Investigation of hysteresis friction in elements under complex stress state

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ABSTRACT

Recent studies have indicated that the best type of damping in vibration isolation system is hysteresis (internal) damping, which effectively reduces vibration amplitudes at resonance, and it does not increase as compared with viscous damping vibration amplitudes at super-resonance. Modern damping investigation methods are based on the experimental determination of the loss factor for a certain form of vibration isolators with fixed dimensions and loading parameters. At the same time, with the emergence of complex nonlinear vibration isolators, such as discussed by the authors, there is a task of expanding this theory on the case of complex designed vibration isolators. In this paper we consider the problem of calculating the loss factor in the beam - columns of variable cross-section under complex loading conditions. A method for calculating the amount of losses in the nonlinear vibration isolators is based on Panovko's energy theory, which consists of experimental loss factor determination in the material of the vibration isolator and subsequent calculation of the loss factor in the isolator in view of obtained data.

Keywords: damping, hysteresis friction, stress state

1. INTRODUCTION

Currently, a lot of work is devoted to the protection of high – precision equipment from both ambient vibration, micro seismic and seismic activity (4, 5, 6). Experiments shows, that damping of structural metals is very low (according to (1): steel 0,0001; brass 9×10^{-5} and aluminum alloy $<10^{-5}$). Therefore, structural damping is obtained through additional materials with high damping (e.g., rubber pads) or specially designed structures (e.g. viscous dampers). Because vibration isolators of precision equipment have to be effective in the range of 1 – 100 Hz (4, 5), their damping characteristics should possess high values at resonance and low values beyond one. This factor defines the use of dampers with high internal friction, which may in particular be achieved by using compressed beams (1, 2, 8). One of the vibration isolator types is an isolator, consisting of compressed steel beam that supports the isolated equipment (7, 8). At the moment, the problem of computing the elastic curve of such isolators is well-studied. However, less attention is given to the problem of calculating the damping characteristics of such isolators. Some authors (1, 7) investigate the influence of internal friction on oscillations of compressed bars, but they measure damping with $\tan\varphi$ parameter. Therefore, they need to perform independent experimental investigations for each type of boundary conditions and cross section type. This paper focuses on the energy model of internal friction, developed in works of Panovko (2). We investigate the influence of internal damping at low vibrations of compressed beam with variable cross section with initial curvature.

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2. PROPOSED ENERGY METHOD

To create a well-posed theory it is important to mention the following experimental fact: energy Ψ_0, dissipated per cycle per unit volume of the material depends only on the amplitude value ε_0 of deformation. Besides this basic qualitative result, experiments have established for a number of materials particular relationships for uniaxial stress state:

$$\Psi_0 = \Psi_0(\varepsilon_0) \tag{1}$$

Usually, the results of experimental investigations are non-dimensionalized and instead of energy dissipation Ψ_0, its ratio to the maximum potential energy of the cycle is used:

$$\psi_0 = \frac{2\Psi_0(\varepsilon_0)}{E\varepsilon_0^2} \tag{2}$$

This ratio, also called the absorption coefficient (2) or loss factor (3), usually depends on the amplitude value of deformation ε_0.

Consider that the vibration mode has the form:

$$X = X(x). \tag{3}$$

We shall normalize it so that the value of X equals unity in a typical section, which deflections are taken as a generalized coordinate. Then the deflection of any section of the beam in the process of oscillations are determined by formula:

$$v(x,t) = X(x)T(t). \tag{4}$$

In Eq. (4) T(t) – is the generalized coordinate, that defines the deflection of the mentioned typical section of the beam. Let the A be the amplitude value of T(t). When the dynamic system is exposed to external excitations, the value of A is constant, but when the system undergo free oscillations, the value of A changes from cycle to cycle. The greatest deviation from the equilibrium position of the system correspond to the deflections:

$$v_{\max} = AX(x). \tag{5}$$

The curvature of the beam in a state of maximum deviation is determined by the approximate expression:

$$\chi = A\frac{d^2X}{dx^2}. \tag{6}$$

Therefore, the amplitude of maximum stress in any point on the beam is determined by:

$$\sigma_0(x,y) = E|\chi y| = EA\left|\frac{d^2X}{dx^2}\right|. \tag{7}$$

As described in (2), further calculations should be based on a specific form of the material dependence – Eq. (1). There are various ways of representing these functions, widely discussed in (2, 3). In our analysis, we will use following simple relation:

$$\Psi_0 = \beta\sigma_0^{n+1}. \tag{8}$$

In Eq. (8) β and n – are the material constants.

Relation (8) can be constructed from the test results, for example from envelope analysis of the beam's free oscillations due to simple constraints, with further approximation of the experimental curve in Curve Fitting Toolbox from MatLab software. According to Eq. (8), Eq. (2) has the form:

$$\psi_0(\sigma_0) = \frac{\Psi_0}{\Pi_0} = 2\beta E\sigma_0^{n+1}. \tag{9}$$

In Eq. (9) $\Pi_0 = \dfrac{\sigma_0^2}{2E}$ - is specific potential energy in the state of maximum deformation. In order to find all the energy dissipated per cycle, substitute in Eq. (8) the expression σ0, given by Eq. (7) and integrate the result over the entire volume:

$$\Psi = \int_V \Psi_0 dV = \beta \left(EA \right)^{n+1} J^* \int_0^l \left| \frac{d^2 X}{dx^2} \right|^{n+1} dx. \quad (10)$$

In Eq. (10) $J^* = J(n) = \int_0^l \int_F |y|^{n+1} dF dx.$ - is a kind of physical – geometric characteristic that depends not only on the shape and cross section size, but also on the beam material. It is important, that the RHS of the equation depends not only on the properties of the material (β and n coefficients), but also on the structure en bloc, besides on the form of the excited vibrations (the latter determines the mode function). In order to get the reduced absorption coefficient, we have to divide Eq. (10) by potential energy of the beam related to the state of the greatest deviation from the equilibrium position, which has the form:

$$\Pi = \int_V \Pi_0 dV = \frac{EJA^2}{2} \int_0^l \left(\frac{d^2 X}{dx^2} \right)^2 dx. \quad (11)$$

After dividing Eq. (10) by Eq. (11) we will get the reduced absorption coefficient for the whole beam:

$$\psi = 2\beta \frac{J^*}{J} E^n A^{n-1} \frac{\int_0^l \left| \frac{d^2 X}{dx^2} \right|^{n+1} dx}{\int_0^l \left(\frac{d^2 X}{dx^2} \right)^2 dx}. \quad (12)$$

The absorption coefficient usually depends not only on the material properties, but also on the considered waveform, which defines the form of the derivative $\frac{dX}{dx}$. In addition, it should be noticed, that when n = 1, we obtain a value of the absorption coefficient of the material $\psi = 2\beta E$. Moreover, Eq. (12) states the fact that the absorption coefficient depends on the amplitude of oscillations.

3. NUMERICAL EXAMPLE

Consider the problem of mass *m* free oscillations on the fixed – fixed compressed beam of variable cross section with initial curvature, which is schematically plotted in Fig. 1.

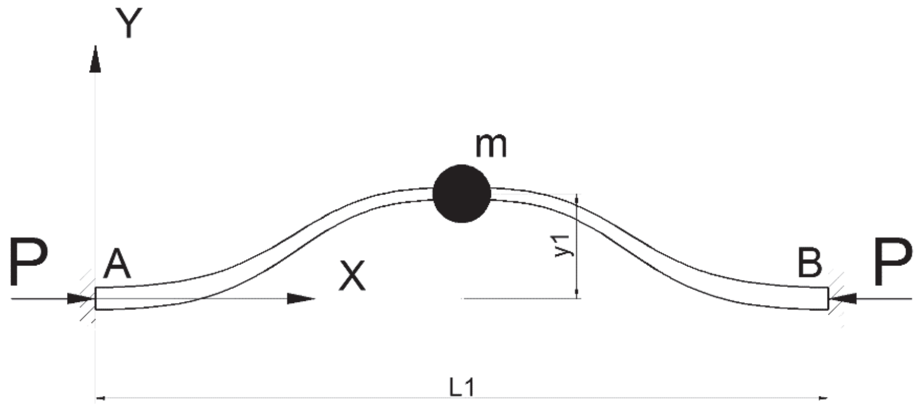


Figure 1 – Scheme of the problem.

Variable beam cross section is described by:

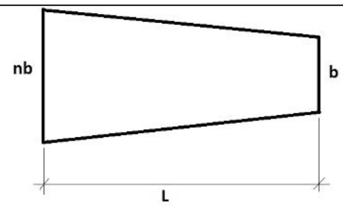
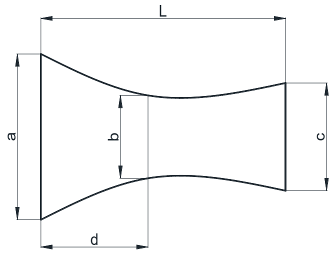
$$EJ(x) = EJ_0 f(x). \quad (13)$$

Consider that the beam's waveform coincides with the curve of static bending subjected to a point force acting in mid span. Since the bending curve in this problem is obtained numerically, it is necessary to represent the Eq. (6) in the finite difference form:

$$A\frac{d^2X}{dx^2} = A\frac{X_{i+1} - 2X_i + X_{i-1}}{h^2}. \tag{14}$$

Then to compute Eq. (12) taking into account Eq. (13) and Eq. (14) we will use formulas for numerical integration like trapezoidal rule or Simpson's rule. Moreover, if the problem involves massive computations, for example, if we need to investigate damping properties for a large number of different beams, parallel integration methods can be applied (9). In the paper, we consider the following types of variable cross-section, presented in Table 1.

Table 1. – Types of variable cross – section.

№№	Description	Scheme
1	Linearly varying cross-section	
2	Parabolically varying cross - section	

The results of calculations are presented on Figs. 2 – 3 for different type of cross section from Table 1. The β and n coefficient were obtained experimentally from beam envelope analysis as stated in (2). For steel alloy we gathered β = 0,0013, n = 1,3.

a) n = 1 b) n = 1.5

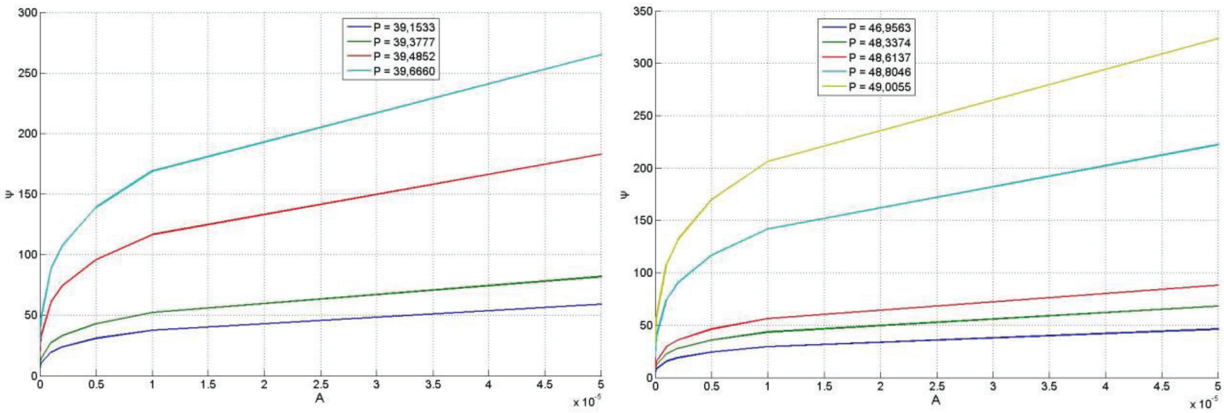


Figure 2 – Curves $\psi - A$ for cross section type №1.

Cross section type №2 from Table 1 can be made symmetrical or asymmetrical. Cross section type influences both the bending curve and absorption coefficient. Curves $\psi - A$ for symmetrical and asymmetrical cross section of type №2 are shown in Fig. 3 in dimensionless form.

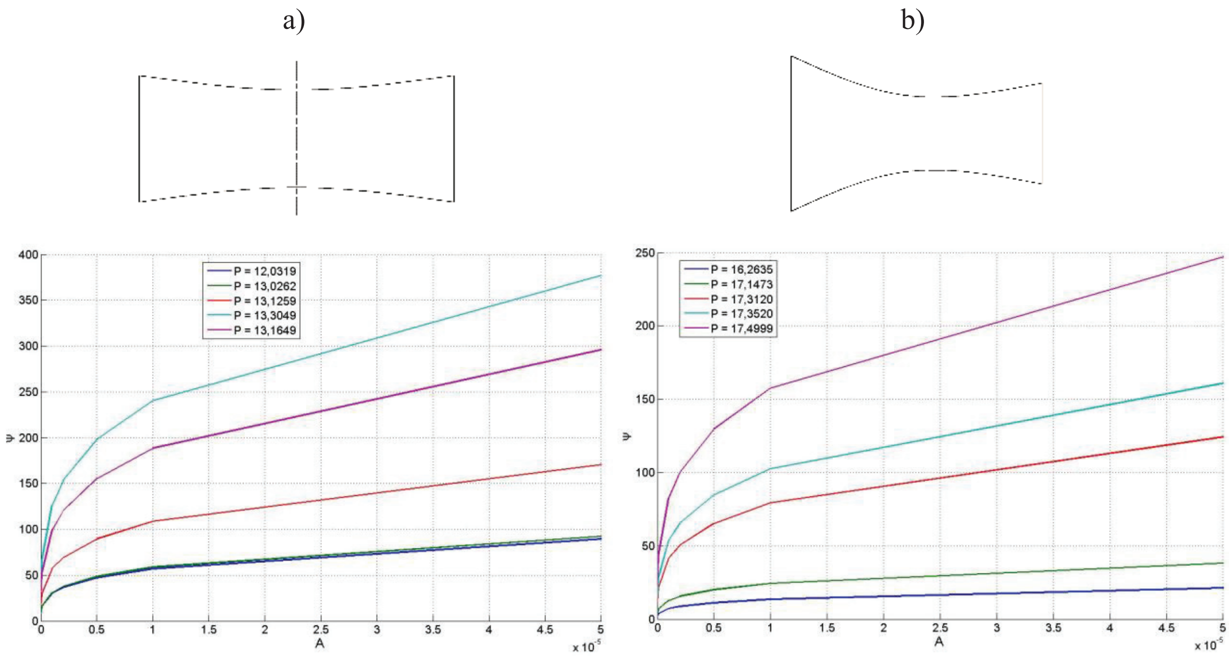


Figure 3 – Curves $\psi - A$ for cross section type №2.

4. RESULTS AND DISCUSSION

The analysis of calculation results shows that the maximum loss factor has a symmetrical beam with variable cross section of type №2 from Table 1. The lowest loss factor has the beam with asymmetrical cross section of type №2. It can also be noted, that the loss factor is greater in those beams, where the compression zone area is larger. Due to the fact, that asymmetrical cross section beams has smaller compression zone area, compression stresses there are lower, which results in lower potential energy of that part of the beam and lower overall loss factor. That is why it is recommended to use symmetrical cross section beams in applications that require higher internal damping. Although beams with variable asymmetric cross section has lower loss factor, their stiffness characteristics can more convenient in some applications, for example they can possess “soft stiffness”, which is favorable for low-frequency vibration isolation purposes. In this case, the process of developing the best isolation system is an optimization problem in which the damping coefficients can be obtained from the calculations presented.

5. CONCLUSION

Energy model for determining the internal friction in the beams takes into account not only the properties of the beam material, but also its boundary conditions and cross section parameters. For this calculation, we do not require numerous experimental studies, which is an advantage of the approach. Thus, the results show that the compressed beam with a variable cross-section have a considerable internal friction, which allows to reduce the amplitude of the resonant oscillation, thereby improving the performance of vibration isolation systems.

REFERENCES

1. Liang Dong, Roderic Lakes. *Advanced damper with high stiffness and nigh hysteresis damping based on negative structural stiffness*. International Journal of Solids and Structures. Vol. 50, pp. 2416 – 2423 (2013).
2. Panovko Ja. G. *Oscillations of elastic systems with internal friction*. FizMatLit (1960).
3. Postnikov V.S. *Internal friction in metals*. Metallurgical Engineering. (1974).
4. Hal Amick, Michael Gendreau, Todd Busch. *Evolving Criteria for Research Facilities: Vibration*. Proceedings of SPIE Conference 5933: Buildings for Nanoscale Research and Beyond, Vol. 5933, pp. 16 – 28 (2005).

5. Eugene I. Rivin. *Vibration isolation of precision objects*. Precision engineering, Vol. 1, pp. 12 – 20 (1995).
6. K. Hasselmann, *A statistical analysis of the generation of micro-seisms*. Review of Geophysics, Vol. 1, issue 2, pp 177–210 (1963).
7. B. Ravindra and A.K. Mallik. *Hard duffing-type vibration isolators with combined coulomb and viscous damping*. Journal of Nonlinear Mechanics. Vol. 28, issue 4, pp. 427–440 (1993).
8. Smirnov V.A. *Nonlinear vibration isolator for kinematic vibration isolation of vibration-sensitive equipment*. Sci-tech journal of Moscow State University of Civil Engineering. Issue 3, Vol. 1, pp. 107 – 112 (2011).
9. Smirnov V.A. *Parallel integration using OpenMP and GPU to solve engineering problems*. Applied Mechanics and Materials. Vols. 475-476, pp. 1190-1194 (2014).