Ultrasonic method for Measuring transport parameters using only the reflected waves at the first interface of porous materials having a rigid frame.

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ABSTRACT

In this paper, acoustic method of direct problem for waves reflected at the first interface of porous materials saturated by air with rigid frame is solved at normal incidence in time domain. The interaction of the sound pulse with the air-saturated porous material is described by an equivalent fluid model. Simulated calculation using inverse problem is presented for determining the porosity, tortuosity, viscous and thermal characteristic length. The advantage of the proposed method is that the four parameters can be determined simultaneously. In addition, no relationship is assumed between the two characteristic lengths and without the required prior knowledge of the thickness of the porous material.

Keywords: Porous medium, Ultrasound, Characterization, Rigid frame.

1. INTRODUCTION

Following the model of Johnson et *al*. (1) The propagation of acoustic waves in a slab of porous material in the asymptotic domain is characterized by four parameters, namely, porosity ϕ, tortuosity α_∞, viscous characteristic length Λ, and thermal characteristic length Λ', the values of which are crucial for the behavior of sound waves in such materials. So, it is of some importance to develop new methods and efficient tools for their estimation. In this work, we present a reflectivity method for measuring these physical parameters describing the propagation of ultrasonic pulses in air-saturated porous materials. This method is based on a temporal model of direct and inverse scattering problems for the propagation of transient ultrasonic waves in a homogeneous isotropic slab of porous material with a rigid frame. A simple expression of reflection coefficient at the first interface is established in high frequency domain, this expression depends on the four physical parameters; porosity, tortuosity, viscous and thermal characteristic lengths. A sensitivity analysis of these four parameters on the wave reflected at the first interface is studied in high frequency-regime. The inverse problem is solved numerically using the least square method by minimizing between the general reflected signal and the reflected signal at the first interface.

2. ACOUSTICAL MODEL

In air-saturated porous media the structure is generally motionless and the waves propagate only in the fluid. This case is described by the equivalent fluid model (2, 3) which is a particular case of the Biot theory (4), in this model the interactions between the fluid and the structure are taken into account in two frequency response factors: the dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson *et al.*(1) and the dynamic compressibility of the air included in the porous material $\beta(\omega)$ given by Allard(2). In the frequency domain, these factors multiply the density of the fluid and its compressibility respectively and represent the deviation from the behavior of the fluid in free space as the frequency increases. In the high frequency approximation their expressions are given by (1,2):

$$\alpha(\omega) = \alpha_\infty\left(1 + \frac{2}{\Lambda}\left(\eta/j\omega\rho_f\right)^{\frac{1}{2}}\right) \tag{1}$$

$$\beta(\omega) = 1 + (2(\gamma-1)/\Lambda')\left(\eta/P_r\rho_f\right)^{1/2}(1/j\omega)^{1/2} \tag{2}$$

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In these equations, Pr is the Prandtl number, γ is the adiabatic constant, η and ρ_f are, respectively, the fluid viscosity and the fluid density. The relevant physical parameters of the model are the tortuosity of the medium α_∞ initially introduced by Zwikker and Kosten (11), the viscous and thermal characteristic lengths Λ and Λ' introduced by Johnson *et al.*(1) and Allard (2).

3. DIRECT PROBLEM

The goal of this work is proposed to study the possibility of an inverse characterization to recognize the porosity, tortuosity, viscous and thermal characteristic lengths of rigid porous medium using only the reflected signal at the first interface. A schematic of the problem is depicted in Figure 1. It consists of a homogeneous rigid porous material that occupies the region $0 \le x \le L$. An incidence acoustical wave impinges normally on the medium. It generates an acoustic pressure field $p(x, t)$ and an acoustic velocity field $v(x, t)$ within the material.

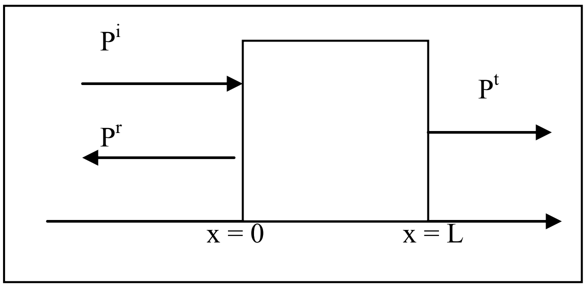


Figure1 - Geometry of the problem

The acoustic fields satisfy the Euler equation and the constitutive equation (along the x axis):

$$\rho_f \alpha(\omega) j\omega v = \partial p / \partial x, \qquad j\omega p \beta(\omega)/K_a = \partial v/\partial x \tag{3}$$

where $j^2 = -1$, ρ_f is the suturing fluid density, and K_a is the compressibility modulus of the fluid. The expression of the acoustic fields and velocity in each medium (1,..,3) are given by the expressions of a pressure wave and velocity arriving at normal incidence in each medium (1,2,3):

$$p_1(x,t) = e^{-j(kx-\omega t)} + R(\omega)e^{-j(-kx-\omega t)} \tag{4}$$

$$v_1(x,t) = 1/Z_f\left(e^{-j(kx-\omega t)} - R(\omega)e^{-j(-kx-\omega t)}\right) \tag{5}$$

$$p_2(x,t) = A(\omega)e^{-j(kx-\omega t)} + B(\omega)e^{-j(-kx-\omega t)} \tag{6}$$

$$v_2(x,t) = 1/Z(\omega)\left(A(\omega)e^{-j(kx-\omega t)} - B(\omega)e^{-j(-kx-\omega t)}\right) \tag{7}$$

$$p_3(x,t) = T(\omega)e^{-j(k(x-L)-\omega t)}, \tag{8}$$

$$v_3(x,t) = 1/Z_f\, T(\omega)e^{-j(k(x-L)-\omega t)} \tag{9}$$

Where $R(\omega)$, $T(\omega)$ are the reflected and transmitted coefficients, $A(\omega)$ and $B(\omega)$ are functions of pulsation used to determine, and:

$z_i(\omega) = \sqrt{\rho_f k_a \alpha_i(\omega)/\beta_i(\omega)}$, $k_i(\omega) = \omega\sqrt{\rho_f \alpha_i(\omega)\beta_i(\omega)/k_a}$ are the characteristic impedance and the wave number, respectively, of the acoustic wave in each medium (i = 1,2,3). k_a is the compressibility module of the fluid. To derive the reflection coefficient in the frequency domain, it is assumed that the pressure field and velocity are continuous at the boundary of the medium

$$p_1(0^-,\omega) = p_2(0^+,\omega) \qquad p_2(L^-,\omega) = p_3(L^+,\omega) \tag{10}$$

$$v_1(0^-,\omega) = \phi v_2(0^+,\omega) \qquad \phi v_2(L^-,\omega) = v_3(L^+,\omega) \tag{11}$$

where ϕ is the porosity of the medium and the $\pm$ superscript denotes the limit from right and left, respectively. Using boundary and initial condition (10)-(11), reflection coefficient can be derived (5):

$$R(\omega) = \frac{(1-Y^2(\omega))sinh(jk(\omega)L)}{2Y(\omega)cosh(jk(\omega)L)+(1+Y^2(\omega))sinh(jk(\omega)L)} \tag{12}$$

The general expression of reflection coefficient Eq. (12) can also be rewritten as follows:

$$R(\omega) = \left(\frac{1-Y(\omega)}{1+Y(\omega)}\right)\left[\frac{1}{1-\frac{2Y(\omega)}{(1+Y(\omega))^2}\left(1-cosh(jk(\omega)L)\right)}\right] \tag{13}$$

Where

$$Y(\omega) = \phi\sqrt{\frac{\beta(\omega)}{\alpha(\omega)}} \quad \text{and} \quad k(\omega) = \omega\sqrt{\frac{\rho\alpha(\omega)\beta(\omega)}{K_a}} \tag{14}$$

The expression of reflection coefficient of the first interface is obtained by letting the thickness L to infinity (L →∞).

$$R(\omega) = \frac{1-Y(\omega)}{1+Y(\omega)} = \frac{1-\phi\sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}}{1+\phi\sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}} \tag{15}$$

The ultrasonic regime corresponds to the range of frequencies such that viscous skin thickness $\delta = \sqrt{2\eta/\omega\rho_f}$ is much small than the radius of the pores *r*, ($\delta/r \ll 1$). This is called also the high-frequency range. In this domain the expressions of the responses factors α(ω) and β(ω) are given by Eq(1) and Eq(2). We replace the responses factors α(ω) and β(ω) with its expressions and we develop asymptotically the relation (15), we obtain the following expression of the reflection coefficient at the first interface:

$$R(\omega) = \frac{1-\frac{\phi}{\sqrt{\alpha_\infty}}}{1+\frac{\phi}{\sqrt{\alpha_\infty}}} + \frac{2\sqrt{\frac{\eta}{\rho_f}}\frac{\phi}{\sqrt{\alpha_\infty}}}{\left(1+\frac{\phi}{\sqrt{\alpha_\infty}}\right)^2}\left(\frac{1}{\Lambda} - \frac{(\gamma-1)}{\Lambda'\sqrt{P_r}}\right)\frac{1}{\sqrt{j\omega}} + \cdots \tag{16}$$

The incident and reflected fields $p^i(t)$ and $p^r(t)$ are related in time domain by the reflection scattering operator R(t). These are integral operators represented by:

$$p^r(x,t) = \int_0^t R(\tau)\, p^i\left(t-\tau+\frac{x}{c_0}\right) d\tau \tag{17}$$

$$= R(t) * p^i(t) * \delta\left(t+\frac{x}{c_0}\right) \tag{18}$$

The lower limit of integration in Eq.(17) is given as 0, which equivalent to assuming that the incident wave front first impinges on the material at t = 0. In Eq. (18) * denotes the time convolution operation, and δ(t) is the delta function. The temporal operator kernel R(t) is independent of the incident field used in the scattering experiment and depend only on the properties of the material its expression is calculated by taking the inverse Fourier transform of the reflection coefficient (Eq.(16)) of slab porous material given by:

$$R(t) = \frac{1-\frac{\phi}{\sqrt{\alpha_\infty}}}{1+\frac{\phi}{\sqrt{\alpha_\infty}}}\delta(t) + \frac{2\sqrt{\frac{\eta}{\rho_f}}\frac{\phi}{\sqrt{\alpha_\infty}}}{\left(1+\frac{\phi}{\sqrt{\alpha_\infty}}\right)^2}\left(\frac{1}{\Lambda} - \frac{(\gamma-1)}{\Lambda'\sqrt{P_r}}\right)\frac{1}{\sqrt{\pi}}\frac{H(t)}{\sqrt{t}} + \cdots \tag{19}$$

Where *H(t)* is the Heaviside function. *R(t)* is the instantaneous response of the porous material corresponding to reflection at the first interface (*x = 0*). The experimental detection of the reflection contribution by the first interface it's independent on the sample's thickness and depends only on acoustic properties (porosity, tortuosity, viscous and thermal characteristic lengths). The convolution of this operator with a transient incident signal gives the reflected signal directly in the time domain.

In order to characterize air-saturated porous materials, it is fundamental to study the sensitivity of each parameter; porosity ϕ, tortuosity α_∞, viscous and thermal characteristic length Λ and Λ' on the reflected waves at the first interface given by Eq. (16) in different high frequencies. This study is useful and we enormously facilitate the determination of these parameters using the inverse problem. Let consider porous sample (S) with physical parameters: Thickness L = 2cm, porosity ϕ = 0.8, tortuosity α_∞ = 1.2, viscous characteristic length Λ = 90μm and thermal characteristic length Λ' = 120μm. Table I summarized the relative variation on the amplitude of the reflected waves at the first interface obtained for ±20% change on the porosity and tortuosity and for ±50% change on the viscous and thermal characteristic length. These results are obtained for various values of frequency (150 KHz, 100 KHz and 50 KHz).

Table 1 - Relative variation on the amplitude of the reflected waves at the first interface for various values of frequency

		ϕ		α∞		Λ(μm)		Λ'(μm)	
		+20%ϕ	-20%ϕ	+20% α∞	-20% α∞	+50%Λ	-50%Λ	+50%Λ	-50%Λ
150 KHz	$\Delta R/R$	-53.82%	63.88%	26.43%	-32.80%	-03.17%	10.14%	01.20%	-03.38%
100 KHz	$\Delta R/R$	-52.97%	62.77%	26.01%	-32.34%	-03.97%	12.37%	01.41%	-04.24%
50 KHz	$\Delta R/R$	-50.99%	60.61%	25.10%	-31.17%	-05.50%	17.55%	01.99%	-05.86%

According to the Table 1 is noted that the influence of the porosity and the tortuosity is important in the variation on the amplitude of reflected wave at the first interface in large frequencies band (50KHz, 100KHz and 150KHz) wherever the influence of the porosity is twice more important than the influence of the tortuosity. Contrariwise, the sensitivity of the viscous and thermal characteristic length is negligible on the reflected wave at the first interface at 100 KHz and 150 KHz, but its influences are remarkable at frequency 50KHz.

4. INVERTED VALUES ON THE THEORETICAL PARAMETERS

As shown in Sec. III, solution of the direct problem involves an operators R(t) expressed as functions of ϕ, α∞, Λ and Λ'. Let consider porous sample (S) previously used in Sec II (L = 2cm, ϕ = 0.8, α∞ = 1.2, Λ = 90μm and Λ' = 120μm). In Fig. 2 is shown the incident signal (dashed line) with central frequency 50 KHz and the simulated reflected signal (solid line) of the sample (S) obtained from Eq. 17 using the general expression of the reflection coefficient given by Eq. 12.

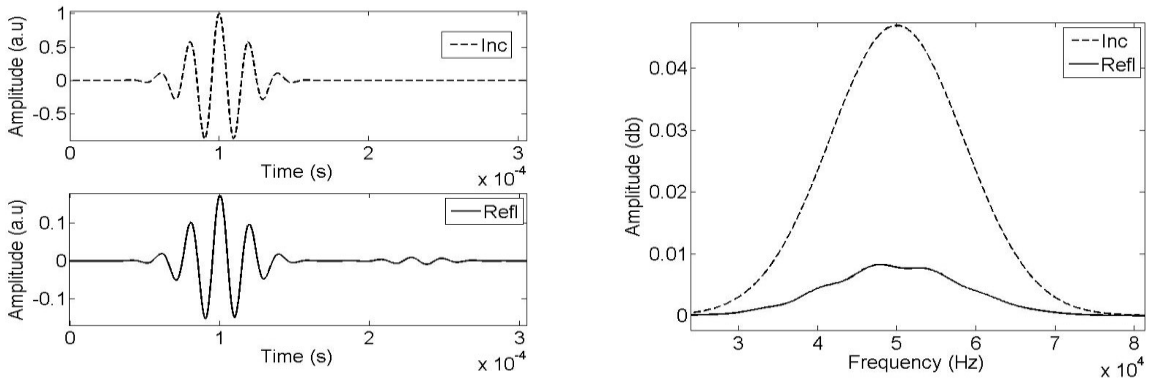


Fig. 2 Incident signal (in dashed line) and reflected signal (in solid line) at left and their spectrum at right with central frequency 50 KHz

Our objective is to reconstruct the values of physical parameters describing the porous medium (sample (S)) by minimizing the cost function $U(\phi, \alpha_\infty, \Lambda, \Lambda')$ defined by:

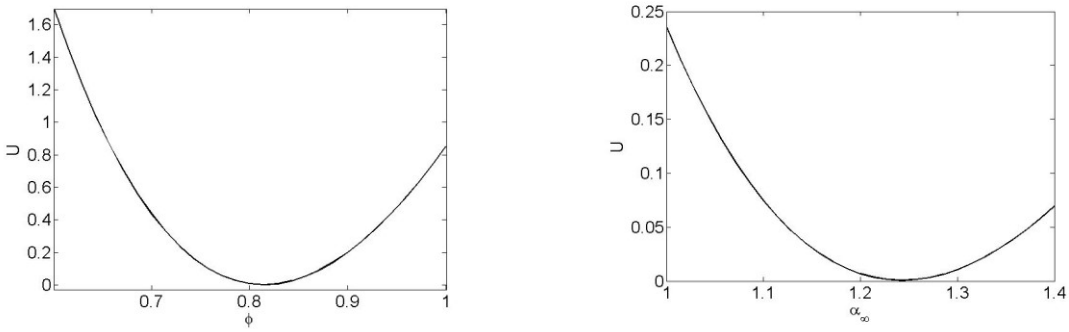
$$U(\phi, \alpha_\infty, \Lambda, \Lambda') = \sum_{i=1}^{N}\left(p_1^r(x, t_i) - p^r(x, t_i)\right)^2 \tag{20}$$

Where $p^r(x, t_i)$ is the reflected signal given by the general expression of the reflection coefficient obtained from Eq. (12), $p_1^r(x, t_i)$ is the reflected signal at the first interface predicted from Eq. (16). The physical parameter values representatives the general reflected signal ($p^r(x, t_i)$) by the sample (S) are kept fixed. The optimized values of the physical parameters are given in Table 2. The inverted values found of the porosity, tortuosity, viscous and thermal characteristic lengths are in good agreement with the values of the same physical parameter describing the sample (S) where the relative error does not exceed 7% for each parameters.

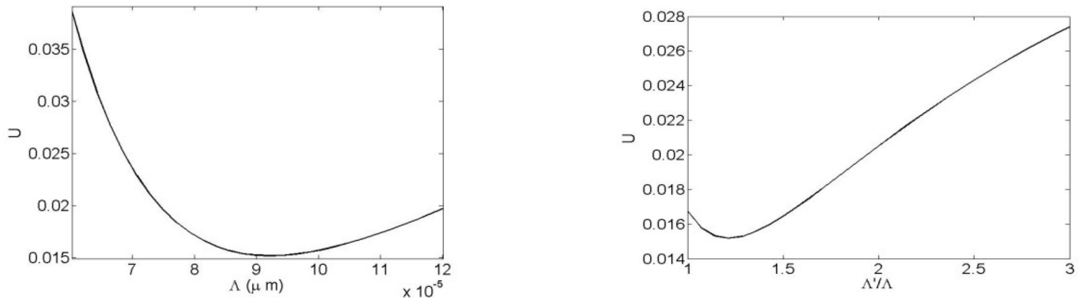
Table 2 - Inverted values of the physical parameters at 50 KHz

ϕ	α∞	Λ(μm)	Λ'(μm)	Λ'/Λ
0.814	1.243	92.14	111.89	1.214

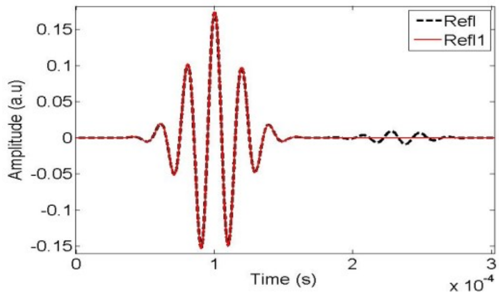
We present in Figs. 3(a), 3(b) the variation in the minimization function U with porosity, tortuosity, viscous and thermal characteristic length. In Fig. 4 we show a comparison between the general simulated reflected signal obtained by Eq. (12) and simulated reflected signal at the first interface given by Eq. (16) for the optimized values of the inverse problem. The correlation between the reflected waves at the first interface of both expressions Eq. (12) and Eq. (16) is good.



Figures 3(a) - Variation in the minimization function U with porosity ϕ (at left) and with tortuosity α_∞ (at right)



Figures. 3(b) - Variation in the minimization function U with the viscous characteristic length Λ.(at left)and with the ratio between the viscous and thermal characteristic length Λ'/Λ (at right).



Figures 4 - Comparison between the general simulated reflected signal (Rfl) and simulated reflected signal at the first interface (Refl1) for the optimized values of the inverse problem.

4. CONCLUSIONS

An inverse scattering simultaneous estimate of the porosity, tortuosity, viscous and thermal characteristic length was given by solving the inverse problem for waves reflected at the first interface; Analytical expressions of general reflection coefficient and reflection coefficient at the first interface are calculated in the frequency regime. The waves reflected by the porous medium are obtained by convolution of the temporal reflection coefficients with the incident signal. The inverse problem is solved numerically using the least-square method by minimizing between the general simulated reflected signal and the simulated reflected signal at the first interface. The reconstructed values of the four parameters are in good agreement. This method is an alternative to the other ultrasonic methods based on transmitted and reflected mode (6-12).

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