



## Static pressure and temperature coefficients of working standard microphones

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### ABSTRACT

The sensitivity of measurement microphones is affected by changes in the environmental conditions, mainly temperature and static pressure. This rate of change has been the object of previous studies focused on Laboratory Standard microphones. The literature describes frequency dependent values for these coefficients which are used for calibration purposes. Working standard microphones are not exempt of these influences. However, manufacturers usually provide a low frequency value of the environmental coefficient. While in some applications the influence of this coefficient may be negligible, in others it may be a significant contribution to the uncertainty of the measurement.

Determining the environmental coefficients of individual specimens of measurement microphones can be a straightforward though time-consuming procedure provided the appropriate facilities are available. An alternative is to determine them using lumped parameter models or numerical calculations. Any of these possibilities require knowledge of the construction details of the microphones, particularly the geometry of the back cavity, and the properties of the membrane.

This paper presents an introductory study of the effect of the environmental coefficients. For this purpose, the environmental coefficients of some commercially available microphones have been determined experimentally, and whenever possible, compared with the coefficients determined numerically using the Boundary Element Method.

Keywords: BEM, metrology, microphones I-INCE Classification of Subjects Number(s): 75.5, 71.1.1

### 1. INTRODUCTION

Pressure and free-field sensitivities of measurement microphones are affected by changes in the environmental conditions during measurements, mainly temperature and static pressure. The determination of static pressure and temperature coefficients of the pressure sensitivity has been the object of previous studies focused on Laboratory Standard (LS) microphones. The literature describes frequency dependent values for these coefficients which are widely used for calibration purposes (1,2). Although Working Standard (WS) microphones are not designed to be used as calibration references, this type is not exempt of these influences; however, manufacturers usually provide only a low frequency value of the environmental coefficients though reference (3) contains data for some microphones of this type. Under some conditions, the effect of the environmental variables may be considered negligible, while in other circumstances they may become a significant contribution to the uncertainty of the measurement, particularly when the frequency dependence of the coefficients is ignored. Figure 1 below presents an example of this influence from a set of free-field reciprocity calibrations on a WS3 microphone performed within few days, in which natural variations of the barometric pressure were occurring. There it can be seen that variations up to 0.5 dB are not uncommon. Such changes, seen from a metrological point of view, can be intolerably large and must be taken into account.

Determining the environmental coefficients for the pressure sensitivity of individual specimens of measurement microphones can be a straight forward though time-consuming procedure provided the

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appropriate facilities are available, typically measurements in plane wave couplers or electrostatic actuator; the determination of the coefficients for free-field sensitivity poses additional practical complications such as the difficulties on changing temperature and static pressure of the large volume occupied by the anechoic room. This makes it unpractical to determine the coefficients from direct measurements, and a procedure based on the relation between free-field and pressure sensitivities is needed.

An alternative to the experimental determination of the coefficients is to estimate the environmental coefficients by using lumped parameter models of the microphone. Numerical calculations are another possibility. However, any of these two methods require knowledge of the construction details of the microphones, particularly the geometry of the back cavity, and the properties of the membrane. Although these data is not openly available for all types of microphones, the use of analytical or numerical calculations based on known types can help to validate values of coefficients obtained experimentally.

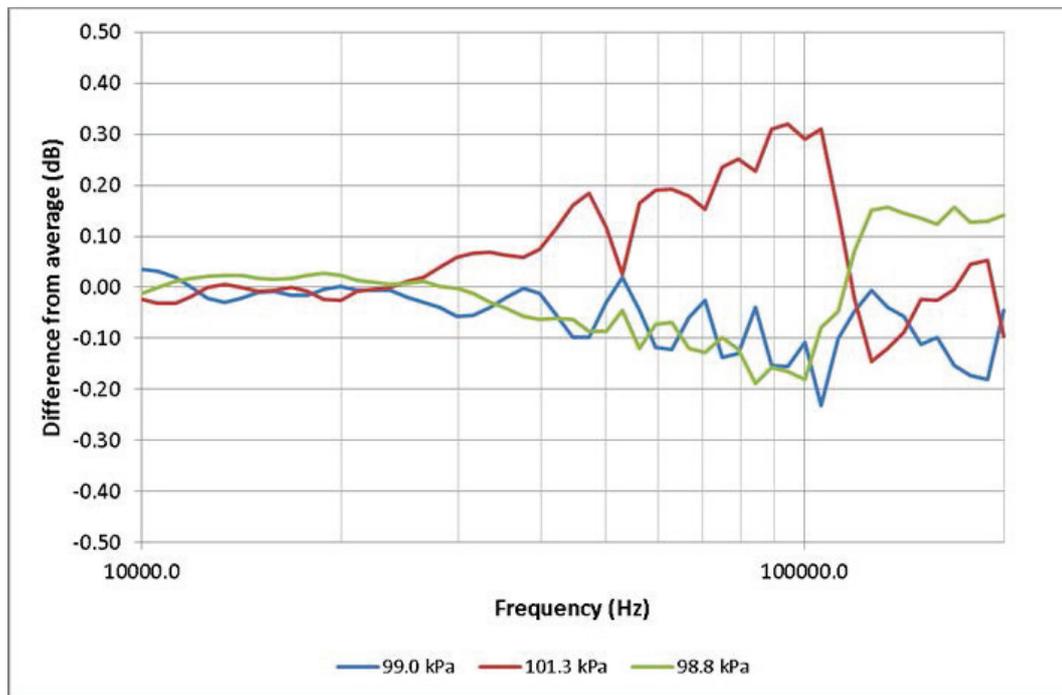


Figure 1 – Example of the variations in the free-field sensitivity measured within a few days interval.

This paper presents an introductory study of the determination of the environmental coefficients of WS3 microphones. For this purpose, the static pressure coefficient of a commercially available WS3 microphone has been determined experimentally, and compared with the coefficients determined numerically using the Boundary Element method. The results demonstrate the importance of knowing the frequency dependence of the coefficients, and illustrate the fact that low-frequency values may not be enough.

## 2. IMPLEMENTATION OF VISCO-THERMAL LOSSES IN BEM

The numerical implementation of acoustic viscous and thermal losses is based on the linearized Navier-Stokes equations: (4)

$$\begin{aligned}
 \frac{\partial \rho}{\partial t} + \rho_o \nabla \vec{v} &= 0 & \rho_o T_o \frac{\partial s}{\partial t} &= \lambda \Delta T \\
 \rho_o \frac{\partial \vec{v}}{\partial t} &= -\nabla p + \left( \eta + \frac{4}{3} \mu \right) \nabla (\nabla \vec{v}) - \mu \nabla \times (\nabla \times \vec{v}) \\
 s &= \frac{C_p}{T_o} \left( T - \frac{\gamma - 1}{\beta \gamma} p \right) & \rho &= \frac{\gamma}{c^2} (p - \beta T)
 \end{aligned}
 \tag{1a-1e}$$

Equations (1a) to (1e) represent respectively conservation of mass, conservation of energy, conservation of momentum and thermodynamic conditions. The symbols are:  $\rho$ , density,  $\rho_o$ , static density,  $\vec{v}$ , particle velocity,  $T$ , temperature,  $T_o$ , static temperature,  $s$ , entropy,  $\lambda$ , thermal conductivity,

$p$ , sound pressure,  $\eta$ , bulk viscosity,  $\mu$ , coefficient of viscosity,  $C_p$ , specific heat at constant pressure,  $\gamma$ , ratio of specific heats and  $\beta$ , rate of increase of pressure with temperature at constant volume. In addition to linear variations, no flow, homogeneous fluid and dimensions of the setup and wavelength larger than the molecular mean free path ( $\sim 10^{-7}$  m) are assumed.

The BEM implementation with losses is based on the Kirchhoff derivation of the Navier-Stokes equations (1a-1e): (4,5)

$$\begin{aligned} (\Delta + k_a^2)p_a &= 0 \\ (\Delta + k_h^2)p_h &= 0 \\ (\Delta + k_v^2)\vec{v}_v &= \vec{0} \quad \text{with} \quad \nabla \cdot \vec{v}_v = 0 \end{aligned} \tag{2a-2c}$$

Eqs. (2a-2c) represent, the so-called acoustic ( $a$ ), thermal ( $h$ ) and viscous ( $v$ ) modes, which can be dealt with independently in the acoustic domain. They are however linked through the temperature and velocity boundary conditions. The pressure is separated into two components  $p = p_a + p_h$ , while the velocity is separated into three components  $\vec{v} = \vec{v}_a + \vec{v}_h + \vec{v}_v$ . The viscous velocity or rotational velocity does not have a corresponding pressure. Correspondingly, the wavenumbers  $k_a$ ,  $k_h$  and  $k_v$  are based on the lossless wavenumber  $k$  and physical properties of the fluid, such as the viscosity, bulk viscosity and thermal conductivity coefficients, air density, and specific heats. Eq. (2a) is a wave equation, while Eqs. (2b-2c) are diffusion equations, given the large imaginary part of  $k_h$  and  $k_v$ . Eq. (2c) is a vector equation and therefore can be split into three components, giving a total of five unknowns.

The implementation in BEM is made by discretizing equations (2a-2c) one by one and combining them into a single matrix equation using the boundary conditions. The matrix equation is solved for the acoustic pressure  $p_a$  and subsequently other variables are obtained on the boundary. From the boundary values, any domain field point can be calculated. (6, 7)

Since the equations (2a-2c) are formally equivalent to the lossless Helmholtz harmonic wave equation, the BEM implementation with visco-thermal losses is built using the OpenBEM research software, which numerically solves acoustic problems in fluids.(8)

In this paper, fully rotational symmetry is assumed, i.e. excitation, boundary conditions and geometry only depend on the  $\rho$  and  $z$  cylindrical coordinates. The boundary to be meshed is defined by the generator line on the  $\rho$ - $z$  plane, thus reducing the computational load.

### 3. THE NUMERICAL MODEL

The Brüel & Kjær type 4939 microphone is a ¼ inch measurement microphone that is meant for free field measurements.(9) This microphone has no holes on the back plate and can therefore be described with an axisymmetrical BEM model with viscous and thermal losses. The nominal parameters of the B&K 4939 are listed in Table 1. A picture of this type microphone is shown in Figure 2. The BEM meshes employed to model the microphone’s internal and external boundaries are also shown.

Table 1 – Design parameters of the Brüel & Kjær 4939 microphone

Membrane radius (mm)	2
Back plate radius (mm)	1,75
Gap thickness ( $\mu\text{m}$ )	18
Membrane tension (N/m)	1039
Membrane density ( $\text{kg/m}^3$ )	8300
Membrane thickness ( $\mu\text{m}$ )	2,35

The full model of the microphone is the result of coupling three domains: i) the interior of the microphone using BEM with losses, ii) the membrane modeled using the Finite Element Method (FEM), and iii) the exterior domain, modeled using lossless BEM. The model can be excited either with a uniform pressure on the membrane, resembling an electrostatic actuator, or with an axial plane wave as in free-field conditions. The actuator excitation will be used for the study in this paper.

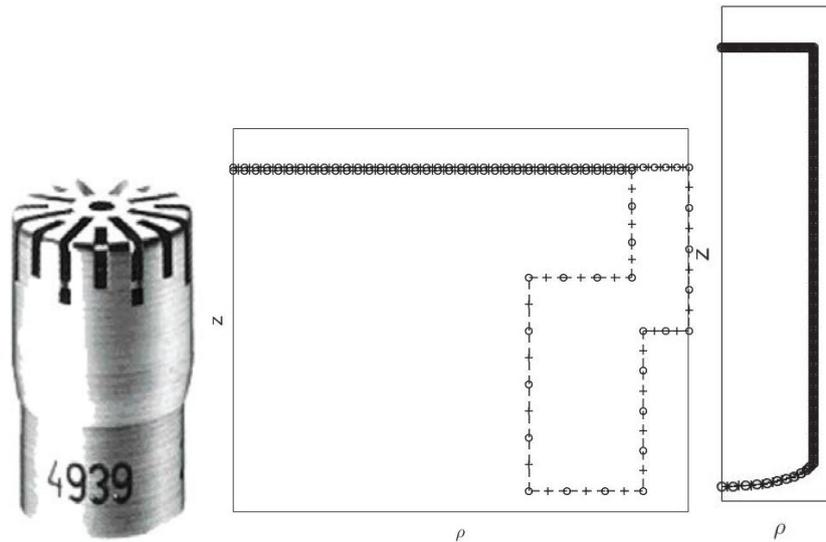


Figure 2 – From left to right, a picture of the type B&K 4939 microphone and the generator meshes used in BEM for the interior and exterior domains (not to scale).

### 3.1 FEM model of the diaphragm

The diaphragm is modeled as a membrane with no stiffness, following the equation:

$$\frac{\partial^2 \varepsilon}{\partial r^2} + \frac{1}{r} \frac{\partial \varepsilon}{\partial r} + K^2 \varepsilon = \frac{P_d}{T} \quad (3)$$

where  $\varepsilon$  is the normal displacement,  $K$  is the wavenumber of the mechanical wave,  $T$  is the membrane tension and  $p_d$  is the sum of sound pressures acting on the diaphragm, internal and external. (10,11) The boundary condition is, as in the actual microphone, no movement at the rim (clamped,  $\varepsilon = 0$ ). This equation is implemented using a one-dimensional Finite Element Method implementation producing the matrix equation:

$$\left( \mathbf{A}_m + K^2 \mathbf{B}_m \right) \varepsilon = \frac{P_d}{T} \quad (4)$$

Note that, as in the BEM implementation of the microphone's interior and exterior domains, no variation over the rotational direction is assumed for the membrane, which is a valid hypothesis for the purpose of this paper.

### 3.2 Calculated microphone sensitivity

Figure 3 shows the calculated frequency response of the B&K 4939 model for the specified parameters. The values are within what should be expected for this microphone type. Actual microphones of the same type differ from unit to unit: each microphone is unique. There is a significant variation and it is not possible to match the model with a specific measurement, since the microphones are adjusted during production and present tolerances.

In this paper we are concerned with the variation of the microphone response with ambient conditions, which will influence air density, speed of sound, and viscous and thermal constants. The model is suitable for this kind of study, since the ambient conditions are part of the model parameters. The equivalence between physical parameters and ambient conditions is calculated using metrological equivalences (12). Figure 4 shows the static pressure and temperature coefficients for the pressure (actuator) sensitivity calculated numerically. . These coefficients are estimated by simulating the microphone response at different static pressures or temperatures, and performing a linear regression between the different microphone responses as a function of the parameter of interest, namely, static pressure or temperature.

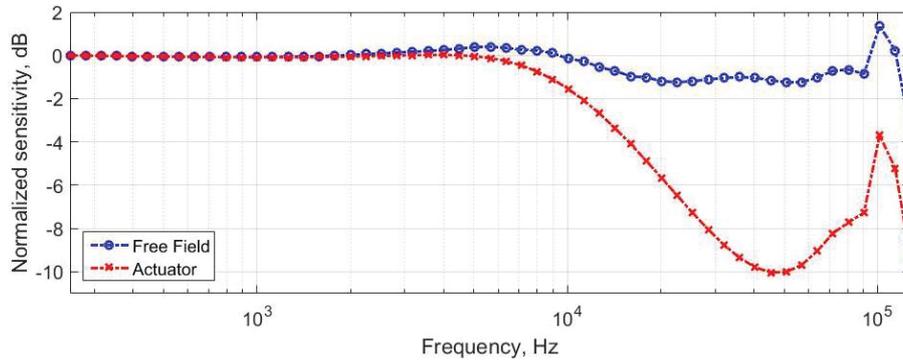


Figure 3 – Calculated sensitivity of the B&K 4939 microphone under free field and actuator excitations.

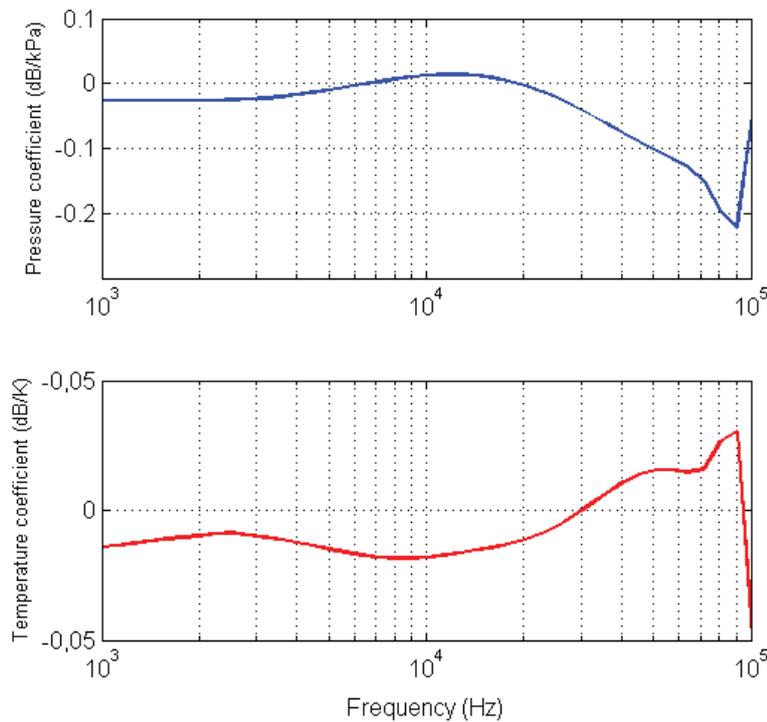


Figure 4. Calculated static pressure and temperature coefficients for the pressure (actuator) response of WS3 microphones Brüel & Kjær type 4939

#### 4. EXPERIMENTAL SET-UP

The environmental coefficients for the pressure sensitivity of Laboratory Standard microphones are determined from reciprocity measurements. This cannot be applied to all types of microphones because of the geometry of the Working Standard microphones though adapters can be made for the purpose. A more readily available method is to use an electrostatic actuator. This method is more versatile than reciprocity in a closed coupler, both in terms of the types of microphones and the frequency range. However, it must be taken into account that the radiation impedance will load the microphone impedance, resulting in a slightly different response around the resonance of the microphone compared to reciprocity. Figure 5 shows a schematic representation of the measurement system used in this exercise. The microphone under test is attached to a preamplifier (Brüel & Kjær type 2669), and exposed to the electrostatic pressure generated by a ½-inch actuator (Brüel & Kjær type UA0033); an adapter is needed when measuring WS3 or ¼-inch microphones. The alternate signal for the actuator, typically a stepped sine, is provided by a sound analyser (Brüel & Kjær PULSE); the frequency range for this exercise is from 251.19 Hz to 100 kHz in 1/12th-octave frequency step. The same analyser is used for measuring the output signal of the microphone under test. The set of preamplifier-microphone and actuator are placed inside a closed chamber that can be

pressurised, or depressurised to reach barometric pressures ranging from 80 kPa to 110 kPa. Eventually, the same set will also be placed within a temperature controlled chamber. The static pressure coefficient at all frequencies can then be calculated using simple linear regression procedures.

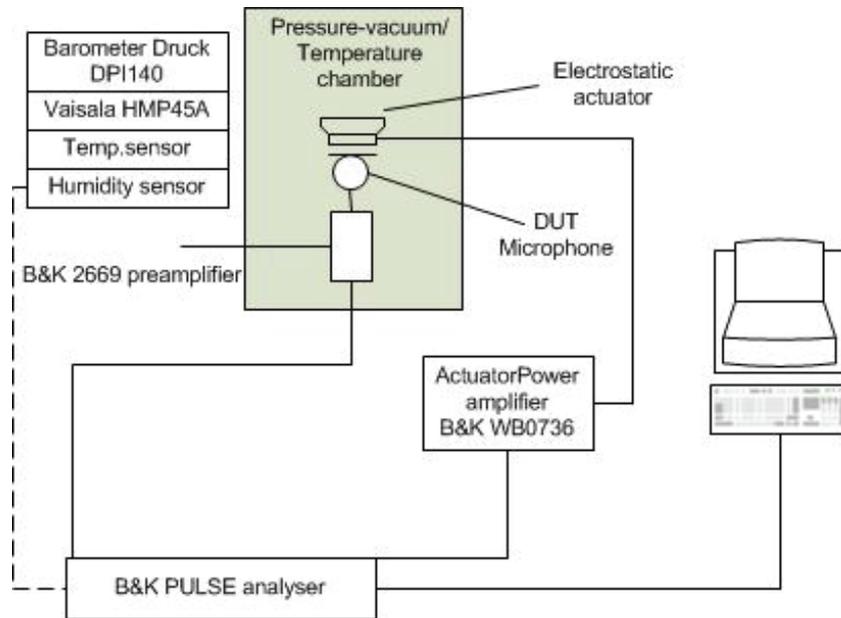


Figure 5. Schematics of the measurement system

Although the environmental coefficients for free-field sensitivity will not be discussed in this paper, their experimental determination is based on the fact that the pressure and free-field sensitivity are related through

$$M_f = M_p S(f, \theta) \frac{Z_a}{Z_a + Z_{a,r}}, \quad (5)$$

where  $S(f, \theta)$  is the diffraction factor,  $Z_{a,r}$  is the acoustic radiation impedance, and  $Z_a$  is the impedance of the microphone. The effect of these additional factors can be used to estimate the environmental coefficients of the free-field sensitivity by estimating their behaviour due to changes in the environmental conditions.

## 5. RESULTS

Figure 6 shows the experimental and numerical actuator response of the microphone. While the agreement is reasonable, it is clear that some parameters of the numerical calculation need to be adjusted in order to improve the fitting. The observed difference can also result in differences in the estimated pressure coefficients.

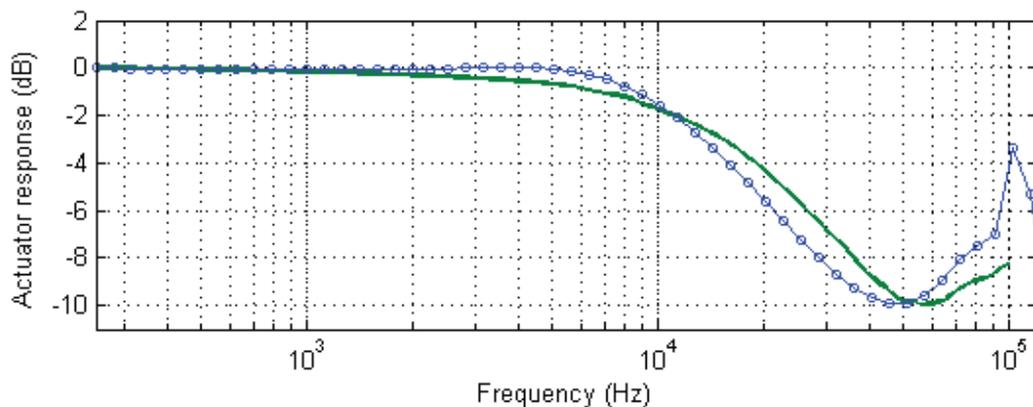


Figure 6 – Experimental and numerical results of the actuator response of the WS3 microphone Brüel & Kjaer type 4939. BEM: Blue line with circle markers; Experimental: continuous green line.

Figure 7 below presents the static pressure coefficient for the actuator response of WS3 microphones Brüel & Kjær type 4939 in the frequency range from 1 kHz to 100 kHz. The figure also contains the pressure coefficient determined from the BEM simulations.

As can be seen in the figure the coefficient has a pronounced frequency dependence, which in turn, indicates that changes in the pressure response may not be negligible for this type of microphones within this frequency range. The values of the coefficients, combined with the changes of the static pressure registered in the measurements shown in Figure 1 may partially explain the differences observed in the responses, and eventually correct them.

The frequency dependence of the coefficient seems also to share some similarities with other types of microphones of other sizes (WS2 and LS2) (3). This is not unexpected because the coefficient depends on the impedance of the microphone, and its frequency dependence.

The agreement between the numerical and experimental results seems to be reasonable considering that the microphone parameters used in the calculations may not be fully accurate. Additional fitting of the parameters may yield a smaller difference between experiments and simulations.

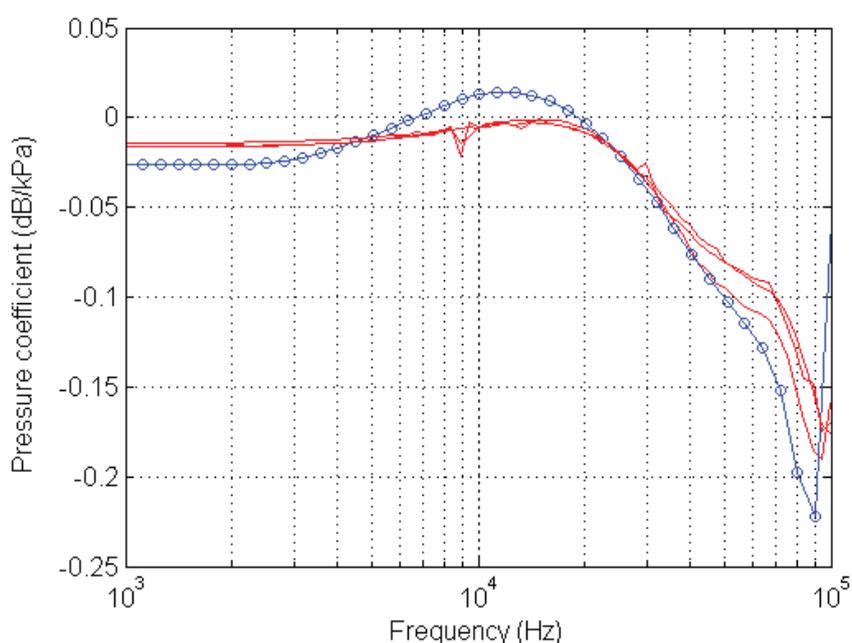


Figure 7 – Experimental and numerical results of the pressure coefficient for the actuator response of the WS3 microphone Brüel & Kjær type 4939. The blue line with circle markers represents the BEM results while the red lines represent the values from three different 4939 microphones.

## 6. CONCLUSIONS

The static pressure coefficients for pressure and free-field sensitivities of WS3 microphones have been determined numerically using a formulation of the Boundary Element Method that includes the viscous losses occurring in the interior of the microphone.

The static pressure coefficient for the pressure sensitivity of WS3 microphones has been determined experimentally using the electrostatic actuator method. There is a good agreement between the numerical and experimental estimates although further refinement of the numerical model may be needed to improve the calculations.

The low frequency value of the coefficients may not sufficiently express the changes occurring in the whole frequency range. Frequency dependence should be made available.

## ACKNOWLEDGEMENTS

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