Some Results of Benchmark Cases in Linear Acoustics

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ABSTRACT
In 2014, the Technical Committee *Computational Acoustics* of the European Acoustics Association has launched its benchmark initiative. For this, a first paper has been published in Acta Acustica united with Acustica 101:811-820 (2015). Three initial benchmark cases have been proposed therein, i.e. the long duct, the cat's eye and the radiatterer. This talk refers to two of these benchmarks and shows how useful a careful investigation of these problems. By analysing the radiatterer, the author discovered a surprising problem in the use of the method of Burton and Miller. Analysing the long duct problem, exhibits the problem that the conventional boundary element formulation in acoustics suffers from numerical damping. Numerical damping is a problem if sharp resonance peaks are expected. However, numerical damping is also causing an additional error which increases with the length of a model and challenges the idea that a certain number of a certain type of elements per wavelength are the only parameter to control the accuracy of a boundary element model.

Keywords: boundary element method, radiatterer, numerical damping
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1. INTRODUCTION

The benchmark initiative the Technical Committee for *Computational Acoustics* of the European Acoustics Association was officially launched at the Forum Acusticum in Krakow in September 2014 and has been described in the paper [1]. The initially proposed benchmark examples comprise three cases of linear acoustics. The whole benchmark initiative, however, comprises examples of linear acoustics, high frequency acoustics, acoustics and vibration and acoustics involving heterogeneous and moving fluids. The official website is found at http://eaa-bench.mec.tuwien.ac.at. Two associate editors are assigned to each of these four fields. A review process will assure that the proposed benchmark examples are reasonable. Proposed benchmark cases will be quipped with doi numbers and, as such, can be cited thereafter.

The three initial examples discussed in [1] comprise the long duct, the cat's eye radiator and an example called the radiatterer. The author of this paper has extensively studied the first two of these examples over the last 20 years, see for example [2–7], also [8, 9], on the duct and [10–12] on the cat's eye model. With the initiation of the benchmark project, the author has investigated these examples again and can report new results which were mainly found because of new and detailed investigations of the radiatterer [13] and the long duct [14]. These results will be very briefly summarized in this short paper.

2. A FLAW REVEALED BY THE RADIATTERER

The radiatterer – see description in [1] – contains a number of resonators with small and large openings. In some first test cases [13], it was considered as a radiator with a unit particle velocity over its surface. Clearly, this had to result in a positive value for the sound power. However, at some particular resonances of a resonator with a very small opening, a negative radiated sound power was yielded. As it was clear that this had been wrong, an investigation began to find the reason for this error. It became clear that the error appeared as a 180 degrees phase shift within the resonator and only within a small frequency band. The situation was quite weird since the author's code had been tested many times before and was always found to be reliable.

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Finally, it turned out that the error had been introduced by the coupling parameter of the Burton and Miller method [13]. While most papers on the Burton and Miller method use a coupling parameter i/k, some formulations seem to require a coupling parameter $-\mathrm{i}/k$. This, however, surprised mathematicians working on this topic since they were convinced that the sign of the coupling parameter should not have an effect on the accuracy of the method and the suppression of the spurious modes, see discussion and references in [13]. Their results were mainly yielded for radiators (and/or scatterers) which are convex or close to convex though. Apparently, the very irregular shape with resonators of the radiatterer has been responsible for this error and the understanding that the sign of the coupling parameter matters indeed.

A review of the literature presenting formulations based on the Burton and Miller method has shown that approximately 50% of the papers use the wrong sign of the coupling parameter. The reason for this consists in the fact that the sign of the optimal coupling parameter depends on the sign of the exponent of the harmonic time–dependence $\mathrm{e}^{\pm \mathrm{i}\omega t}$ and the sign of the Green's function. Choice of the correct sign of the coupling parameter is also resulting in an improved condition such that the number of iterations for the solution of the system of equations decreases substantially.

3. NUMERICALLY DAMPED RESONANCES IN BEM AND THE LONG DUCT

The author has already mentioned that the conventional boundary element method in acoustics seems to produce numerical damping [2]. The benchmark case of the long duct was used to confirm this observation [14]. Observation of numerical damping is relevant for any weakly damped resonances in room acoustics.

Another, maybe even more far–reaching, observation is the fact that numerical damping leads to an increasing error if the duct becomes longer while elements of the same size are used. This observation, however, is in contrast to the generell idea that a fixed number of elements per wavelength is the only criterion for a suitable mesh size in boundary elements. It would be similar to the pollution effect known from the finite element methods. In particular, the author's papers [3, 4] indicated that BEM is free of the pollution effect. Therefore, further investigation of this numerical damping effect is really a task for future research in this field. The proposal of an extended benchmark case involving more duct examples to better exhibit this pollution–like error is desired.

4. CONCLUSION

While the original idea of the benchmark initiative consisted in comparison of methods and codes, the authors's participation in it guided the author to new findings about the boundary element method and his own code. It revealed that many authors use the wrong coupling parameter of the Burton and Miller method and it further indicated that BEM seems to produce an additional discretization error which increases if the size of the domain increases.

Therefore, the author wishes to encourage other researchers to participate in the benchmark initiative, not only (but also) to compare their results with those of others. A suitable benchmark problem can even lead to a new understanding of a method or even to entirely new results.

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