A new generalized approach to Statistical Energy Analysis (SEA)
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ABSTRACT
SEA is widely used for the prediction of aircraft interior noise. In a traditional SEA model a system is partitioned into a number of subsystems and expressions are obtained for the transmission of reverberant energy between those subsystems. The accuracy of an SEA model depends on the accuracy of the library of subsystems that are available in a given SEA code. Traditional SEA codes often have somewhat limited libraries of subsystems that are based on analytical derivations of wave propagation and scattering in simplified cross-sections. Such cross-sections are adequate for very simple systems but they are often not general enough to rigorously describe the vibro-acoustic behavior of modern aircraft constructions. This paper describes recent work by the authors that has focused on generalizing SEA theory in order to describe wave propagation through arbitrary cross-sections. The new SEA theory has been implemented in the commercial vibro-acoustics software wave6 and the current paper provides an overview along with a number of validation examples.

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1. INTRODUCTION

1.1 Applications of interest
Consider the problem of predicting the interior noise and vibration in structures such as aircraft, trains and launch vehicles across a broad frequency range. These systems consist of many components and each component is typically large compared with the wavelengths of interest. The systems therefore often contain millions of structural and acoustic modes across the frequency range of interest. Deterministic methods such as finite elements and boundary elements are useful for modeling the low frequency responses of these systems (when the system has less than a few thousand modes) but are typically not practical for modeling the response across a broad frequency range. This is partly because of the computational expense associated with describing short wavelength behavior deterministically, but more fundamentally because such behavior tends to be very sensitive to small perturbations in the properties and boundary conditions of a system. The use of statistical methods such as Statistical Energy Analysis (SEA) then become appropriate instead. In SEA a system is described in terms of a series of coupled subsystems and expressions are derived for the input, transmission and dissipation of energy within each subsystem of the system. By adopting a statistical description of the local dynamic properties of each subsystem it is possible to predict the “ensemble average” response of a complex system across a broad frequency range. For more information about SEA the reader is referred to [1,2].

1.2 Limitations of existing SEA methods
The original derivations of SEA were based on observations of the energy flow between coupled oscillators [2,3]. Such derivations provide physical insight into the way in which energy is transmitted between coupled dynamic systems. However, the derivations have also perhaps led to some misperceptions about the applicability of SEA over the years (for example, that SEA is restricted

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to weakly coupled systems or that SEA can only be applied to systems with high modal overlap). The SEA equations can, however, be derived using a wave approach that does not require specific assumptions about coupling strength or modal overlap [4].

The parameters in an SEA model are usually obtained by estimating the transmission of waves at different types of junctions. For example, in one of the earliest papers on SEA, Lyon and Eichler [5] derived expressions for the coupling loss factors (CLFs) between various semi-infinite plates based on the wave transmission coefficients between the plates (the use of a wave approach to calculate CLFs was motivated in part by Heckl’s prior observations that wave mechanics expressions encountered in room acoustics are equally applicable to structural problems). The initial derivations of the SEA parameters were restricted to very specific idealized types of subsystems and junctions. For example, the wave transmission coefficients between simple thin beams and plates coupled at 90 degrees. Over the next 20 years the SEA literature expanded to include extensions of these analytical derivations to describe wave scattering for different types of cross-sections and junctions. The SEA codes of the early 1990s typically included large libraries of such modal density and coupling loss factor (CLF) formulations, each based on a disparate collection of analytical derivations (each with their own assumptions and ranges of applicability). During the early 2000’s the use of more general wave scattering formulations became available that accounted for subsystems connected at different angles. For example, wave scattering at point junctions [6], line junctions [7] and acoustic radiation at area junctions [8]. However, such analytical derivations are still of somewhat limited applicability. For example, consider the cross-sections shown in Figure 1. The use of such lightweight constructions is becoming increasingly common in many industries (along with increased use of composite materials).

![Figure 1. Examples of more general cross-sections. Left figure is from [9], right figure is from [10]](image)

In principle, it is possible to derive analytical expressions to model wave propagation in some of the cross-sections shown in the figure (the literature is not lacking in analytical derivations of wave propagation in simplified laminates described by thin/thick plate theory for example). However, as the cross-sections become more complicated such analytical derivations tend to become increasingly unwieldy and simplifying assumptions are usually required in order to ensure an analytical derivation remains tractable. For example, it is quite common to encounter assumptions that only three wavetypes exist in such cross-sections and/or that the variation of the dispersion curves of these wavetypes with heading can be represented by simplified equivalent orthotropic flat plates. Such assumptions lack rigor and the resulting analysis methods are typically not sufficiently detailed to distinguish between the different vibro-acoustic behavior of the cross-sections shown in Figure 1 or to be able to guide design decisions about the detailed construction of such cross-sections.

### 1.3 Limitations of existing periodic SEA methods

The limitations discussed in the previous section led the authors to look at alternative methods for
modeling wave propagation within SEA models. A periodic structure approach was previously presented by the authors in [9]. In this approach a cross-section is described by a unit cell modeled with structural finite elements. Periodic boundary conditions are then applied to the cell (using fixed values of phase constants in the x and y directions) and an eigenvalue analysis is performed in order to determine the frequencies at which the homogeneous equation of the unit cell is satisfied (i.e., extraction of phase constant surfaces). Expressions can then be obtained for the modal density, radiation efficiency and transmission loss of a given structural unit cell as discussed in [9]. Such an approach provides a more general way to describe the vibration response of a complex cross-section. However, there are also a number of limitations with the approach previously presented by the authors that affects its use for practical problems. One limitation is that the equations are not solved at a fixed frequency. This means, for example, that it is difficult to include general structural, acoustic and foam finite elements within the unit cell using such an approach (due to the frequency dependence of the dynamic stiffness matrices of such elements). As can be seen from Figure 1, many of the applications of interest are not just restricted to structural elements within the cross-section. A second more fundamental problem is that while the previous derivation enables radiation efficiencies and modal densities to be “updated” or “overridden” in an existing analytical SEA model, the approach does not really form a foundation for a general SEA code. For example, it is not easy to predict general CLFs at point, line and area junctions using such an approach. As noted by Lyon and Eichler back in the early 1960’s [5], indirect mechanical excitation of structural components through structural junctions with other subsystems is an essential part of many practical applications.

2. New approach to SEA based on general periodic structure theory

The issues discussed above have led the authors to derive more general periodic FE methods for modeling wave propagation in 1D, 2D and 3D waveguides. The work forms part of a generalized wave approach to SEA and enables an arbitrary amount of detail to be included in the cross-section of each subsystem (including general structural and acoustic finite elements within the unit cell). General expressions have also been derived for the modal density of the unit cells and for the coupling loss factors at point, line and area junctions at various locations within these unit cells. The methods can accurately account for the effects of curvature and stress stiffening (associated with, for example, combined pressurization and tension loading effects). Unlike older SEA methods, the new approach does not pre-assume a fixed number of wavetypes or a fixed number of headings within an SEA subsystem (i.e., it is not pre-assumed that the wavetypes are restricted to flexure, extension and shear waves at headings of 0, 45 and 90 degrees). Instead the methods accurately account for the actual wavetypes of a subsystem at each frequency of interest and fully account for the variations of the dispersion curves with heading. The methods have been implemented in the commercial vibro-acoustic software wave6 [11]. The following sections present a number of validation examples to illustrate the methods.

3. Simple validation examples

3.1 Modal density (curved shell)

Consider the 2D SEA subsystem shown in Figure 2. The subsystem consists of a doubly curved shell of radius 2m that references a 1mm thick steel cross-section. The cross-section of the subsystem is described by a unit cell with a single QUAD8 shell element. Periodic boundary conditions are applied to the cell and the modal density is calculated as a function of frequency. The modal density is compared with an analytical result from the literature [12] and good agreement is observed. The analytical result is only valid above the ring frequency of the shell, while the periodic approach is valid across the entire frequency range.
3.2 Pressurization and stress stiffening (flat plate)

Consider a 2D SEA subsystem consisting of a flat plate that references a 1mm thick steel cross-section. The cross-section of the subsystem is described by a unit cell with a single QUAD8 shell element. An in-plane tension is applied to the plate in the x-direction. Periodic boundary conditions are applied to the cell and the dispersion curve of the shell is calculated as a function of frequency. The wavenumber in the direction of the applied tension is compared with an analytical result from the literature [13] and plotted in Figure 3. Good agreement is observed. The stiffening effects of in-plane tension are clearly evident at low frequencies. A polar plot of the dispersion curves of the plate at a fixed frequency also illustrates the non-isotropic effect of the tension on wave propagation at different headings (such effects are accurately accounted for in the current method).

Figure 3 – Left figure, dispersion curve of bending wave of flat plate, with and without in-plane tension applied in one direction (current method shown with solid line, reference result from [13] shown with dots). Right figure, variation of dispersion curve with heading at a fixed frequency. Note the non-isotropic behavior of the dispersion curves with heading as shown by the figure.
3.3 Point junction (CLFs between 1D beams)
Consider the 1D SEA subsystems shown in Figure 4. The subsystems have different lengths and different damping loss factors. Excitation is applied to one of the subsystems and interest lies in predicting the modal energy of each of the subsystems. The subsystems each reference a 5mm x 15mm steel cross-section. The cross-sections are described by unit cells that contain 9 HEX20 solid elements. 1D periodic boundary conditions are applied to each cell and the point CLFs are calculated for a point connection between the unit cells (located at the centerlines of the beam axes). The modal energies of the subsystems are compared with those obtained when the junction is modeled using an analytical point CLF derivation (from [6]). Good agreement is observed. It should be noted that (unlike the analytical derivation) the periodic approach is more general and does not pre-assume specific wavetypes in the subsystems. The periodic approach can also be used to predict the CLFs between general coupled structural and acoustic wavetypes (for example, for point connections to fluid filled pipes etc).

![Subsystems](image)

Figure 4 – Modal energy of a number of beams coupled at a point when input power is applied to the first beam. Current method shown with solid line, reference result from [6] shown with dots.

3.4 Line junction (CLFs between flat isotropic plates)
Consider the three SEA subsystems shown in Figure 5 coupled together at a line junction. The cross-section of each subsystem is described by a unit cell with a single QUAD8 shell element (one of the plates has a 1mm thick steel cross-section while the other plates have a 2mm thick steel cross-section). Excitation is applied to one of the plates and interest lies in predicting the modal energies of the plates. The modal energies of the subsystems are compared with those obtained when the junction is modeled using an analytical line CLF derivation (from [7]). Good agreement is observed. It should be noted that (unlike the analytical derivation) the periodic approach is more general and does not pre-assume specific wavetypes in the subsystems.
Figure 5 – Modal energy in three line coupled plates when first plate excited by a 1W input power. Each plate references a single QUAD8 element. Current method shown as solid line, reference result from [7] shown with dots.

3.5 Line junction (CLFs between curved isotropic plates)

Consider the two 2D SEA subsystems shown in Figure 6 connected by a curved line junction. The first subsystem is a 1mm thick aluminium flat plate while the second subsystem is a 3mm thick singly curved shell with a radius of curvature of 1m. Excitation is applied to the flat plate subsystem and interest lies in predicting the space averaged velocity of the two subsystems. The cross-section of each subsystem is described by a unit cell with a single QUAD8 shell element. The space average velocity predicted by the SEA model is shown in the Figure. An FE model was also used to predict the space average velocity response of the plates. In principle, a Monte Carlo simulation should be used when comparing against an ensemble average SEA prediction (as discussed in [4]), however, a single realization of the FE model was used in this example. There is good agreement between the FE and SEA predictions.
3.6 Area junction : Radiation efficiency

Consider the two SEA subsystems shown in Figure 7. A 5mm thick flat aluminium plate is connected to an acoustic cavity over a finite area connection region (i.e., a baffled aperture) of size 1.35m x 1.2m. The cross-section of the 2D SEA subsystem is described by a unit cell with a single QUAD8 shell element. Interest lies in predicting the resonant radiation efficiency of the plate when radiating into the cavity. The radiation efficiency predicted by the periodic approach is compared with the radiation efficiency from an analytical formulation ([8]). Good agreement is observed. The small differences in radiation efficiency observed in the example are due to different assumptions about the baffle boundary conditions applied to the plate between the two methods.

Figure 7 – Resonant radiation efficiency of an apertured flat plate radiating into an acoustic cavity (current method shown by smooth line, reference result from [8] shown by markers).
4. Numerical examples

The previous sections have demonstrated that the new periodic theory gives very similar results to existing analytical SEA derivations when a simple unit cell is used to describe the cross-section of an SEA subsystem (for many of the examples, the unit cell consisted of a single QUAD8 shell element). However, the benefit of the new approach lies in its generality. In particular, when more complex cross-sections are encountered it is not necessary to try to fit something to the nearest cross-section available in a limited library of analytical SEA subsystem derivations. Instead, it is possible to accurately model the response of the cross-section by adding more detail to the unit cell used in the SEA model. This is illustrated in the following sections using a number of numerical examples.

4.1 Wavetypes of a 1D hydraulic line

Consider the 1D SEA subsystem shown in Figure 8. The subsystem represents a hydraulic line which contains a heavy hydraulic fluid in a flexible walled pipe (the pipe contains a liner and braiding material and has a soft rubber outer section). The cross-section is modeled with a unit cell that references various structural and acoustic solid elements. The wavetypes of the cross-section were calculated along with their dispersion curves. It can be seen that the number of wavetypes varies with frequency as different waves cut-on. The dispersion curve of the fluid wave is often of particular interest for hydraulic lines (since this wave is readily excited by pressure pulsations within the fluid). The dispersion curve of a fluid within a rigid pipe was calculated and compared with the fluid wave in the more general representation of the hydraulic line. It can be seen that the flexibility of the pipe wall causes a significant reduction of the wavespeed of the fluid wave. This is accurately captured by the periodic approach. The current method can also be used to determine the effective material properties of a hydraulic line (using an inverse approach in which the material properties are determined by fitting the wavespeed of the fluid wave to a measured wavespeed).

![Figure 8 – Wavetypes and dispersion curves of a hydraulic line modeled using a 1D periodic model. Comparison of dispersion curve of fluid dominant wave when pipe wall is elastic or rigid.](image)

4.2 Wavetypes of a 2D curved orthotropic fairing

Consider the 2D SEA subsystem shown in Figure 9. The subsystem consists of a curved orthotropic sandwich panel with a radius of curvature of 2m. The core of the sandwich panel is described by two...
HEX20 elements (that reference an orthotropic material). The skins are described by QUAD8 elements (that reference PCOMP physical properties). The dispersion curves of the cross-section are shown at a frequency of 350Hz. It can be seen that below the ring frequency there are only two wavetypes of the section (not three as is sometimes assumed in older SEA methods). The combined effects of the non-isotropic material layup and curvature are also clearly seen in the dispersion curve (it should be noted that the dispersion curve is very different from that of an orthotropic flat plate, an assumption that is often made in older SEA methods).

Figure 9 – Dispersion curves of wavetypes of an orthotropic curved sandwich panel. Note the combined effects of curvature and material anisotropy on the dispersion curve.

4.3 Wavetypes of a 2D fuselage

Consider the 2D SEA subsystem shown in Figure 10. The subsystem references a unit cell that represents a typical fuselage construction. A built up SEA model could be created in which the fuselage is coupled to other components via point, line and area junctions. However, in the current example the wavetypes of the subsystem were calculated at a frequency of 1kHz. An example of a propagating wave within the fuselage is shown in the Figure. It can be seen that the wave propagation involves a complex deformation of the stringers that cannot be represented by thin beam theory. Simplified ribbed panel models that are based on simple thin plates with orthogonally stiffened thin beams are therefore not expected to be able to accurately describe this type of response.
5. New methods for modeling 3D SEA cavities

The previous examples demonstrated the use of new generalized SEA methods based on periodic structure theory. The authors have also been developing new methods for modeling the response in SEA acoustic cavities when there is significant absorption within the cavity. Figure 11 is taken from [14] and shows the predicted sound pressure level variation within a Daimler A-class at 4kHz due to windnoise loading on the sideglass of the vehicle (the latter is described by CFD). The plot shows a 20dB variation in the sound pressure level within the vehicle (due to the absorption of the cavity and the blockage effect of the seats). In a traditional SEA model it is common to subdivide the interior cavity of a vehicle into a number of SEA subsystems in order to describe spatial variations in the response. However, such partitioning is somewhat empirical and often does not result in a particularly accurate prediction of the interior SPL variations when compared with test data. The authors have therefore developed and implemented an alternative approach in which a single SEA subsystem can be used to model the interior cavity of the vehicle. As discussed in [14], this subsystem can be used to accurately account for the spatial gradients in the direct and reverberant fields within the cavity.
6. Summary

This paper has described a number of new SEA methods for more accurate modeling of noise and vibration in large complex systems. In particular, a new generalized approach to SEA has been derived for modeling wave propagation in 1D and 2D SEA subsystems that have complex sections. The methods accurately calculate the wavetypes of the sections and do not pre-assume a fixed number of wavetypes in each SEA subsystem. Methods have been developed for modeling the transmission of noise and vibration between such subsystems at point, line and area type connections. It was shown that these methods are generalizations of existing analytical methods for calculation of SEA coupling loss factors at point, line and area junctions. The new methods are, however, also more general and do not pre-assume a fixed number of wavetypes or a fixed number of headings at which waves can propagate. The generality of the approach was demonstrated through numerical examples which showed the wavetypes of a general hydraulic line containing a heavy fluid within an elastic walled pipe, the wavetypes of a fairing that referenced an orthotropic curved sandwich panel and a wavetype within a typical fuselage construction. Finally, an example was presented that showed new methods for modeling spatial gradients in SEA cavities (such methods are well suited to applications in which there is a significant variation in the SPL within a cavity due to spatial gradients in the direct and reverberant fields of the cavity). The methods described in this paper have been implemented in the commercial vibro-acoustic software wave6.

REFERENCES