Evaluation of uncertainty in the free-field calibration of hydrophones by the three-transducer spherical wave reciprocity method

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ABSTRACT

The primary method adopted for free-field calibration of underwater acoustic transducers in the frequency range 250 Hz to 500 kHz by the UK National Physical Laboratory is the method of three-transducer spherical wave reciprocity. This paper describes the evaluation of measurement uncertainty associated with estimates of the transducer transmitting and receiving sensitivities provided by NPL's implementation of this calibration method. Consideration is given to formulating the problem of uncertainty evaluation in terms of a measurement model, which is developed from a sequence of sub-models that describe different aspects of the measurement and the available information about the input quantities. A comparison is made of the results obtained using conventional 'uncertainty propagation' and a Monte Carlo method as approaches to solving the formulated problem. The sensitivity of the results obtained to assumptions about the correlations associated with influence quantities in the calibration and the reliability of the available information about those quantities is investigated.

Keywords: 77 Sampling, quality control procedures and measurement uncertainty, 71.9 Calibration; acoustical and electrical performance verification, 81.2 International standards, 73.5 Laboratory facilities (design and construction).

1. INTRODUCTION

The most common absolute method used for the calibration of underwater acoustic transducers relies on the principle of acoustic reciprocity (1-3). The three-transducer spherical-wave reciprocity calibration method requires the use of three hydrophones, at least one of which must be a reciprocal transducer; that is, its transmitting and receiving sensitivities are related by the so-called reciprocity parameter, depending only on frequency and the density of water. In the measurements, the hydrophones are paired off in three measurement stages, at each of which one device is used as a transmitter and the other as a receiver. For each pair of hydrophones, a measurement is made of the ratio of the voltage across the terminals of the receiving device to the current driving the transmitting device. Using the reciprocity principle as applied to the reciprocal hydrophone, the sensitivity of any one of the hydrophones can be determined from the purely electrical measurements described above, given knowledge of the geometry of the acoustic field propagating between the two devices (in this case, a spherical wave geometry).

This method is adopted by the UK National Physical Laboratory (NPL) for calibrating underwater acoustic transducers in the frequency range 250 Hz to 500 kHz is three-transducer spherical- wave reciprocity (1). This paper is concerned with the evaluation of measurement uncertainty associated with this calibration method. The aims are as follows:

(i) To give a formulation of the problem of uncertainty evaluation in terms of a measurement model that defines the mathematical relationship between all quantities involved in the measurement, and the available information about those quantities;

(ii) To compare the results obtained from different approaches to solving the formulated problem and, in particular, to validate the results obtained using the conventional approach of 'uncertainty propagation' with those obtained from a Monte Carlo method that provides a more generally-applicable approach;

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(iii) To investigate the impact on the results of different assumptions about the correlations associated with input quantities in the measurement model used to describe corrections applied as part of the calibration. In practice, it can be difficult to quantify such correlations, and an uncertainty analysis can be undertaken for different assumptions in order to investigate the sensitivity of the results to those assumptions;

(iv) To investigate the sensitivity of the results to different assumptions about the 'reliability' of the available information about the input quantities in the measurement model.

The overall objective is to give confidence in the reported uncertainties that accompany results obtained from the calibration method.

2. CALIBRATION METHOD

2.1 Calibration method

In the calibration method, three measurements are made, each involving the use of two of the three transducers, with one transducer acting as a transmitting device and the other as a receiving device. One transducer is labelled P (and is typically used only as a projector or transmitting device), a second is labelled H (and is typically used only as a hydrophone or receiving device), and a third transducer, labelled T, is used both as a transmitting and receiving device, and is assumed to be reciprocal. The measurement set up is shown schematically in Figure 1. The sensitivity of any one device may be determined from the calibration (1).

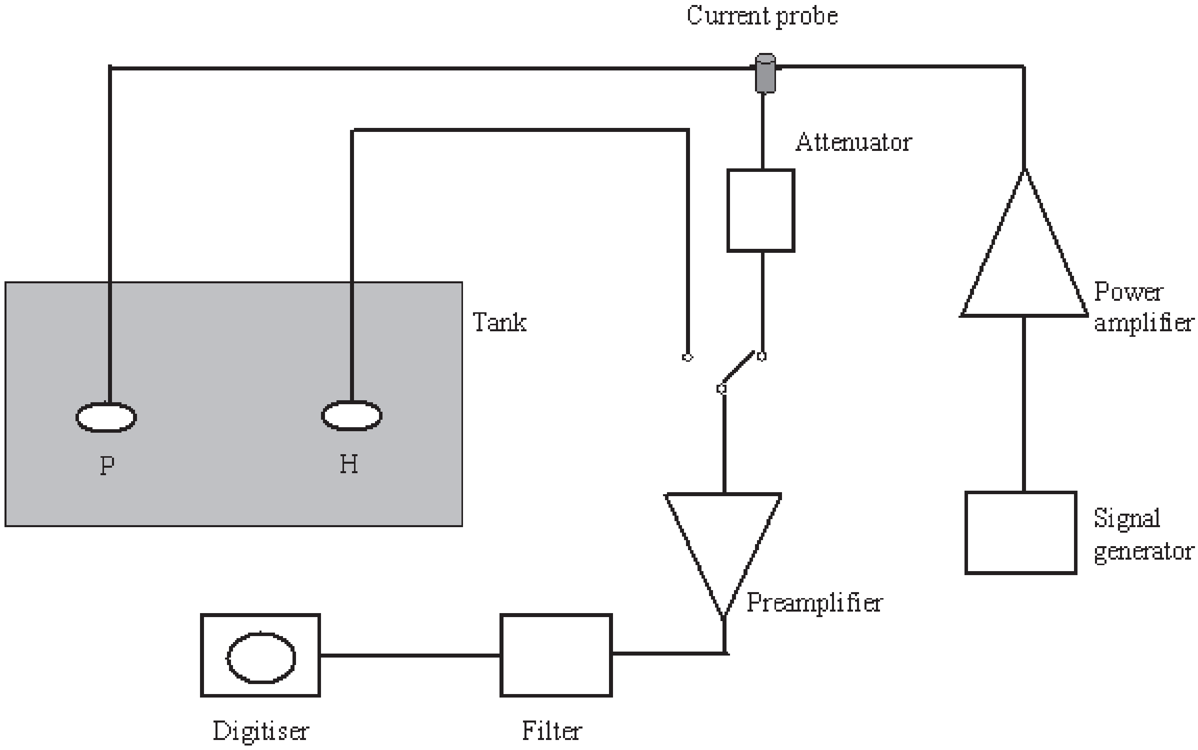

Figure 1 - Schematic diagram showing the arrangement of the instrumentation in a calibration. P is the projector or transmitting device and H is the receiving device.

Free-field calibration of underwater electroacoustic transducers requires that the measurements be made in a free-field acoustic environment. However, calibrations are commonly undertaken in laboratory test tanks that are quite reverberant (1-3). To enable free-field acoustic conditions to be realised, gated sinusoidal signals are employed to make measurements at discrete acoustic frequencies. In the arrangement, measurements are made on the steady-state portion of the received signal, with time-gating techniques used to isolate the direct-path signal from reflections from the tank boundaries and the water surface. In such measurements, the steady-state signal available for analysis is limited in time both by the arrival of boundary reflections, and by start-up transients caused by the resonant behaviour of the transducers under test. If the signal received at the measuring hydrophone does not reach steady-state conditions during the available echo-free time, it is not possible to observe the steady-state directly. This means that for a given tank size and transducer Q-factor, there will be a lower limiting frequency below which it is not possible to make measurements by conventional means.

2.2 Sources of uncertainty

In general, the sources of Type B uncertainty specific to free-field reciprocity calibrations include the following (1, 4):

- uncertainty of any assumptions about the acoustic field, e.g. that the field is a spherical-wave field (this may be checked by varying the separation distance between transducers and checking that the product of electrical transfer impedance and distance is invariant);
- non-reciprocal behaviour by transducers (can be evaluated by checking the equivalence of the Z_{PT} and Z_{TP} electrical transfer impedances);
- uncertainties in the measurement of the separation distance;
- uncertainties in the values for acoustic frequency and water density (required to calculate the reciprocity parameter);
- lack of steady-state conditions, especially where bursts of single-frequency sound waves are used (the resonance frequency and Q-factors of the transducers and the echo-free time of the test tank will influence this contribution);
- interference from acoustic reflections, leading to a lack of free-field conditions;
- lack of acoustic far-field conditions;
- the spatial averaging effects of the hydrophones under calibration due to their finite size and the lack of perfect plane-wave conditions (typically, only an issue at very high frequencies);
- misalignment, particularly at high frequencies where the hydrophone response may be far from omnidirectional;
- acoustic scattering from the hydrophone mount (or vibrations picked up and conducted by the mount);
- uncertainty in measurement of the receive voltage (including uncertainty due to the measuring instrumentation (voltmeter, digitizers, etc.);
- uncertainty of the gains of any amplifiers, filters, and digitizers used (the use of a common measurement channel and the measurement of voltage ratios may reduce this component);
- uncertainties in the measurement of the drive current or voltage;
- uncertainties due to the lack of linearity in the measurement system (the use of a calibrated attenuator to equalize the measured signals may significantly reduce this contribution);
- uncertainty of any electrical signal attenuators used;
- electrical noise include RF pick-up;
- uncertainty of any electrical loading corrections made to account for loading by extension cables and preamplifiers

3. MEASUREMENT MODEL

3.1 Basic measurement model

To assess the uncertainties requires the measurand or output quantity intended to be measured to be identified, as well as the input quantities on which the output quantity depends, and the measurement model that defines the mathematical relationship between the output quantity and the input quantities (5). The focus is on the determination of the free-field sensitivity M_H of the hydrophone H for a single acoustic frequency f. The measurement model for the calibration of an underwater acoustic transducer by the method of free-field reciprocity is based on the equation for the sensitivity:

$$M_H \quad = \quad \sqrt{J\,\frac{d_{PH}\,d_{TH}}{d_{PT}}\,\frac{Z_{PH}\,Z_{TH}}{Z_{PT}}} \tag{1}$$

where d is the separation of the transducers and Z is the electrical transfer impedance and is equal to the voltage developed by the receiving hydrophone divided by the drive current in the projecting hydrophone. The subscripts refer to which hydrophones are used for the transfer impedance measurement (eg P → H), the transducers being designated P, T and H.

The reciprocity parameter, J, is given by $J = 2d_0/\rho f$ where d_0 is the reference distance in the definition of the transmitting response (usually equal to 1 metre and so omitted from the equation), and ρ is the water density and f is the acoustic frequency. Expanding the above equation for M_H in terms of the receive voltage, V, and the drive currents, I, gives:

$$M_H \quad = \quad \sqrt{\frac{2}{\rho f} \frac{d_{PH}\, d_{TH}}{d_{PT}} \frac{V_{PH}\, V_{TH}\, I_{PT}}{I_{PH}\, I_{TH}\, V_{PT}}} \qquad (2)$$

3.2 Type B uncertainties (systematic bias)

The approach adopted is that where corrections are made (eg for instrument calibrations), a correction factor is applied for the effect under consideration, and the uncertainty on that correction is included in the uncertainty budget. Where a correction is not made but an uncertainty for an effect must be included, an assumption is made that the “correction factor” for the effect is unity and an appropriate uncertainty on this factor is included. All uncertainties are *relative uncertainties* (usually expressed in percent).

3.2.1 Uncertainties from electrical measurements

The drive current is measured using a calibrated current transformer which has a calibration factor denoted by C (derived from a calibration certificate). The calibration is performed at only a selection of frequencies and interpolation is used to derive the calibration at a specific frequency. In addition, the current probe is not perfectly linear in response. Hence the calibration factor is actually given by $C = C_P.C_I.C_L$ where C_P is the calibration factor at the frequency specified on the certificate, C_I is the interpolation factor and C_L is the linearity factor. The current probe produces an output voltage, E, corresponding to the measured current. Since this “current” voltage is in general larger than the voltage produced by the receiving hydrophone, it is attenuated before measurement by a factor of A dB by use of a calibrated attenuator such that it is within 6% (0.5 dB) of the hydrophone receive voltage. So, the drive current is given by:

$$I \quad = \quad E\,C\,10^{(\frac{A_{PH}}{20})} \quad = \quad E\,C_P\,C_I\,C_L\,10^{(\frac{A_{PH}}{20})} \qquad (3)$$

If the hydrophone voltage and current probe voltage are then measured through the same measurement channel consisting of preamplifier of gain G, filter of insertion loss factor F, and digitizer of calibration factor D, and since we are calculating the voltage ratio to get the transfer impedance, errors from G, F and D are perfectly correlated and cancel out. The expression for transfer impedance then becomes:

$$Z_{PH} \quad = \quad \frac{G\,F\,D\,V_{PH}}{G\,F\,D\,E_{PH}\,C_P\,C_I\,C_L\,10^{(\frac{A_{PH}}{20})}} \quad = \quad \frac{V_{PH}}{E_{PH}\,C_P\,C_I\,C_L\,10^{(\frac{A_{PH}}{20})}} \qquad (4)$$

Note that for the current probe calibration factors, the certificate calibration and the interpolation factor depend on frequency but are identical for each current at a given acoustic frequency. However, the linearity may vary for each current since I_{PH} is not necessarily identical to I_{TH} etc. Note that in the calculation of MH, there will be some cancellation of factors C_P and C_I because they are perfectly correlated, and so will cancel because they appear in a ratio. The electrical impedance of the hydrophone is used to make a correction to the received voltage to account for the presence of extension cable and/or the electrical loading by the pre-amplifier, and this correction is also ascribed an uncertainty.

3.2.2 Uncertainties from acoustic factors

(i) *Lack of steady state*

Each hydrophone is a resonant device and as such will take some time to reach steady-state when driven by a tone-burst (gated sine wave). Typically, a device takes Q cycles of its resonance frequency to reach steady-state (where Q is the Quality factor). Gated signals are used in order to eliminate the effect of reflections from the tank boundaries (the direct signal arrives before the reflected signals, giving each tank a characteristic echo-free time). At low frequencies, the longer period of the cycles in the signal leaves little time for steady state to be reached, and the uncertainty from this component is greater (6). Although this effect is primarily caused by the transmitter, the effect is observed primarily in the receive voltage, V. The uncertainty is greater at low frequencies, reducing when several cycles of steady-state are available for measurement. A uniform distribution is assumed, but it could be argued that at low frequencies the distribution should be one sided since the effect is more likely to lead to an underestimate of the value. The effect depends most strongly on the properties of the transmitting device, so the uncertainty for V_{PH} will be correlated strongly with V_{PT} but very weakly with V_{TH}.

(ii) *Lack of spherical spreading*

The method assumes that the acoustic wave spreads spherically, so that the acoustic pressure falls away inversely with distance from the source. This is inherent in the definition of the transmitting response of the projector. If this is not the case, for example if the separation between source and receiver is not sufficient to enable acoustic far-field conditions to be achieved, the receive voltage V will be modified by a factor due to the lack of spherical-wave conditions (uniform distribution). There will be strong correlation between the uncertainties for each of the received voltages.

(iii) *Non reciprocal transducer*

The method depends upon at least transducer T being reciprocal (linear, passive and reversible). There is no way of knowing this without testing against another transducer. If P is also reciprocal, this can also be used as a receiver, allowing a fourth measurement to be made, that of T → P. If both P and T are reciprocal, the value of Z_{PT} and Z_{TP} should be identical, and the degree to which they are not acts as a test on the uncertainty of the assumption. The value for M_H can now be calculated using either Z_{PT} or Z_{TP} on the denominator of the formula in equation (1), with the mean of the two values being the best estimate. In the calibration at NPL, the fourth measurement of T → P is always measured as an additional check, and half the value of the disagreement obtained is used as the uncertainty. Typically, at the agreement is better than 1% in the middle of the frequency range, and the agreement can be tested for every frequency of measurement and an uncertainty assigned accordingly. A typical uncertainty of 1.5% is ascribed with a uniform distribution.

(iv) *Wetting*

The device sensitivity can depend on how long the hydrophone is soaked in the tank. This is because any air clinging to the surface if the hydrophone is not properly wetted will attenuate the sound (air layers and bubbles are strong reflectors of sound). Transducers are initially soaked overnight and then for 30 minutes between measurements. Typically, the wetting and soaking time used is stated with the results and the calibration is only valid for this. However, a residual uncertainty may be included in the budget.

(v) *Misalignment*

The transducers and hydrophones may be aligned in two ways: (a) visually with an alignment mark; or (b) acoustically be looking for a maximum signal. Method (a) is used for nominally "omnidirectional" hydrophones with elements made from small spheres or cylinders; method (b) is used for directional transducers such as "piston" devices. Both these are subject to an error which increases with frequency. Method (a) has a symmetric uncertainty distribution; in method (b) misalignment can only lead to a reduction in signal, so the distribution is really one-sided. The value is very frequency dependent, and increases at high frequencies.

(vi) *Separation*

The uncertainty in positioning the hydrophones at the correct separation is normally less than 1% with a uniform distribution assumed. For a separation of 1.5 m, this equates to 15 mm, which is easily achieved with a precision positioning system.

(vii) *Density and frequency*
Usually known very accurately from calculations. Typically known to better than 0.2% (uniform).

3.2.3 Type A uncertainties

(i) *Signal* n*oise (electrical)*
Electrical noise adds to the uncertainty when the measured voltage signal is low. This occurs for frequencies away from the resonance frequency of the transmitting device (so this tends to be worse at very low or very high frequencies). This is also worse for calibration of low sensitivity receivers. The noise is usually considered to be Gaussian. The signal to noise ratio may be improved by coherent averaging of repeated acquisitions (that is for multiple "pings"), and for poor signal to noise ratio on-board averaging on the digitiser may be used (perhaps 50 or 100 averages). The standard deviation obtained from the measurement of multiple pings can be considered a measure of the noise. This would have a normal distribution and typically has a standard deviation value of less than 0.5% for a good signal. This is be considered a Type A uncertainty (see next section) and applies to both the receive voltage and drive current.

(ii) *Calibration repeatability*
In addition to the repeated measurement of the electrical voltages and currents, the *whole experiment* to calibrate the hydrophones (and so produce values for M) is repeated *at least* four times. This produces an overall value for M (by averaging the values obtained from repeated calibrations) and produces a Type A uncertainty for M. Between each repeated calibration run, the transducers are removed from the test tank and re-mounted. This allows any imprecision in the mounting, alignment and wetting of the hydrophones to influence the repeatability.

4. MEASUREMENT UNCERTAINTY EVALUATION

4.1 The propagation of probability distributions

The basis for the evaluation of measurement uncertainty is the propagation of probability distributions. In order to apply the propagation of probability distributions, a measurement function is formulated of the generic form $Y = f(X_1, \ldots X_N)$ relating input quantities $X_1, \ldots X_N$, about which information is available, and the output quantity Y, about which information is required. Additionally, information concerning the input quantities is encoded as probability distributions for those quantities, such as rectangular (uniform), Gaussian (normal), etc. The information can take a variety of forms, including a series of indication values, data on a calibration certificate, and the expert knowledge of the metrologist. An implementation of the propagation of probability distributions provides a probability distribution for Y, from which can be obtained an estimate of Y, the standard uncertainty associated with the estimate, and a coverage interval for Y corresponding to a stipulated (coverage) probability. Particular implementations of the approach are the GUM uncertainty framework (7) and a Monte Carlo method (8, 9).

4.2 GUM uncertainty framework

The primary guide in metrology on uncertainty evaluation is the 'Guide to the expression of uncertainty in measurement' (known as the GUM) (7). It presents a framework for uncertainty evaluation based on the use of the law of propagation of uncertainty and the central limit theorem. The law of propagation of uncertainty provides a means for 'propagating uncertainties' through the measurement function, i.e., for evaluating the standard uncertainty $u(y)$ associated with the estimate $y = f(x_1, \ldots x_N)$ of Y given the standard uncertainties $u(x_i)$ associated with the estimates x_i of X_i (and, when they are non-zero, the covariances $u(x_i, x_j)$ associated with pairs of estimates x_i and x_j). The central limit theorem is applied to characterize Y by a Gaussian distribution (or, in the case of finite effective degrees of freedom, by a t-distribution), which is used as the basis of providing a coverage interval for Y. For the case of input quantities that are independent, the law of propagation of uncertainty takes the general form

$$u^2(y) = c_1^2u^2(x_1) + c_2^2u^2(x_2) + \cdots c_i^2u^2(x_N) \tag{5}$$

where c_i is the sensitivity coefficient for X_i given by the first order partial derivative of f with respect to X_i evaluated at the estimates of the input quantities. For the measurement function of concern here, the sensitivity coefficients are straightforward to calculate and the law of propagation of uncertainty takes a particularly simple form. For example, consider the following measurement function:

$$Y = X_1 \sqrt{(X_2/X_3)} \tag{6}$$

of $N = 3$ input quantities, which is indicative of the measurement function in equation (1), with estimates $x_1 = x_2 = x_3 = 1$. Then,

$$c_1 = \sqrt{\frac{x_2}{x_3}} = 1, \qquad c_2 = \frac{x_1}{2}\sqrt{\frac{1}{x_2 x_3}} = \frac{1}{2}, \qquad c_3 = -\frac{x_1}{2}\sqrt{\frac{x_2}{x_3}} = -\frac{1}{2} \tag{7},$$

and hence

$$u^2(y) = u^2(x_1) + \left(\frac{1}{2}\right)^2 u^2(x_2) + \left(\frac{1}{2}\right)^2 u^2(x_3) \tag{8}.$$

4.3 Monte Carlo method

A Monte Carlo method for uncertainty evaluation is based on the following consideration (8, 9). The estimate y of Y is conventionally obtained, as in section 4.2, by evaluating the measurement function for the estimates x_i of X_i. However, since each X_i is described by a probability distribution, a value as legitimate as x_i can be obtained by drawing a value at random from the distribution. The method operates in the following manner. A random draw is made from the probability distribution for each X_i and the corresponding value of Y is formed by evaluating the measurement function for these values. Many Monte Carlo trials are performed, so that the process is repeated many times, to obtain M, say, values y_k, $k = 1, \ldots M$, of Y. The values y_k are used to provide an approximation to the probability distribution for Y, and thence an estimate of Y, and the standard uncertainty associated with the estimate and a coverage interval for Y.

4.4 Comparison of methods

Although the GUM uncertainty framework can be expected to work well in many circumstances, it is generally difficult to quantify the effects of the approximations involved, which include the linearization of the measurement function in the application of the law of propagation of uncertainty, the evaluation of effective degrees of freedom using the Welch-Satterthwaite formula, and the assumption that the output quantity can be characterized by a Gaussian distribution (or t-distribution). In contrast, the Monte Carlo method has a number of features (9), including (a) that it is applicable regardless of the nature of the measurement function, i.e., whether it is linear, mildly non-linear or highly non-linear; (b) that there is no requirement to evaluate effective degrees of freedom; and (c) that no assumption is made about the distribution for Y, for example, that it is Gaussian.

In consequence, the Monte Carlo method provides results that are free of the approximations involved in applying the GUM uncertainty framework, and it can be expected to provide an uncertainty evaluation that is reliable for a wide range of measurement problems. Furthermore, the method can be used, as here, as a basis for validating the results obtained using the GUM uncertainty framework and testing the assumptions made in applying the GUM uncertainty framework as an approach to measurement uncertainty evaluation.

An important distinction between the two approaches to measurement uncertainty evaluation is that the GUM uncertainty framework only makes use of expectations and standard deviations (and possibly covariances and degrees of freedom) that are used as summaries of the probability distributions used to characterize the input quantities. In contrast, the Monte Carlo method uses the complete information about the input quantities encoded by these distributions.

5. RESULTS

The results presented in this section are indicative of a calibration undertaken at an acoustic frequency $f = 40$ kHz only.

The GUM uncertainty framework returns the relative standard uncertainty of 2.02% (the expanded uncertainty for a coverage factor of k=2 is given by 4.03%, equivalent to a confidence interval of 95%). The Gaussian probability density function that characterizes the sensitivity M_H on the basis of this information is shown as the solid (red) curve in Figure 1. The endpoints of a 95 % coverage interval for the output quantity are shown as dashed (red) vertical lines.

An application of a Monte Carlo method with 107 trials returns the relative standard uncertainty of 2.09 %. An approximation to the probability density function that characterizes the output quantity provided by a Monte Carlo method is shown as the histogram (scaled frequency distribution) in Figure 2. The endpoints of a 95 % coverage interval for the sensitivity M_H are shown as solid (blue) vertical lines. There is very good agreement between the results provided by the two approaches to uncertainty evaluation.

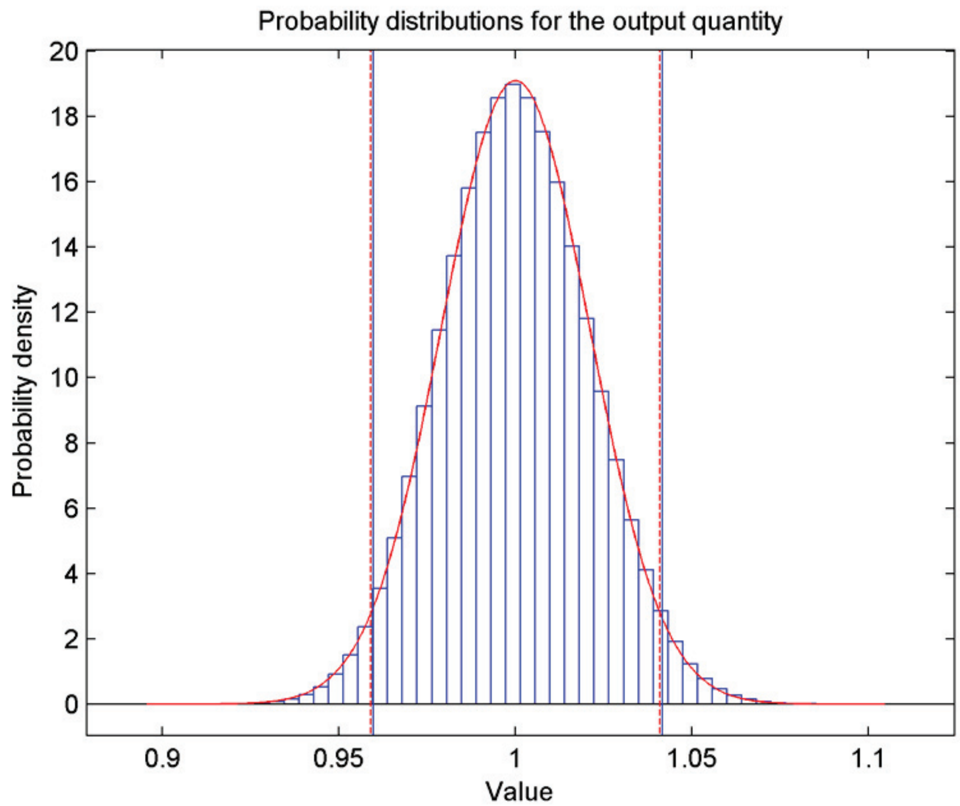

Figure 1. Probability density functions for M_H obtained using the GUM uncertainty framework (solid curve) and a Monte Carlo method (histogram). The endpoints of the 95 % coverage intervals provided by the two approaches are also indicated.

Table 1 lists the input quantities, the relative standard uncertainties associated with the estimates and the contributions of each input quantity to the (combined) relative standard uncertainty associated with an estimate of the sensitivity M_H provided by the GUM uncertainty framework.

The impact on the results provided by the GUM uncertainty framework of a positive correlation associated with pairs of estimates of the corrections for (a) lack of spherical spreading, (b) lack of steady state, and (c) electrical loading is to reduce the (combined) relative standard uncertainty, with the reduction greatest for correlation associated with pairs of estimates of the corrections for lack of steady state. Consequently, treating the quantities as independent provides a ‘safe’ value for u(y) in the case that pairs of quantities are positively correlated but it is difficult to quantify the correlation coefficients in practice. Positive correlation associated with pairs of quantities leads to the ‘cancellation’ of quantities in the measurement function (compare to the cancellation of the quantity G in the sub-models of equation (4) – see Section 3.2.1). On the other hand, negative correlation will, in this example, tend to ‘reinforce’ quantities, leading to an increase in $u(y)$.

The impact of accounting for imprecise information about quantities characterized by rectangular distributions (such as those for which the information is based on ‘expert knowledge’) is quite modest. Furthermore, the impact is quantifiable. For example, if the semi-widths of those distributions are regarded as ‘reliable’ to 50 %, the standard uncertainty provided by a Monte Carlo method remains unchanged when reported to two significant decimal digits.

Table 1 – Input quantities, relative standard uncertainties, and contributions obtained using the GUM uncertainty framework and a Monte Carlo method.

Source of uncertainty	**value ± %**	**Probability distribution**	**Divisor**	**c_i**	**u_i ± %**	**v_i or v_{eff}**
Acoustic frequency	0.2	rectangular	1.73	0.5	0.06	infinity
Water density	0.2	rectangular	1.73	0.5	0.06	infinity
Non-reciprocal behaviour	1.5	rectangular	1.73	0.5	0.43	infinity
Transfer impedance Z_{PH}:						
Separation	1.0	rectangular	1.73	0.5	0.29	infinity
Non-spherical field	2.0	rectangular	1.73	0.5	0.58	infinity
Non steady-state	2.0	rectangular	1.73	0.5	0.58	infinity
Orientation & alignment	0.5	rectangular	1.73	0.5	0.14	infinity
Noise (current)	0.50	normal	1.00	0.5	0.25	infinity
Noise (hydrophone voltage)	0.50	normal	1.00	0.5	0.25	infinity
Wetting	0.5	rectangular	1.73	0.5	0.14	infinity
Electrical loading	0.5	rectangular	1.73	0.5	0.14	infinity
Attenuator calibration	0.8	normal	2.00	0.5	0.20	infinity
Current probe calibration	1.0	normal	2.00	0.5	0.25	infinity
Transfer impedance Z_{TH}:						
Separation	1.0	rectangular	1.73	0.5	0.29	infinity
Non-spherical field	2.0	rectangular	1.73	0.5	0.58	infinity
Non steady-state	2.0	rectangular	1.73	0.5	0.58	infinity
Orientation & alignment	0.5	rectangular	1.73	0.5	0.14	infinity
Noise (current)	0.50	normal	1.00	0.5	0.25	infinity
Noise (hydrophone voltage)	0.50	normal	1.00	0.5	0.25	infinity
Wetting	0.5	rectangular	1.73	0.5	0.14	infinity
Transfer impedance Z_{PT}:						
Separation	1.0	rectangular	1.73	0.5	0.29	infinity
Non-spherical field	2.0	rectangular	1.73	0.5	0.58	infinity
Non steady-state	2.0	rectangular	1.73	0.5	0.58	infinity
Orientation & alignment	0.5	rectangular	1.73	0.5	0.14	infinity
Noise (current)	0.50	normal	1.00	0.5	0.25	infinity
Noise (hydrophone voltage)	0.50	normal	1.00	0.5	0.25	infinity
Wetting	0.5	rectangular	1.73	0.5	0.14	infinity
Type A uncertainty (repeatability)	1.0	normal	1.00	1.0	1.00	3 (n-1)
Combined uncertainty		normal			**2.02**	
Expanded uncertainty		normal (k=2)			**4.03**	

6. CONCLUSIONS

This paper has reported the evaluation of measurement uncertainty associated with estimates of the receiving sensitivities of an underwater acoustic hydrophone provided by NPL's implementation of the three-transducer spherical-wave reciprocity calibration method.

Consideration has been given to formulating the problem of uncertainty evaluation and comparing the results obtained using conventional 'uncertainty propagation' and a Monte Carlo method as approaches to solving the formulated problem. The sensitivity of the results obtained to assumptions about the correlations associated with influencing quantities in the calibration and the reliability of the available information about those quantities has been investigated.

The results obtained using the GUM uncertainty framework (or conventional 'uncertainty propagation') are validated by those obtained using a Monte Carlo method. Consequently, it is appropriate to use the GUM uncertainty framework as an approach to uncertainty evaluation for the problem described as well as for sufficiently similar problems.

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