FEA based sound power estimation with non-equidistant frequency steps
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ABSTRACT
The radiated sound power is a common objective for the design of thin-walled and lightweight components with dynamic loading. For the numeric determination there are different approaches based on the normal surface velocity e.g. the equivalent radiated sound power. Thus, the FEA of structural dynamics in frequency domain is used for an efficient power estimation. In detail, steady state dynamics based on modal superposition offer less computational time and the opportunity of composite modal damping. The number of frequency steps is optimised using mode based increments with biasing leading to non-equidistant step sizes throughout the frequency domain. In addition, the number of frequency steps is depending on the number of modes that can be changing during optimisation processes with structural or material modifications. Thus, there are different aspects to consider ensuring the comparability of FEA based sound power estimates. In this study, we present an approach for the determination of the total sound power in the frequency domain based on the power spectral density. The estimate is independent from the number of modes and thus allows the comparison of different structural components, materials or shapes. Moreover, this approach opens the prospects of accessing optimisation algorithms with sound power objectives.

Keywords: Radiated Sound Power, Finite Elements, Frequency Domain, Optimisation

1. INTRODUCTION
The determination of the radiated sound power is a common design criterion especially for lightweight and thin-walled components with dynamic loading. The numerical estimation relies on different approaches based on the structure’s normal surface velocity [1]. Thus, the Finite-Element-Analysis in frequency domain is an appropriate tool for the power estimation.

Steady state FEA in frequency domain is quite efficient but still significant concerning computational time. Thus, the sound power optimisation still is computationally expensive [2] and therefore limited in application on large structures in a wide frequency range.

In addition, frequency domain FEA with modal superposition can be used to integrate anisotropic damping of composites by an energy related approach [3–6] and thus is suitable for acoustic laminate optimisations.

Moreover, the previous modal analysis can be used for a mode based frequency spacing. This assures an accurate representation of the resonances but leads to non-equidistant frequency steps. Next, the total number of frequency steps depends on the number of modes within the treated frequency range. Changing geometry or material parameters within optimisation procedures may result in more or less contributing modes.

Reducing the radiated sound power by numerical optimisation methods requires a robust objective [2, 7], typically as a scalar value. In contrast, there is a strong dependency of the radiated sound power on the frequency wherein resonances contribute the most.

Within this study, an approach to determine the average power in the given frequency range based on the power spectral density is presented. The determination is independent from the number of modes and frequency steps and valid for varying frequency step sizes. This enables the comparison of different components, materials or geometries as well as the implementation as an optimisation objective.

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2. FEA-BASED SOUND POWER ESTIMATION

The radiated sound power is commonly used to express the behaviour of radiating surfaces, components and machines and is formulated as the integral of the intensity over the surrounding and radiating surface. Exact calculations of the sound power are limited to a few academic cases. Thus, numerical approximation methods are commonly used but computationally expensive as fluid-structure-interaction has to be solved in one or both directions. The boundary element method (BEM) of large-scale problems became a very popular approach but is limited for a large frequency range or modified structures within optimisation loops (e. g. [8]). This study is based on either the mechanical input power or a velocity based approach of the sound power, the lumped parameter model (LPM) [1]. Other approximation methods such as the equivalent radiated sound power (ERP) or the volume velocity give comparable results [1, 9, 10], but are not treated here.

These simplified methods are restricted by some assumptions. For hard reflecting surfaces such as stiff thin-walled structures, particle velocity and structure normal velocity are identical. Moreover, an evaluation of the sound pressure on the structure’s surface is needed.

\[
p \approx \rho_f c_f v_n
\]  

The relation between particle velocity and sound pressure is reduced to the fluid’s characteristic impedance \(\rho_f c_f\).

The most accurate approximation is the lumped parameter model by KOOPMANN and FAHNLINE [11–13]. It is a simplification of the RAYLEIGH-integral including a TAYLOR series for the GREEN’s function as a multi-pole expansion. This yields to a formulation for a source at \(x_\mu\) and a receiver at \(y_\nu\)

\[
P_{LPM} = \frac{1}{2} k \rho_f c_f \sum_{\mu=1}^{N_s} \sum_{\nu=1}^{N_s} S_{\mu \nu} \frac{\sin(k|x_\mu - y_\nu|)}{2\pi|x_\mu - y_\nu|} \text{Re}\{v_\mu v_\nu^*\}
\]

also weighting the interacting sources. The LPM predictions are exact for dipole modes. Besides, it generally gives appropriate results in the low and mid frequency range. The power levels are related to a reference of \(P_0 = 10^{-12} W\)

\[
L_W = 10 \log \left( \frac{P}{P_0} \right) dB.
\]

Within steady state finite element simulations, the energy balance consisting of different elastic and dissipating components is estimated implicitly. Therein, the kinetic energy of the whole system or several components is given by the integral over the volume \(V\)

\[
E_{kin} = \int_V \frac{1}{2} \rho_s v v^T dV
\]

with the density of the solid surface \(\rho_s\) and the surface velocity \(v\). According to common standards in acoustics the energy level

\[
L_E = 10 \log \left( \frac{E_{kin}}{E_0} \right) dB
\]

is referred to a level of \(E_0 = 10^{-12} J\).

Sound radiation is depending on frequency with most relevant contributions at the resonances. Changing material or geometric parameters in optimisation processes may result in a different number of modes contributing within the considered frequency range

\[
\Delta f = f_u - f_l
\]

Next, scalar values are more likely to be implemented as an objective [14]. This mean power value is estimated by frequency integral of \(N_f\) frequency steps.

\[
\bar{P} = \frac{1}{f_u - f_l} \sum_{n=1}^{N_f} (\Delta f_n P_n)
\]
with

\[ \Delta f_n = \frac{f_{n+1} - f_{n-1}}{2} \quad \text{for} \quad 2 \leq n \leq N_f - 1 \]  \hspace{1cm} (8)

\[ \Delta f_1 = \frac{f_2 - f_1}{2} \]  \hspace{1cm} (9)

\[ \Delta f_{N_f} = \frac{f_{N_f} - f_{N_f-1}}{2} \]  \hspace{1cm} (10)

referring to figure 1.

![Diagram](image)

Figure 1. Mean power estimation by frequency integral

As another comparative figure, the total energy within the frequency range is determined by

\[ E_{tot} = \int_{f_i}^{f_u} P(f)df = \bar{P} \cdot \Delta f . \]  \hspace{1cm} (11)

3. NUMERICAL PARAMETER STUDY

3.1 Model description

For a FEA parameter study a model of a rectangular plate of approximately 278 \cdot 234 \cdot 2 mm with 1,600 quadratic shell elements (4,961 nodes) has been used. Linear isotropic material behaviour has been considered (\( E = 200 \, GPA, \nu = 0.3, \rho = 7.89 g/cm^3 \)) and a viscous damping (\( \eta = 0.0001 \)) implied. The plate is excited in normal direction with 1N at an arbitrary point apart form diagonals or symmetry axes.

With an aspect ratio of 1 : 1.188 of the given dimensions the plate is tuned to have its first (torsion) mode at 100 Hz as well as a maximum frequency difference of the first five modes (figure 2).

![Diagram](image)

Figure 2: Natural frequencies of a rectangular plate: frequencies (red) and modal density per third octave band (blue)
3.2 Frequency spacing parameters

The number and distribution of the frequency steps can be controlled by different parameters of the steady state FEA. For illustration of the different frequency step options steady state FEA models with 136 frequency steps in a range from 10 Hz to 1 kHz have been used. Figure 3 shows the results up to 300 Hz with a shift of 5 dB between the curves due to readability. A linear and logarithmic spacing within the frequency range this is hardly underestimating the significant contribution of the modes due to the low frequency resolution. In contrast, a mode based spacing forces one frequency step exact at the resonance. There, the frequency steps depend on the distribution of the modal density. Last, a frequency spread of $f_s = 0.1$ has been used distributing the frequency steps linear within $\pm 10\%$ of the related eigenfrequency. Spread based step definitions might fail for high modal densities with overlapping frequency spreads. Than, the local mode based frequency intervals lead to a high frequency resolution between the modes but not close to the resonances and thus are inefficient (see figure 3 between modes 4 and 5 exemplary).

![Figure 3. Sound power estimation with different frequency step definitions](image)

For a convergence analysis of the determined power and energy levels, only the first mode has been used further on. Compared to the damping determination by the full width at half maximum, the evaluated frequency spread is related to the half power width (hpw) within a range of 1 hpw to 1,000 hpw. Thus, this study is independent from the specific material damping (0.0001 here). In addition, the number of frequency steps $N_f$ within $\Delta f$ has been varied from 3, which is the least valid value, to 10,000. Last, biasing allows a compression of the points near the resonance as well as an expansion in the frequencies in higher distance. The bias parameter $b$ is an exponent and thus chosen between 1 (equidistant spacing) to 10.

3.3 Results

The averaged sound power obviously decreases with the used frequency range (figure 4). Therein, more frequency steps are necessary to achieve convergence within larger frequency ranges. In general, biasing by 10 instead of 1 decreases the number of required frequency steps about one order of magnitude. Besides, biasing breaks the continuity of the convergence for less than 10 frequency steps, which is of no practical relevance.

For a better impression of the influence of $N_f$, the difference of the power levels related to the simulation with less number of steps is shown in figure 5. Assuming 0.1 dB as satisfying convergence criterion, the required number of frequency steps is about 0.5 the number of treated half power width with bias 1.0 (e.g. 100 hpw with 50 points).

There, biasing helps reducing convergent $N_f$ for wide frequency ranges whereas more steps are required in narrow band analysis. In detail, bias 2.0 requires equal efforts than 1.0 above 50 hpw and is recommended to reduce computational efforts for 200 hpw or more. Bias 10.0 can help to reduce CPU time from 1,000 hpw on, whereas it gives appropriate results already for 200 hpw. As a rule of thumb, the product of bias and $N_f$ should be at least $hpw/2$. 

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Figure 4: Average sound power of the first mode of a rectangular plate: dependency on frequency range and number of frequency steps for different bias parameters

Figure 5: Convergence of the average sound power of the first mode of a rectangular plate: difference of the power level related to less frequency steps
The figures are plotted for the LPM only. The kinetic energy achieves exactly the same convergence results.

Concerning computational costs, only the number of points is relevant. Due to figure 6, the efforts exponentially increase with the number of steps. For qualitative studies on thin-walled components, the computationally expensive post-processing with about 70% of the total time can be saved by using the kinetic energy only [9].

Figure 6. Total cpu time for FEA and acoustic post-processing

Next, the total power levels (11) are determined assuring to treat a frequency range being broad enough to represent almost all of the radiated energy (figure 7). The most accurate determination for 1,000 hpw, bias 10 and 10,000 points leads to a reference value of 72.3 dB. Again a deviation of 0.1 dB is required to achieve convergence. The figure illustrates a contradictory convergence: narrow band analysis up to 50 hpw underestimate the total power. In contrast, broad band frequency studies tent to an overestimation for low $N_F$ followed by a convergence against the exact solution. For a bias of 1.0, the results are satisfying when $N_f < N_{hbw}/2$ if at least a frequency range of 100 hpw has been under investigation. Bias 2.0 again requires at least 100 hpw for convergence but only half the number of frequency steps. A higher bias might help to achieve convergence from 10 to 50 hpw but than requires numerous frequency steps.

Figure 7: Total energy radiated by the first mode of a rectangular plate: dependency on frequency range and number of frequency steps for different bias parameters
Figure 8 summarises the energy study with a limited number of frequency ranges. It is clearly to be seen, that broad band frequency studies are required and need sufficient number of steps. Bias can help to reduce the computational costs but has to be handled with care for narrow band investigations.

Last, an optimal configuration can be found for a frequency range of 100 hpw with only 20 points and a bias of 2.0. Almost as good is determining 200 hpw with 50 points and a bias of 2.0 to 5.0.

With the given damping of 0.0001, the requirement of 100 hpw leads to a frequency spread of 0.01. If the relative distance between the modes is higher than 0.01, the spread based definition is most efficient. For a relative frequency distance below 0.01, which is quite common in the high frequency range with high modal densities, the mode based linear step definition places the points closer to the points of interest.

4. CONCLUSIONS

In this study an approach to determine the mean radiated sound power of a given frequency range is presented. It is independent from equidistant frequency steps of mode based steady state FEA solutions. There, different frequency step definitions can be used. Basic step definitions within the given range may cause a significant underestimation of the sound power by missing the resonance peaks. Spread based step definitions might be inefficient for high modal densities with overlapping frequency spreads.

The approach has been used to determine the frequency step parameters of the first torsional mode from a rectangular plate. There, the frequency range and number of frequency steps are linear depending.

For a convergence analysis, the total radiated power of the single mode has been determined based on the mean radiated sound power. As a result, a frequency range of at least 100 hpw should be defined to cover almost all the radiated energy.

Further studies will address common thin-walled parts besides the academic plate case and include the acoustic measure in optimisation objectives.

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