaluating the modal damage on a mode-by-mode basis and summing up over all modes to determine the total damage. The method is particularly useful when the structural modes of the system are widely spaced as a closed form solution of the modal damage can be obtained. To demonstrate the simplicity of the method, the analysis is applied to vibration measurements taken on a plant.

The method is easy to apply, but requires more validation. This can be achieved by comparing the results with a real case scenario. The method can also be validated by comparing the results of a structure having a limited number of modes, e.g. two degrees of freedom, where a closed form solution of response is available. This is being tested at the moment.

Keywords: Fatigue, Vibration, Vibration Measurements.

1. INTRODUCTION

This paper provides a method for analyzing vibration induced fatigue failure from vibration measurements. Dynamic stress measurements and fatigue life predication are traditionally carried out using strain gauges. This approach however can sometimes encounter problems prohibiting the use of strain gauges.

The method requires modal analysis to be carried out and vibration measurements taken at few locations. The method is applied to excitation of beam type modes. However, if the structure or part of experience wall type deformation, the method can be adapted to analyze shell type deformation-provided sufficient number of circumferential measurements are taken and the structure is adequately modelled.

Pipework energy carrying excitation is usually characterized by sparsely spaced energy carrying modes. When this is the case, a closed form solution of fatigue damage of a single degree of freedom system can be derived and this simplifies the calculations - see Appendix A for more details. The method is also applied to a broad band random excitation case. The purpose of the test case given in here is demonstrate how the method can be applied.

2. THEORY

It is assumed that the pipework (or steel structure) is computer modelled using beam, shell or a combination of both and that a number of time histories at certain locations are taken for sufficient length of time (condition monitored). The steps to calculate fatigue damage are then as follows:

Carry out modal analysis and calculate the structure modal properties, namely

- Natural frequencies, $\{f_{n\,m}\}$ Hz (1-a)
- Modal masses, $\{m_m\}$ kg (1-b)
- Mode shapes, $[\varphi_m]$ $1/kg^{1/2}$ (1-c)

If the mode shapes are normalized with respect to the mass matrix, the modal masses are given by

- $$\{m_m\} = \Sigma(\{\varphi_x^2\} + \{\varphi_y^2\} + \{\varphi_z^2\})^{-1/2} \qquad (1\text{-d})$$

Modal parameters can also be obtained from measurements [5-8].

[1] sfahmy@talk21.com

If necessary, correct measurements data to account for digitization errors or for errors occurring from numerical integration - see Appendix B.

- For *shock type excitation*, calculate the modal velocities by applying,

$$\{V_m\} = [\Phi]\ \{V_L\} \qquad \text{mm/s} \qquad (2)$$

Where $\{V_L\}$ is a vector containing the peak values at measurement locations, L.

- Calculate the static stress per unit force $\{S_L\}$ at locations where stresses and fatigue damage need be assessed. $\{S_L\}$ is a geometrical property vector containing the static stress per unit force at desired locations and can be easily obtained from computer model or from design-by-rule method (e.g. ASME VIII).

- For shock excitation, calculate **the modal** stress per unit force from,

$$\{S_m\} = [\Phi]\ \{S_L\} \qquad \text{MPa/N} \qquad (3)$$

- Fatigue Damage and fatigue life are then calculated. Two cases are considered. The first is for a transient type loading which could be a one-off event or a multiple of. The second case is for a broad band random excitation. Both excitations occur in practice.

- Damage from shock type excitation is addressed in Section 3, while continuous random excitation is given in Section 4.

The author was involved in at least two incidents whereby shock type excitation led to catastrophic failure of a small bore connection. In the first, the transient resulted from a malfunction that led to inadvertent operation of safety valve. In the second incident, the origin of the transient was unidentified. In the two cases, investigation carried out after concluded that failure occurred as a result of fatigue.

3. FATIGUE DAMAGE FROM SHOCK EXCITATION

Fatigue damage resulting from shock excitation is derived from first principles in Appendix A. The analysis assumes shock excitation of small duration. This enables a closed form solution be obtained.

A vibration mode can be viewed as a simple oscillator having the following properties,

- Frequency, f_n
- Modal Mass m_m
- Modal damping ζ_m

These quantities are obtained from modal analysis carried out as discussed above. The modal fatigue damage D_m resulting from of a single shock excitation is given by - see Appendix A,

$$D_m = (f_{nm}T/bc).\ (S_m\,\omega_{nm}\,m_m)^b\,V_m^{\ b} \qquad (4)$$

where,

D_m	is the fatigue damage per mode	non-dimensional
f_{nm}	is the pipework natural frequency	Hz
ω_{nm}	$= 2\,\pi\,f_{nm}$	rad/s
m_m	is the modal mass	kg
T	is the response time, discussed below	s
S_m	is the modal hot spot stress per unit force	$N/mm^2/N$
V_m	is the modal velocity	mm/s
b & c	are constants associated with fatigue material properties	

Equation (4) therefore provides the ***modal*** fatigue damage occurring as a result of each transient at location of interest. The total damage is obtained by summing up over all the modes,

$$\{D_L\} = [\Phi]\ \{D_m\} \qquad (5)$$

Where $\{D_m\}$ is single column array containing the modal damages $\{D_m\}$.

For a number of transients N_T , the total fatigue damage at any location, D_{total} is calculated from,

$$\{D_{total}\} = N_T\ \{D_L\} \qquad (6)$$

Failure occurs when the total accumulative damage, D_{total} at any location is equal to **one.**

Substituting $D_{total} = 1$, the following relation is obtained.

$$N_T = bc / [V_m^b (S_m \omega_n m)^b f_n T] \quad (7)$$

It can be easily shown that the product $f_n T$ is equal to $1/2\pi\zeta$, while the product $(S_m \omega_n m)$ is equal to $(K S_m / \omega_n)$. Substituting, the following equation is obtained:

$$N_T = 2 \pi bc \zeta / [(V_m K S_m / 2 \pi f_n)^b] \quad (8)$$

Damping therefore has a large effect on fatigue life. As can be seen, if the amount of damping is doubled, fatigue life is quadrupled

Numerical values of b & c are as follows [9],

b	$= 3$	
c	$= 1.726 \times 10^{12}$ (MPa)^3	for Class weld F - mean
	$= 0.630 \times 10^{12}$ (MPa)^3	for Class weld F - (mean-2σ)
	$= 0.380 \times 10^{12}$ (MPa)^3	for Class weld F - (mean-3σ)

4. FATIGUE DAMAGE RESULTING FROM RANDOM EXCITATION

For broad band random excitation, assuming that the structure is lightly damped such that it peaks steeply at resonance, the modal mean square response $< \mathbf{x_B} >$, can be calculated-in terms of the power spectral density of excitation $G_F(f)$, as follows [10]

$$< x^2_B > = \pi^2 m^2 f_n G_F(f) / \zeta \quad (9)$$

Where

$f_n = \omega_n / 2\pi$

$G_F(f) = < F^2 > / \Delta f_F$ (10)

Δf_F is the band width of excitation, and

$< x >$ signifies temporal average of the variable **x**.

The expression derived above gives the mean square modal response for stationary random excitation. For non-stationary excitation, a time response function enveloping the spectral excitation can be applied.

Equation (9) shows that the response is proportional to the equipment mass, m**,** the square root of the natural frequency, f_n and inversely proportional to the square root of the damping ratio, **ζ.**

The stiffening of the pipework therefore would lead to an increase in the force transmitted to the steel structure, a problem that sometimes eludes pipework designers.

The rms stress value is obtained by calculating the stress due to unit force and multiplying the stress value obtained by,

$$[(\pi/2) \eta m^2 f_n / (\Delta f_H \zeta)]^{1/2} \quad (11)$$

Where $\mathbf{\Delta f_H}$ is set equal to $\mathbf{1/ G_H (f)}$ for unit rms acceleration. The factor $\boldsymbol{\eta}$ is introduced to allow for finite bandwidth of excitation and is a function of system parameters and excitation bandwidth [9].

5. CASE STUDY

The method is applied to a pipework system where a number of vibration channels, totaling 12, are taken over a duration of 72 hours as transient events leading to pipework vibration were not identified. Only one of the twelve channels captured a response to what appears to be a water hammer transient. The measured data indicated excitation up to 80Hz. However, only frequencies up to 50 Hz were analyzed as excitation over and above this range were considered be resulting from shell modes excitation, and these were not included in the analysis.

Peak velocity responses were identified. These are given in Table (2). Modal damping was calculated from measurements and found to range between 5 – 8% of critical damping. Two locations on the pipework were identified being critical. Values of S_0 in the out- plane and in-plane direction at piping connections, these are given in Table (3). Results of the transient event are given in Table (4). 'Fatigue damage' occurs if it approaches a value 1.00, which depends on number of transients per unit time. All calculations were carried out on MATLAB.

6. DISCUSSION AND CONCLUSIONS

A method for calculating the dynamic stress and fatigue damage in pipework from vibration measurements is proposed. The method relies on decomposing the system into its natural modes and obtaining the overall damage by summing the modal damages applying Minor's rule. When the response is characterized by widely spaced modes, a closed form solution can be obtained. This simplifies the calculations to a great extent. The analysis is also applied to a broadband random excitation. The method looks promising and easy to implement, but requires further validation by comparing the results with an alternative approach. This is currently being investigated by comparing the results with those obtained from a two-degrees-of freedom system.

7. TABLES

Sensor Designation	1	2	4	5	6	7	8	9	10	11	12
Peak Velocity	0.45	0.40	0.60	0.40	0.20	0.45	0.90	0.40	0.45	0.48	0.95

Table 1: Peak Measured Velocity Responses (mm/s)

Mode No.	Freq.	Mode No.	Freq.	Mode No.	Freq	Mode No.	Freq	Mode No.	Freq
1	5.6	101	16.2	201	24.4	301	32.8	401	41.8
2	5.7	102	16.1	202	24.5	302	33.0	402	41.8
3	6.0	103	16.1	203	24.5	303	33.0	403	42.1
4	6.2	104	16.1	204	24.7	304	33.1	404	42.1
5	6.2	105	16.3	205	24.7	305	33.1	405	42.2
98	15.9	198	24.1	298	32.4	398	41.5	484	49.7
99	15.9	199	24.1	299	32.5	399	41.6	485	49.8
100	16.0	200	24.2	300	32.6	400	41.7	486	49.9

Table 2: Pipework Natural Frequencies 1-50 Hz

Node number	In-plane stress per unit force	Out-plane stress per unit force
12150	0.026	0.040
15100	0.452	0.302

Table 3: S_0 Values (Pk-Pk -MPa/N)

Node number	In-plane	Out-plane
12150	2.43E-006	0.83E-005
15100	1.44E-003	1.10E-004

Table 4: Fatigue Damage Resulting from One Event

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APPENDIX A
Derivation of Fatigue Damage of a Simple Oscillator resulting from Shock Excitation

1.0 INTRODUCTION

The equation of motion of **a single degree of freedom** system subject to base excitation is given by:

$$m\, d^2x/dt^2 + c\, dx/dt + k\, x = -\, m\, a_H \qquad \text{(A.1)}$$

where,

m	is the secondary system modal mass	kg
c	is the secondary system damping coefficient	N/mm/s
k	is the secondary system stiffness	N/mm
x	is the *relative* displacement	mm
a_H	is the primary structure acceleration	mm/s^2

The solution of the equation of motion (A.1) due to a transient excitation, assuming small damping, is given by [12]

$$x = (V_0 / \omega_n) \,.\, \exp(-\zeta\, \omega_n t) \,.\, \sin(\omega_n t) \qquad \text{(A.2)}$$

where,

V_0	is the initial velocity due to impact	mm/s
ω_n	is the system circular frequency = $2\,\pi\, f_n$	rad/s
ζ	is the damping ratio	

The force F is given by the product of the stiffness k and the displacement x, i.e. $F = k\, x$, which from equation (A.2), can be written as

$$F = k\, (V_0 / \omega_n) \,.\, \exp(-\zeta\, \omega_n t) \,.\, \sin(\omega_n t) \qquad \text{(A.3)}$$

Noting that $k = m\, \omega_n^{\,2}$, equation (A.3) can be written as

$$F = (m\, \omega_n V_0) \,.\, \exp(-\zeta\, \omega_n t) \,.\, \sin(\omega_n t) \qquad \text{(A.4)}$$

If we denote **the stress per unit force S_0,** the stress, S can be written as,

$$S = S_0\, F \qquad \text{(A.5)}$$

which from equation (A.3) is

$$S = S_0 \,.\, k \,.\, x = S_0\, k\, (V_0 / \omega_n)\, \exp(-\zeta\, \omega_n t) \,.\, \sin(\omega_n t) \qquad \text{(A.6)}$$

or from equation (A.4),

$$= S_0\, (m\, \omega_n V_0).\, \exp(-\zeta\, \omega_n t) \,.\, \sin(\omega_n t) \qquad \text{(A.7)}$$

It should be noted that in equation (A.5), the definition of the stress **S** as amplitude, range or rms value follows the definition of the force **F** with no effect on the value of the value S_0.

Equations (A.6) or (A.7) are equivalent and can be used interchangeably depending on ease of obtaining the value of the stiffness or of the modal mass. Usually the mass m, is easier to determine in practice than the stiffness k. In subsequent development therefore, equation (A.7) will be used in favour of equation (A.6)

If we let $S_1 = S_0\, (m\, \omega_n V_0)$ (A.8)

it follows from equation (A.7) that

$$S = S_1\, \exp(-\zeta\, \omega_n t) \,.\, \sin(\omega_n t) \qquad \text{(A.9)}$$

The maximum stress excursion due to an acceleration of short duration occurs at time = $\pi/2\omega_n$.

Substituting into equation (A.9), it follows that,

$$S_{max} = S_1 \exp(-\zeta\, \omega_n \pi / 2\omega_n) \sin (\omega_n \pi /2\, \omega_n) = S_1 \exp(-\zeta \pi/2) \qquad (A.10)$$

In practice, the value $\zeta\pi/2$ is in the range 0.02 -0.05 and thus the value $\exp(-\zeta\pi/2)$ is in the range 0.9 to 0.95. It follows that a conservative estimate of the maximum value of stress during any excursion, $(S_D)_{max}$ is equal to S_1 and the following simplified equation will be used in subsequent analysis:

$$S_{max} \;\; = S_0\, (m\, \omega_n\, V_0) = S_0\, (2\, \pi\, m\, f_n\, V_0) \qquad (A.11)$$

Fatigue damage is assessed using the method below, bearing in mind that crack initiation occurs in the first few cycles while crack propagation continues thereafter with diminishing effect.

At each event, the system will accumulate a certain amount of fatigue damage which can be reasonably estimated according the Palmgren-Miner rule.

According to the Palmgren-Miner rule, the incremental damage δD due to an incremental number of cycles $\delta n(S)$ occurring between stress level S and $S + \delta S$, is given by

$$\delta D = \omega_n\, (S)/N(S) \qquad (A.12)$$

where N(S) is the number of cycles at stress level S sufficient to induce fatigue failure in a constant-amplitude fatigue test with stress amplitude S, which can be estimated from a typical S-N empirical relationship,

$$NS^b = C \qquad (C=3) \qquad (A.13)$$

The total damage resulting from one event therefore is given by integrating the incremental damage δD resulting from an elemental number of cycles $\omega_n(S)$, over the number of cycles at all stress levels, i.e.

$$D = \int dD = \int dn(S) / N(S) \qquad (A.14)$$

To evaluate this integral, the differential dn(S) is written as (dn(S)/dS) dS and the distribution of the number of cycles n for different stress levels S is sought.

Substituting S^b/C for N from equation (A.13), the evaluation of equation (A.14) therefore reduces to,

$$D = \int dD = \int S^b\, (dn(S)/dS)\, dS /C \qquad (A.15)$$

The distribution of the number of cycle's n of the system at any stress level S due to transient motion is worked out as follows:

We know that there is zero number of cycles over and above the maximum value determined by equation (A.11), namely

$$(S)_{max} = S_0\, (2\, \pi\, m\, f_n\, V_0)$$

There are also an infinite number of cycles at zero stress level. A function that satisfies these two conditions is

$$n(S) = -\, f_n T\, \mathbf{In}\, [S/(S)_{max}\,] \qquad (A.16)$$

$$\text{thus} \quad (dn(S)/dS) = -\, f_n \mathbf{T}/S \qquad (A.17)$$

where **T** is the response duration and is proportional to $1/ f_n\, \zeta$.

Substituting (A.17) into (A.15) and integrating between S=0 and S= $(S_D)_{max}$, the following equation is obtained,

$$D = \int dD = -\, (f_n \mathbf{T}/C) \int S^{b-1}\, dS \qquad = (f_n \mathbf{T}/bC).\, (S)_{max}^{\;b}$$

Therefore, fatigue damage the system would suffer as a result of single impact or shock is given by

$$D = (f_n \mathbf{T}/bC).\, S_0^{\,b}\, (2\, \pi\, m\, f_n\, V_0\,)^b \qquad (A.18)$$

which can be written as

$$\mathbf{D = (f_nT/bC). (S_0\, \omega_n\, m)^b\, V_0^{\,b}} \qquad \mathbf{(A.19)}$$

Equation (A.19) gives an estimate of the fatigue damage occurring as a result of one shock transients. For a total number of transients N_T, the total fatigue damage D_{total}, is given by

$$D_{total} = N_T\, D = (N_T\, f_nT/bC). (S_0\, \omega_n\, m)^b\, V_0^{\,b} \qquad (A.20)$$

According to the Palmgren-Miner hypothesis, failure will occur when D_{total} approaches the value 'one'. Substituting D_{total} = 1.00 in equation (A.20), the following functional relationship is obtained between the SBC response velocity and number of transients, N_T

$$N_T \quad = \quad bC / \{V_0^{\,b}\, (S_0\, \omega_n\, m)^b\, f_nT\} \qquad (A.21)$$

It should be noted that the product $\mathbf{f_nT}$ is equal to $1/2\pi\zeta$, thus:

$$N_T = 2\pi\, b\, C\, \zeta\, /\, (V_0\, S'_0)^b \qquad (A.22)$$

APPENDIX B
DISRETISATION ERRORS

Digitization errors can occur in the frequency or in the time domain. This is a *probable* type of error and its inclusion should ensure conservatism of the results.

B.1 Discretization Error in the Time Domain

If the response of the pipework in the vicinity of one mode is assumed to be harmonic at frequency f_n, then the maximum *possible* discretization error in the vibration amplitude A, can be shown to be:

$$(e_{max}/A) = 1 - \sin \pi\ (1-2\ \Delta t\ f_n)/2 \qquad (B.1)$$

where Δt is the time step.

For Δt = 1 ms, the % error say at f_n=100 Hz, is

$$(e_{max}/A)_t = 4.90\% \qquad (B.2)$$

while for Δt = 0.5 ms, the % error at the same frequency, f_n=100 Hz, is only,

$$(e_{max}/A)_t = 1.23\ \% \qquad (B.3)$$

B.2 Discretization Error in the Frequency Domain

If the response of the pipework in the vicinity of one mode once more is assumed to be harmonic of frequency f_n, the maximum *possible* discretization error in the measured amplitude A can be shown to be:

$$(e_{max}/A)_f = = 1 - 1/\ (1+ \Delta f/2\ \zeta\ f_n)^{0.5} \qquad (B.4)$$

where Δf is the resolution of the frequency spectrum and ζ is the damping ratio

For example, if Δf = 0.5 Hz, f_n = 100 Hz and ζ = 5%, the error is,

$(e_{max}/A)_f$ = 2.4% and is independent of the sampling rate (B.5)

It is worth noting that, reducing the error in the time domain results in an increase in error in the frequency domain and vice versa