A numerical method for determining the radial wave motion correction in plane wave couplers

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ABSTRACT
Microphones are used for realising the unit of sound pressure level, the pascal (Pa). Electro-acoustic reciprocity is the preferred method for the absolute determination of the sensitivity. This method can be applied in different sound fields: uniform pressure, free field or diffuse field. Pressure calibration, carried out in plane wave couplers, is the most extended. Here plane wave propagation is assumed. While this assumption is valid at low and mid frequencies, it fails at higher frequencies because the membrane of the microphones is not moving uniformly, and there are viscous losses. An existing solution is an analytical expression that estimates the difference between the ideal plane wave sound field and a more complex lossless sound field created by a non-planar movement of the microphone’s membranes.

Alternatively, a correction may be calculated numerically by introducing a full model of the microphone-coupler system in a Boundary Element formulation. In order to obtain a realistic representation of the sound field, viscous losses must be introduced in the model. This paper presents such a model, and the results of the simulations for different combinations of microphones and couplers. The results are compared to experimental data, and the existing analytical solution.

Keywords: BEM, metrology, microphones I-INCE Classification of Subjects Number(s): 75.5, 71.1.1

1. INTRODUCTION

Condenser microphones are preferred for acoustic measurement and calibration due to its stability, frequency range and sensitivity. The sensing element is a metallic membrane that is stretched over a cavity and very close to a back electrode plate, forming a capacitor. The capacitor is polarized with a DC voltage, either using an external source or an electret film, in order to avoid transduction non-linearities. (1) The membrane has its first resonance on the high part of the device’s frequency range, and it is damped by the viscous and thermal losses in the gap between membrane and back plate. Damping and resonance are carefully balanced by controlling membrane tension, gap width and by boring holes in the backplate. It is therefore a highly coupled device, which poses a challenge when creating models that can describe its behavior in detail. (2, 3)

Microphone calibration using the reciprocity technique in couplers assumes plane wave propagation, but, in reality, radial wave motion will exist and affect the final calibration uncertainty. Although analytical corrections have been proposed in order to reduce the influence of this effect (4), these corrections are derived assuming a velocity distribution of the microphone diaphragms that might deviate from the actual velocity distribution of the measured microphones. Alternatively, the present study investigates the potential of modeling the microphone-coupler system using a Boundary Element formulation. In this way, one can avoid the use of a generalized velocity distribution for describing the radial motion of the microphones’ diaphragms and account for the individual velocity fluctuations.

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The Boundary Element Method (BEM) solves the acoustic harmonic wave equation in its integral form: the Helmholtz Integral Equation (HIE). The HIE relates the acoustic pressure on any point in the domain to the pressure and normal particle velocity on the domain’s boundaries. By discretizing the HIE, the BEM can solve the acoustic variables on the boundary, given a set of Dirichlet, Neumann or Robin boundary conditions. The sound field in the whole domain can be calculated by using the HIE again with the calculated boundary values.

In the past few years both BEM and Finite Element Method (FEM) implementations including viscous and thermal losses in acoustic simulations have been proposed. The BEM implementation in its three-dimensional version is employed in this paper.

2. IMPLEMENTATION OF VISCO- THERMAL LOSSES IN BEM

The numerical implementation of acoustic viscous and thermal losses is based on the linearized Navier-Stokes equations:

$$\frac{\partial \rho}{\partial t} + \rho \nu \nabla v = 0 \quad \rho \nu \frac{\partial T}{\partial t} = \lambda \Delta T$$

$$\rho \nu \frac{\partial v}{\partial t} = -\nabla p + \left( \eta + \frac{4}{3} \mu \right) \nabla \left( \nabla v \right) - \mu \nabla \times \left( \nabla \times v \right)$$

$$s = \frac{C_p}{T} \left( T - \frac{\gamma - 1}{\beta \gamma} p \right) \quad \rho = \frac{C_p}{\gamma} (p - \beta T)$$

Equations (1a) to (1e) represent respectively conservation of mass, conservation of energy, conservation of momentum and thermodynamic conditions. The symbols are: $\rho$, density, $\rho_0$, static density, $v$, particle velocity, $T$, temperature, $T_0$, static temperature, $s$, entropy, $\lambda$, thermal conductivity, $p$, sound pressure, $\eta$, bulk viscosity, $\mu$, coefficient of viscosity, $C_p$, specific heat at constant pressure, $\gamma$, ratio of specific heats and $\beta$, rate of increase of pressure with temperature at constant volume. In addition to linear variations, no flow, homogeneous fluid and dimensions of the setup and wavelength larger than the molecular mean free path ($\sim 10^{-7}$ m) are assumed.

The BEM implementation with losses is based on the Kirchhoff derivation of the Navier-Stokes equations (1a-1e): (9, 10)

$$\left( \Delta + k_a^2 \right) p_a = 0$$
$$\left( \Delta + k_h^2 \right) p_h = 0$$
$$\left( \Delta + k_v^2 \right) \vec{v}_v = 0 \quad \text{with} \quad \nabla \cdot \vec{v}_v = 0$$

Eqs. (2a-2c) represent, the so-called acoustic ($a$), thermal ($h$) and viscous ($v$) modes, which can be dealt with independently in the acoustic domain. They are however linked through the temperature and velocity boundary conditions. The pressure is however coupled into two components $p = p_a + p_h$, while the velocity is separated into three components $v = v_a + v_h + v_v$. The viscous velocity or rotational velocity does not have a corresponding pressure. Correspondingly, the wavenumbers $k_a$, $k_h$ and $k_v$ are based on the lossless wavenumber $k$ and physical properties of the fluid, such as the viscosity, bulk viscosity and thermal conductivity coefficients, air density, and specific heats. Eq. (2a) is a wave equation, while Eqs. (2b-2c) are diffusion equations, given the large imaginary part of $k_h$ and $k_v$. Eq. (2c) is a vector equation and therefore can be split into three components, giving a total of five unknowns.

The implementation in BEM is made by discretizing equations (2a-2c) one by one and combining them into a single matrix equation using the boundary conditions. The matrix equation is solved for the acoustic pressure $p_a$ and subsequently other variables are obtained on the boundary. From the boundary values, any domain field point can be calculated.

Since the equations (2a-2c) are formally equivalent to the lossless Helmholtz harmonic wave equation, the BEM implementation with visco-thermal losses is built using the OpenBEM research software, which numerically solves acoustic problems in fluids. The implementation used in this paper is fully three-dimensional. Some examples can be found in (12).
3. THE NUMERICAL MODEL

Primary pressure calibration by reciprocity makes use of a set of plane-wave couplers of different lengths. Two microphones of the same diameter as the coupler are mounted on both ends, as shown in figure 1. The microphones are assumed to be reciprocal, acting either as sources or detectors. In a given measurement, one microphone (emitter) is excited as a source and the other produces an output (receiver).(13)

In metrology, the microphone sensitivities are determined considering a simplified model of the interactions between the different parts of microphone-coupler system. The numerical model in this paper, however, takes no assumption: full coupling, where all parts interact, is considered.

Figure 1 – Sketch of the setup in the BEM model. It contains five coupled domains: emitter microphone, receiving microphone, their two membranes, and the coupler cavity connecting them. The cavity has the same radius as the membranes and four different lengths.

The setup is fitted with two one-inch B&K type 4160 microphones, specifically designed for this purpose. In practice, one of the microphones may be replaced by another one-inch microphone to be calibrated. The BEM model of the B&K 4160 has been described in detail in (14), where it was modeled with an exterior unbounded domain (free-field conditions). Figure 2 shows the microphone and the model of the internal back electrode, which has 19 holes of different sizes.

Figure 2 – B&K type 4160 calibration microphone. Left: exterior appearance. Right: detail of the back electrode, taken from the BEM mesh.
The membrane is modeled using a two-dimensional Finite Element Method (FEM) model written for this purpose in Matlab; no stiffness is assumed. The equation of the membrane is: (2)

\[ T\left(\Delta + K^2\right)\varepsilon = p_d \]  

(3)

In (3) \( \varepsilon \) is the normal displacement, \( K \) is the wavenumber of the mechanical wave, \( T \) is the membrane tension and \( p_d \) is the sum of sound pressures acting on the diaphragm, internal and external. After discretization using FEM, a matrix equation is obtained:

\[ \left(A_m + K^2B_m\right)\varepsilon = \frac{p_d}{T} \]  

(4)

The membrane deflection due to the static polarization voltage is calculated separately using this same FEM model, and added as a deformation to the membrane mesh in contact with the B&K 4160 interior domain. Figure 3 shows the shape of the deformation, where the change of the condenser capacitance has been taken into account. The deflection is close to 3 µm at the center, which is significant as compared with the membrane-back plate gap of around 20 µm. The size and holes in the back plate are taken into account when setting up the electrostatic force excitation in the model.

![Figure 3 – Calculated membrane static deflection in two plots: as a function of the radius (upper) and as a tree-dimensional shape (lower).](image)

The coupler domain is a cylinder with a radius of 9.3 mm, a little larger than the membrane radius of 9 mm. Four different coupler lengths have been used, corresponding to the actual couplers employed in the laboratory: 3.5 mm, 5.5 mm, 7.5 mm and 9.5 mm. Each of the B&K 4160 microphones has a cylindrical front cavity of the same radius and 1.95 mm deep that adds to the cylindrical space formed by the coupler.

The emitter microphone is excited by prescribing a harmonic pressure on the parts of the membrane that are directly over the back plate, that is, excluding the holes and rim. The membrane movement in the receiver microphone is averaged on the same parts of the membrane, thus imitating the transduction principle of the condenser microphone.

The results of the simulation should be expressed in a way that can be compared with measurement results. In practice, it is only input voltages and currents that are available to the experimenter, whereas the numerical model can show pressures and particle velocities anywhere in all modeled domains. In this paper, the ratio (volume velocity on the emitter membrane) over (average pressure on the receiver membrane) inside the coupler has been chosen as a measure of the model performance.
This quantity, commonly referred to as the acoustic transfer admittance of the coupler, can be derived from the measurements, as will be shown later on. Figure 4 shows a BEM simulation of the acoustic transfer admittance between two microphones B&K type 4160, when they are calibrated by reciprocity technique in four couplers with different lengths. As can be seen, the length of the cavity (together with the front cavity of each of the microphones) has a strong influence on the absolute value of the acoustic transfer admittance. The fluctuations observed around 8 kHz are possibly the consequence of some meshing errors that were discovered late in the numerical model development. It should be noted that no velocity distribution of the microphone membranes has been assumed in these numerical calculations.

Figure 4 – BEM calculation of the acoustic transfer admittance between two microphones B&K type 4160 in four different couplers.

In practice, this fully coupled BEM model of a pressure calibration system is on the limit of what is achievable with numerical methods of acoustics with losses. Refinement and testing is cumbersome due to the heavy computational load, and the fact that it is too difficult or impossible to measure inside the actual setup. As will be shown in the following, only voltages or currents at the terminals are available, which must be complemented with analytical models.

4. ANALYTICAL SOLUTION

The transmission line theory used to model the acoustic transfer admittance in a coupler assumes that the microphones behave as ideal, rigid pistons that have the same diameter as the cavity of the coupler. Under such conditions, only plane waves can exist inside the coupler. However, the microphones are neither ideal pistons nor match perfectly the dimensions of the coupler, meaning that radial motion might occur inside the coupler. This introduces uncertainty when relating the electrical measurements of the reciprocity technique with the pressure sensitivities of the microphones. Figure 5 shows the idealized acoustic transfer admittance obtained with the conventional transmission line theory in four different couplers. In order to compensate for the possible influence of radial motion, reference (4) suggests an analytical correction for the acoustic transfer admittance that is based on a Bessel-function velocity distribution. The resulting acoustic transfer admittance can be seen in Figure 6, and the actual analytical correction as a function of frequency is shown in Figure 7. As can be seen, the proposed analytical correction becomes more important at high frequencies and for short couplers.
Figure 5 – Ideal acoustic transfer admittance based on conventional transmission line theory.

Figure 6 – Acoustic transfer admittance compensated for the radial motion inside the coupler. The correction assumes that the microphone membranes follow a Bessel-function velocity distribution.
5. PRESSURE CALIBRATION SETUP

Pressure sensitivity determined using the reciprocity technique is a function of the electrical transfer impedance, that is the ratio of the open-circuit voltage at the terminals of the microphone acting as receiver to the current flowing through the microphone used as transmitter (or sound source). As mentioned before, the microphones are coupled using a cylindrical coupler of about the same internal diameter as the diameter of the membrane. The determination of the pressure sensitivity requires an estimate of the acoustic transfer impedance which is typically calculated as described in Section 4. The underlying theory relates the sensitivities of the transmitter and receiver microphone to the two transfer impedances by

\[
M_1M_2 = \frac{1}{Z_{12}} \frac{u_2}{i_1}.
\]

Assuming that the sensitivities of the microphones are known, the acoustic transfer impedance (or admittance) can be determined from

\[
\frac{1}{Z_{12}} = \frac{i_1}{u_2} M_1M_2.
\]

The electrical transfer impedance can be measured using the reciprocity calibration system. A schematic representation of the measurement set-up is shown in Figure 8. Details of the reciprocity calibration method can be found elsewhere. (13)

The system microphone-coupler-microphone is placed inside a bell that can be pressurised if needed, and that effectively shields the coupled system from the background noise. Temperature, relative humidity and static pressure sensors are also located inside the bell. The measurement actions and instructions are controlled by a computer via IEEE interface, and TCP/IP protocols. The current flowing though the transmitter microphone cannot be measured directly, but on the terminals of a reference capacitor connected in series with the microphone. The ratio of the voltage on the receiver to the voltage on the terminals of the reference capacitor is measured using a signal analyser and sinusoidal signals in the frequency range from 20 Hz and up to 25 kHz in 1/12th-octave steps depending on the coupler length. The couplers used in a typical calibration have nominal lengths of 3.5 mm, 5.5 mm, 7.5 mm, and 9.5 mm.

Figure 7 – Analytical correction based on Bessel-function velocity distribution.
6. DISCUSSION OF RESULTS

Using Equation (6) for determining the acoustic transfer admittance is apparently straightforward, however, there are some considerations to be made. On the one hand, the measured ratio of voltages contains information about the sound field generated inside the coupler and hence about the deviations of the sound field with respect to a plane wave field. This information cannot be extracted directly by any means. However, working around some assumptions on the pressure sensitivity of the microphones, some observations can be made.

For instance, if the sensitivities used for the calculation are the true pressure sensitivities, that is, sensitivities obtained when a true plane wave is propagating inside the coupler, the acoustic transfer admittance corresponds to the acoustic transfer admittance corrected for radial wave motion. Clearly, the true pressure sensitivity cannot be known as such, but a good approximation is the pressure sensitivity determined using the transmission line acoustic transfer admittance corrected for radial wave motion in the reciprocity calibration, or, alternatively using the microphone pressure response obtained with an actuator. In this paper the former approach is used; this is shown in Figure 9.

One can normalize the acoustic transfer admittance determined above to the ideal transfer admittance in order to compare it with the analytical solution. This comparison is shown in Figure 10, where good agreement is observed with the analytical result for the shorter coupler, while the corrections differ for the other couplers around and above the resonance frequency. This can be due to the fact that the microphone impedances used in the calculation of the analytical correction are determined from a lumped-parameter model, as opposed to the ratio of voltages, which contains information about the true impedances. Another matter is that viscous losses are not considered in the analytical solution.
Figure 9 – Acoustic transfer admittance determined from the ratio of voltages measured in four different couplers and true pressure sensitivities. This admittance includes the influence of the radial motion effect inside the coupler. This figure should be compared to Figure 6, as the latter attempts at accounting for the radial motion effect using a Bessel-function velocity distribution.

Figure 10 – Correction for the acoustic transfer admittance determined analytically (solid lines) and experimentally (dashed lines) with respect to the idealized acoustic transfer admittance defined by the transmission line theory. It should be noted that the analytical correction does not account for heat conduction effects and other possible effects beyond the radial motion problem.

On the other hand, if the sensitivity used in the calculation is a raw sensitivity or imperfect sensitivity, for example, one determined from reciprocity measurements without compensating for any imperfection in the theory, including radial wave motion, the resulting acoustic transfer admittance will correspond to the one determined using the conventional transmission line theory; this is
comparable with the results shown in Figure 5. The experimental result is shown in Figure 11.

Finally, the radial wave motion correction determined from the numerical simulations is shown in Figure 12. The results show that the numerical model requires additional fitting in order to reach a better approximation. This can certainly be due to inaccuracies in the provided geometry of the microphone under study. Unfortunately, this does prevent the reaching of any conclusion but leaves room for improvement.

Figure 11 – Acoustic transfer admittance determined from the ratio of voltages measured in four different couplers and raw or imperfect pressure sensitivities. This admittance does not include the influence of the radial motion effect inside the coupler. This figure should be compared to Figure 5.

Figure 12 – Difference between the acoustic transfer admittance determined by BEM and the idealized acoustic transfer admittance obtained with the transmission line theory. The BEM formulation needs to be further adjusted in order to achieve reliable results.
7. **CONCLUSIONS AND FUTURE WORK**

The acoustic transfer admittance between two LS1 microphones Brüel & Kjaer type 4160 has been determined using analytical, experimental and numerical methods.

The numerical model shows the potential of becoming a valid tool in acoustic primary calibration, but it still needs refinement. It is hoped that an improved implementation will be useful to advance understanding of the calibration system in details that are not available to the experimenter. Eventually, the analytical models will benefit, leading to an improvement of the uncertainty of acoustic measurements.

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**REFERENCES**


