Study on Stability Analysis of Acoustic Resonance in Heat Exchanger Tube Bundles

[ Identification of feedback type between vortices and sound field ]

Eiichi NISHIDA\(^1\); Hiromitsu Hamakawa\(^2\)

\(^1\) 1-1-25 Tsujido Nishikaigan, Fujisawa, Kanagawa, Japan
\(^2\) Ohita University, 700 Tannohara, Ohita, Ohita, Japan

ABSTRACT

Acoustic resonance may occur in heat exchangers such as gas heaters or boilers which contain tube bundles. This resonance is classified in self-excited oscillation, and feedback effect between vortex shedding and sound field plays important role. The final goal of our study is to develop a method by which to predict the resonance attack critical gas flow velocity and maximum resonance amplitude at the design stage. In order to reach this goal, it is essential to formulate the feedback effect between vortex shedding and a resonance mode concerned, and to execute a stability analysis of the resonance mode. There are two mechanisms in the feedback process: as the acoustic resonance grows, vortex strength is increased and vortex shedding synchronization grows.

This paper is concerned with the proposal of phenomenological model suitable to explain this mechanism and the formulation of these kinds of feedback mechanism with the use of this model. The model adopts vortex shedding wake oscillator model which is effective for tube vibration problems. Tube vibration movement is replaced by acoustic particle movement. Another improvement of our study is the introduction of statistical modeling of the wake oscillator to express vortex shedding synchronization effect. Here, the randomness of vortex shedding is explicitly modeled by a probability density function of the phase of the oscillator, and this function depends on the level of acoustic resonance. Based on these ideas to express the vortex/acoustics interaction, the formulas of stability analysis were derived.

1. INTRODUCTION

Acoustic resonance may occur in heat exchangers such as gas heaters or boilers which contain tube bundles. High level noise generated by resonance has concerned heat exchanger designers for a long time. The key research issues include prediction of resonance and estimation of the critical gas flow velocity for a given tube bundle configuration and acoustic space geometry in ducts. In order to investigate these issues, many studies have been conducted. Since the early 1950’s there have been many research articles on resonance phenomena in heat exchangers. The work of Y. N. Chen\(^{[1]}\) in 1968 is well-known as a breakthrough in this field. He identified the relationship between resonant noise and Karman vortex shedding, and proposed parameters which can be used to predict potential resonances at the design stage\(^{[2]}\). Following his work, research works have been done which include resonance prediction methods at the design stage, countermeasures for existing resonance cases, mechanism of resonance generation with a focus on feedback phenomena et. al. and they are mentioned in the review papers by Weaver\(^{[3]}\), Paidoussis\(^{[4]}\), Blevins\(^{[5]}\), and Eisinger\(^{[6]}\).

As for the acoustic resonance prediction methods, they are classified into two kinds; empirical ones and feed-back model based ones. Empirical design rules have been proposed based on accumulated experimental or in-situ plant data on the critical flow velocity for a variety of tube bundle configurations. Representative works have been done by Grotz\(^{[7]}\), Y. N. Chen\(^{[1]}\), Fitzpatrick\(^{[8]}\), Ziada\(^{[9]}\), Blevins\(^{[10]}\) and Eisinger\(^{[11]}\) et. al. As for the latter methods, including recent works which investigate the vortex/acoustics interaction\(^{[19-20]}\), many studies have been executed\(^{[12-15]}\), but there are many issues to be resolved.

\(^1\) nishida@mech.shonan-it.ac.jp
\(^2\) hamakawa@oita-u.ac.jp
The final goal of our study is to develop a method by which to predict the resonance attack critical gas flow velocity and maximum resonance amplitude for an arbitrary designed tube bundle and duct configuration. In order to reach this goal, it is essential to formulate the feedback effect between vortex shedding and a resonance mode concerned, and to execute a stability analysis of the resonance mode. There are two mechanisms in the feedback process: as the acoustic resonance grows, vortex strength is increased and vortex shedding synchronization grows. Here, the former is referred to as ‘Gain Feedback’, and the latter, ‘Synchronization Feedback’.

This paper is concerned with the proposal of phenomenological model suitable to explain this mechanism and the formulation of these kinds of feedback mechanism with the use of this model. The model adopts vortex shedding wake oscillator model which is effective for tube vibration problems. The wake oscillator is the semi-analytical model of tube vibration induced by vortex shedding and one of its advantages might be its compatibility with experiments. It seems that the effects of tube vibration on vortex shedding behavior are similar to those of acoustical oscillation. Based on this hypothesis, vibration movement may be replaced by acoustic particle movement. There are many studies on this issues by Hartlen et.al.[23] or by Blevins et.al.[16], and so on. Among them, the work of Faccinetti et.al.[22] attracted our attention and following their work, we incorporated wake oscillator model to this resonance problem.

Another improvement of our study is the introduction of statistical modeling of the wake oscillator to express vortex shedding synchronization effect[17]. Here, the randomness of vortex shedding is explicitly modeled by a probability density function of the phase of the oscillator, and this function depends on the level of acoustic resonance. This method gives the result that when the resonance level increases, the synchronization level in the tube bundles also increases, which seems to be a reasonable conclusion. The results of the application of this method to a two dimensional heat exchanger experiment model seem to support the validity of the proposed modeling method.

Our study here is the extension of these ideas, and the formulas of stability analysis were derived.

2. NOMENCLATURE

- General
  - \( \theta \) : amplitude.
  - \( i \) : location or node \( i \), namely, \( q_i = q(x_i) \)
  - \( m \) : parameter related to resonant mode
  - \( t \) : transpose of matrix or vector
  - \(*\) : complex conjugate
  - \( t, r, n \) : total, resonant and noise components: deterministic part of parameter.
  - \( \gamma^2_x(\omega) \) : coherence spectrum related parameter \( x \).
  - \( \mu \) : mass ratio.
  - \( p(\theta) \) : probability density function
  - \( pt, pl \) : transverse and longitudinal pitch ratio of tube bank.
  - \( \text{sinc}(x) \) : sinc function
  - \( t \) : dimensionless time
  - \( u_g \) : reduced gap flow velocity.
  - \( z \) : vector of which elements are \( z_i \).
  - \( D \) : tube diameter
  - \( E<> \) : ensemble average
  - \( S_x(\omega) \) : auto or cross spectrum related to parameter \( x \)
  - \( T \) : time
  - \( M \) : mass number.
  - \( U \) : uniform flow velocity
  - \( U_g \) : gap flow velocity
  - \( Z \) : matrix of which elements consist of \( z_{ij} \).
- Sound Field
  - \( \Psi_m \) : resonant mode shape vector
  - \( \zeta_m \) : modal damping ratio of resonant mode.
\( \Omega_r \): angular natural frequency of resonance. 
\( c \): sound speed 
\( p \): acoustic pressure 
\( y \): acoustic particle displacement. 
\( M,C,K \): sound field mass, damping, stiffness matrices. 
\( H_r \): matrix which is composed of frequency response functions of acoustic field 
\( R \): resonance index 
• Wake Oscillator 
\( \alpha \): parameter which shows the sensitivity of vortex shedding synchronization to acoustic resonance. 
\( \beta \): index of synchronization level. 
\( \phi_w \): phase lag of wake oscillator to acoustic excitation. 
\( \theta_w \): statistical phase component of wake oscillator. 
\( \Theta_w \): scattering range of \( \omega \) 
\( \Omega_f \): angular frequency of vortex shedding. 
\( w \): dimensionless displacement of wake oscillator. 
\( W \): displacement of wake oscillator. 
• Interaction Parameters 
\( \varepsilon \): van der Pol parameter. 
\( \Omega \): dimensionless angular frequency of coupled system. 
\( \Omega^* \): angular frequency of coupled system. 
\( f_{a,i} \): dimensionless coupling force on wake oscillator at node i. 
\( f_{r,i} \): dimensionless coupling fluid force at node i. 
\( f_{a,m}, f_{r,m} \): generalized dimensionless coupling forces. 
\( A \): scale factor of coupling force \( f_a \). 
\( F_{v,i} \): fluid force at node i. 
\( S_{f,\omega}(\omega) \): sound source cross spectrum matrix of which elements are \( E < f_{r,i}(\omega)f_{r,j}(\omega) > \) 
\( S_t \): Strouhal number. 
\( S_{f,\omega}(\omega) \): generalized sound source cross spectrum. 
\( C_L \): lift force coefficient in resonant condition. 
\( C_{LS} \): lift force coefficient in stable condition.

3. OUTLINE OF PROPOSED MODEL

3.1 Mechanism Underlying Resonance

When the gas flow rate in the duct surpasses a critical level, generation of high level noise by acoustic resonance may occur. Fig. 1 shows the mechanism leading to acoustic resonance. As the gas flow rate increases, Karman vortex shedding frequency increases, eventually reaching the natural frequency of an acoustic mode within the duct. The resulting resonance can potentially generate high levels of noise which is enough to damage the duct.

The acoustic resonance phenomenon is classified as a self-excited oscillation induced by the vortex/acoustics interaction. Fig. 2 shows the basic mechanism. In the stable state, the acoustic pressure is low and has random characteristics in time and space. In this state, the vortices have a unique shedding frequency but they are not synchronized in space as shown in Fig. 2 (a). On the other hand, under resonance conditions, as shown in Fig. 2 (b), the acoustic mode generates high level synchronized pressure fluctuations. Therefore, acoustic resonance affects the vortex shedding in two ways; first, the vortex strength is increased. Second, the spatial correlation of vortex shedding (region of vortex shedding synchronization) is expanded in three dimensions. These feedback mechanisms are essential to simulate and predict this resonance attack, and they were incorporated in the proposed model.
3.2 Outline of Modeling Method

Many studies[16][22-23] mention that vortex shedding is described by the wake oscillator model shown in Fig. 3. The most common sound field variable is acoustic pressure, but in order to describe vortex/sound interaction, acoustic particle description is more suitable than acoustic pressure.

In the description of sound field and vortex shedding, we introduce dimensionless time(t), displacement amplitudes of acoustic particle and wake oscillator(y, w), fluid force (fv, ), two kinds of frequencies(δ,ω) and reduced flow velocity(ur) as follows;

\[ t \equiv \Omega_j T \]  
\[ y(t) \equiv Y(T) / D, \quad w(t) = W(T) / D \]  
\[ f_v(t) = (D\Omega_j^2M)^{-1}F_v(t) \]  
\[ u_r = \frac{2\pi U_g}{\Omega_j D} \]  
\[ \delta \equiv \Omega_j / \Omega_f, \quad \omega \equiv \Omega / \Omega_f \]  
\[ U_g : \text{Gap velocity} \quad S_j : \text{Strouhal number} \]  
\[ \Omega_j = 2\pi S_j U_g / D \]  
\[ \Omega_f : \text{Natural frequency of acoustic mode} \]  
\[ \Omega : \text{Natural frequency of coupling system} \]

The term 'fluid force' means the force on the tube generated by vortex shedding and the tube applies the reaction force, ‘-fv’ to the sound field. Fig. 4 shows a block diagram of the feedback between vortex shedding and sound field expressed by acoustic particle velocity.

Here, a modeling method to introduce statistical characteristics of vortex shedding is described. The synchronization of vortex shedding becomes marked as the acoustic resonance increases. In order to express this feature incorporation of a statistical expression is necessary. Here, we suppose that statistical factors are expressed by the randomness of the phase of the wake oscillator, and also suppose that the phase is divided into two parts; a deterministic part (shown by ‘φω’) and a statistical part (shown by ‘θw’). Based on this expression, the displacement of wake oscillator becomes;

\[ w(x_i, \omega) \equiv \overline{w}(\omega) \cdot \exp \{ j \theta_{w,i}(\omega) \} \]  
\[ \theta_{w,i}(\omega) : \text{statistical phase component} \]  
\[ \overline{w}(\omega) = w_{0,i}(\omega) \exp \{ -j \phi_{w,i}(\omega) \} : \text{deterministic part} \]  
\[ w_{0,i}(\omega) : \text{amplitude} \]  
\[ \phi_{w,i}(\omega) : \text{deterministic phase component} \]

Based on these expressions, an interaction model between sound field and vortex shedding is shown in Fig. 5. Acoustic resonance amplitude depends on a location (or node) xi. At the location of large resonance amplitude, wake oscillator amplitude increases and Ωw,i, which means a scattering range of statistical phase component θw,i, decreases. Note that ‘φw,i(ω)’ means phase lag to acoustic particle excitation.

Hereafter, for simplicity, acoustic modes perpendicular to the flow direction are concerned. The same approach may be applied to the acoustic modes which are dominant in the flow direction.

4 FORMULATION OF MODEL

4.1 Description of Vortex Shedding

Description of wake oscillators are along with Faccinetti et. al.[22] which mentions tube vibration...
problems. Here this vibration is replaced by acoustic particle oscillation. The response of wake oscillator (referred to as w) excited by acoustic oscillation might be derived as follows:

$$\ddot{w}(x_i,t) + \varepsilon(w(x_i,t)^2 - 1) \ddot{w}(x_i,t) + \ddot{w}(x_i,t) = \ddot{f}_w(x_i,t) \tag{8}$$

The right hand term of equation (8) means the excitation effect of acoustic particle on the wake oscillator. As for the feedback from tube vibration to wake oscillator, Faccinetti et al. [22] considers the three type of feedback mechanism named displacement, velocity and acceleration feedback. Based on the comparison between simulated and experimental results, Faccinetti et al. [22] adopted the acceleration feedback shown in the equation (9). Here there is no tube motion, but rather the acoustic field. Nevertheless, as a trial basis, we adopted this relation.

$$\ddot{f}_w(x_i,t) = A \ddot{y}_w(x_i,t) \tag{9}$$

There is a non-linear term in the left-hand side of the equation (8), which includes the squared term of wake oscillator displacement. This non-linear term is needed to explain two phenomena: first, without acoustic excitation, vortices are shed. And second, there is the upper limit in sound pressure which means a limit cycle in the amplitude of the wake oscillators. This equation is suitable to describe models of a self-sustained, stable and nearly harmonic oscillation of finite amplitude. In literature of the Faccinetti et al. [22], only the main harmonic contribution of the nonlinearities is treated using the sinusoidal solution as follows:

$$\ddot{w}(x_i) = w_0 \cdot \exp(-j \phi_{w,i}) \tag{10}$$

Here is the problem of huge DOFs of wake oscillators. Fig. 5 suggests the method to solve this problem. In order to reduce DOFs of wake oscillators we thought that deterministic parts of wake oscillators might be expressed by modal coordinate with the resonance mode, assumed that this coordinate satisfies a van der Pol equation.

$$\ddot{w}_n(t) + \varepsilon(w_n^2(t) - 1) \dot{w}_n(t) + w_n(t) = f_{a,n}(t) \tag{11}$$

$$\ddot{f}_{a,n}(t) = A \ddot{y}_n(t) \tag{12}$$

The feedback from sound field to wake oscillator shown in equation (9) becomes as follows:

$$f_{a,n}(t) = A \ddot{y}_n(t) \tag{13}$$

Based on these assumptions, a wake oscillator in any location is expressed as follows;

$$\ddot{w}(x_i,t) = w_n(t) \cdot \psi_m(x_i) = w_{0n} \cdot \exp(-j \phi_{w,n}) \cdot \psi_m(x_i) \tag{14}$$

Actually, based on equation (8), phase lag of a wake oscillators varies depending on its amplitude, but in equation (14) means that phase lag of wake oscillators are unique shown by $\phi_{w,m}$.

In the next, we mention the statistical behaviors of wake oscillators shown by phase component $\theta_{w,i}$. In order to describe the statistical characteristics of these components, we introduce a probability density function of the form expressed by the equation as follows:

$$p(\theta_{w,i}) = \frac{1}{2\Theta_{w,i}} \text{ in } -\Theta_{w,i} \leq \theta_{w,i} \leq \Theta_{w,i}$$

$$= 0 \text{ in others} \tag{15}$$

Here, as shown in Fig. 6, the statistical part of the phase, $\theta_{w} (-\pi \leq -\Theta_{w} \leq \theta_{w} \leq \Theta_{w} \leq \pi)$, has equal probability over the specified range, and the width of this range depends on the amplitude ratio of the resonance component to the total component of the acoustic particle displacement at the location.
of vortex shedding as shown in Fig. 7. This ratio is referred to as 'resonance index', \( R \), and is defined as follows:

\[
R(x_i) = \frac{y_i(x_i)}{y_i(x_i)} \quad 0 \leq R(x_i) \leq 1 \quad (16)
\]

At the actual work, \( R \) may be determined from the power spectrum of acoustic pressure data. Using this expression, the feedback effect in the synchronization of vortex shedding may be expressed as follows:

\[
\Theta_{w,i} = \pi(1 - R(x_i))^\alpha \quad 0 \leq \Theta_{w,i} \leq \pi \quad (17)
\]

This equation means that an increase of \( R \) from zero to one leads to a decrease of \( \Theta_w \) from \( \pi \) to zero which means that the scattering of phase of vortex shedding in any location decreases. This feature seems to be rational because it coincides with the experimental fact that as the acoustic resonance level increases, the synchronization of vortex shedding becomes marked. And the parameter \( \alpha \) implies the sensitivity of vortex shedding to the acoustic resonance level. It is demonstrated in Fig. 8. This figure shows how the relationship between \( R \) and \( \Theta_w \) is affected by the parameter \( \alpha \). A large value of \( \alpha \) means a high sensitivity of \( \Theta_w \) to the change of \( R \). Based on this characteristic of \( \alpha \), this parameter might be used as the index which shows the sensitivity of the tube bundle configuration to resonance attack. It may be interesting to evaluate this value for tube bundles of various tube pitches or tube arrangement and to investigate the relationship between the value of this parameter and the likelihood of resonance attack.

### 4.2 Description of Sound Field and Sound Source

Firstly, we mention the expression of sound source. The tube which sheds vortices is approximated as a dipole-type sound source. This source is considered as a reaction of the fluid force on the tube \( f_v \). Here, this source is referred as \(-f_v\) and it is composed of the force on the node \( i \), \( \{-f_v(x_i)\} = \{-f_v,i\} \). Hereafter it is referred to as 'sound source vector'.

As the feedback formulation, following the literature of Faccinetti et al.[22], the dimensionless wake variable \( w \) is associated to the fluctuating lift coefficient on the structure as follows:

\[
w_s(x_i) = 2 \frac{C_{L_s}(x_i)}{C_{L_s}} \quad (18)
\]

Here, \( C_{L_s} \) means lift coefficient in the lock-out or non resonant stable condition. Using this interpretation, the relation between the sound source and wake oscillator behavior becomes as follows:

\[
\tilde{f}_{w,i}(\omega) = M \cdot \tilde{w}(x_i) = M \cdot w_s(x_i) \cdot \exp(-j \phi_{w,i})
\]

\[
= f_{w,i}(x_i) \cdot \exp(-j \phi_{w,i}) \quad (19)
\]

\[
f_{w,i}(\omega) = \tilde{f}_{w,i}(\omega) \cdot \exp(j \phi_{w,i}) = M \cdot w_s(\omega) \quad (20)
\]

Here, equation (19) and (20) are related to deterministic and statistical expression of \( w \), respectively. \( M \) is referred to as mass number and expressed as follows;

\[
M = \frac{C_{L_s}}{2} \frac{1}{8\pi s_i^2 \mu} \quad \mu = p_i \cdot p_i \quad (21)
\]

In the case where the deterministic source applies to sound field, adopting discrete form, the governing equation of sound field becomes as follows:

\[
M \ddot{y}_r + C \dot{y}_r + K y_r = \vec{T}_v \quad (22)
\]

On the other hand, in the case of statistical source expressed by equation (20), the governing
Equation becomes as follows;

\[
S_y(\omega) = H_r(\omega) \cdot S_{sh}(\omega) \cdot H^H_r(\omega) \quad (23)
\]

\(H^H_r\): Hermitian operator

\(S_{fr}(\omega) = \{S_{fr,ij}(\omega)\}\) : sound source cross

spectrum matrix.

\(S_y(\omega) = \{S_{y,ij}(\omega)\}\) : crossspectrum of

acoustic particle displacement.

\(H_r(\omega) = \{H_{r,ij}(\omega)\}\) : matrix of frequency

response functions of sound field.

Equation (23) does not yield the phase information which is necessary for estimation of limit cycle

closed loop of vortex-acoustic oscillation. The method to solve this problem is as follows;

considering that in usual cases of resonance attack, only one mode is dominant, modal coordinate is

convenient to express this phenomenon. It is introduced as follows;

\[
y_r(x_i,t) = y_m(t) \cdot \psi_m(x_i) \quad (24)
\]

\[
\ddot{y}_m(t) + 2\zeta_m \delta \dot{y}_m(t) + \delta^2 y_m(t) = -f_{v,m}(t) \quad (25)
\]

In order to incorporate the reduction of net force generated by statistical characteristics of wake

oscillators, we adopt the generalized force as follows;

\[
f_{v,m}(t) = \sqrt{\beta(y_{m0})} \cdot \psi_{m}^T \cdot \vec{f}_m(t)
\]

\[
= \sqrt{\beta(y_{m0})} \cdot M \cdot \psi_{m}^T \cdot \vec{W}(t) \equiv \sqrt{\beta(y_{m0})} \cdot M \cdot w_m(t) \quad (26)
\]

\(w_m(t) \equiv \psi_{m}^T \cdot \vec{W}(t)\)

Here, the parameter \(\beta\) express the reduction effect and have the value between 0 and 1 depending

on the scattering range of statistical phase components of wake oscillator, \(\Theta_{w,i}\). Here this parameter is

defined as follows;

\[
\beta(y_{m0}) = \frac{S_{fr,m}}{S_{fr,m}} \quad (0 \leq \beta(y_{m0}) \leq 1) \quad (27)
\]

The numerator is shown as follows;

\[
S_{fr,m} = E < f_{v,m}^* (\omega) \cdot f_{v,m}(\omega) >= \psi_m^T \cdot S_{fr} \cdot \psi_m
\]

\[
= \sum_i \sum_j \psi_{mj} \cdot S_{fr,ij}(\omega) \cdot \psi_{mj}
\]

\[
= \sum_i \sum_j \psi_{mj} \cdot \psi_{mj} \cdot \vec{f}_{v,i} \cdot \vec{f}_{v,j} \cdot \text{sinc}(\Theta_{w,i}) \cdot \text{sinc}(\Theta_{w,j}) \quad (28)
\]

The denominator \(\overline{S_{fr,m}}\) is achieved by setting \(\Theta_{w,j} = 0\) for any location which means

full-correlation of vortex shedding.

4.3 Solution of Coupled System and Flow of Numerical simulation

The governing equations of the vortex-resonance coupled system are as follows;

\[
\ddot{w}_m(t) + 6 \{w_m^2(t) - 1\} \dot{w}_m(t) + w_m(t) = f_{a,m}(t)
\]

\[
\ddot{y}_m(t) + 2\zeta_m \delta \dot{y}_m(t) + \delta^2 y_m(t) = -f_{v,m}(t) \quad (29)
\]
The coupling terms become as follows;

\[ f_{a,m}(t) = A \cdot y_m(t), \quad f_{w,m}(t) = \sqrt{\beta(y_m)} \cdot M \cdot w_m(t) \] (30)

Considering harmonic motion, a solution of modal responses of resonance and wake oscillators is sought in the form;

\[ y_m = -y_{m0} \cos(\omega t) \quad w_m = w_{m0} \cdot \cos(\omega t - \phi_{w,m}) \] (31)

Substituting in equation (29) and (30), and considering only the main harmonic contribution of the nonlinearities, elementary algebra finally yields the equation on the angular frequency \( \omega \) as follows;

\[ \omega^6 - [1 + 2\delta^2 - (2\zeta_m \delta)^2] \omega^4 - [2\delta^2 + (2\zeta_m \delta)^2 - \delta^4] \omega^2 - \delta^4 + G = 0 \] (32)

\[ G = AM \sqrt{\beta(\omega^2 - \delta^2)} \omega^2 \]

This equation is bi-cubic polynomial and yields three roots. Corresponding to these roots, three sets of solution on the amplitude \( w_{m0}, y_{m0}, \) phase delay \( \phi_{w,m} \) are achieved as follows:

\[ w_{m0} = 2 \left[ 1 + \frac{AM \sqrt{\beta}}{\varepsilon} \cdot \frac{C}{(\delta^2 - \omega^2)^2 + (2\zeta_m \delta)^2 \omega^2} \right] \]

\[ C = (2\zeta_m \delta) \omega^2 \]

(33)

\[ y_{m0} = M \sqrt{\beta[(\delta^2 - \omega^2)^2 + (2\zeta_m \delta)^2 \omega^2]}^{-0.5} \cdot w_{m0} \] (34)

\[ \phi_{w,m} = \tan^{-1} \left( \frac{-2\zeta_m \delta \omega}{\delta^2 - \omega^2} \right) \] (35)

Fig. 9 is the flow of numerical calculation and includes two loops: the inner loop iteration to seek a value of \( \beta \), and outer, to proceed to next step of flow velocity. The parameters which should be given beforehand are shown in double lined boxes.

5 EXAMPLE OF SIMULATION

In order to show that the proposed method has the ability to realize the features of the resonance phenomena such as limit cycle of acoustic oscillation, lock-in, or wake oscillator’s phase delay to acoustic excitation, we applied our method to one-dimensional resonance problem as shown in Fig. 10. Target is 2nd resonance mode. Each mesh element includes a single tube in its center, namely node, and lift forces are applied to these nodes. The parameters which should be given beforehand are listed in Table 1. The values of parameters are rather arbitrary. Especially the values of van del Pol parameter \( \varepsilon \), coupling force scaling parameter \( A \), the reference lift coefficient \( CLS \) are taken from the literature of the Faccinetti et. al. [22] which deals with VIV of single tube. The values of the other parameters are fixed based on the experimental results [24].

Figure 11 shows three coupling frequencies \( \omega_1-\omega_3 \). Looking at the lowest graph in which all three roots are shown, envelope of three lines slightly deviate from the value of one near the reduced flow of 5, which shows the occurrence of lock-in. The other representative response data are shown in Fig. 12. Limit cycle of modal coordinates for wake and acoustic particle oscillations are realized. Phase jump near the reduced flow of 5 is also achieved. Figure 13 shows the influence of a parameter \( \alpha \) on the limit cycle of wake and acoustic oscillations. As \( \alpha \) increase, limit cycle amplitudes also increases. Considering that the parameter \( \alpha \) indicates the sensitivity of the tube bundle configuration to resonance attack, this trend seems reasonable.

6 EXPERIMENTAL VERIFICATION

The stochastic wake oscillator model leads to the equation of coherence of fluid force between
locations \(i\) and \(j\) as follows;

\[
\gamma^2_{f_{ij}}(\omega) = \frac{S_{f_{ij}}(\omega) \cdot S_{f_{ij}}^*(\omega)}{S_{f_{ij}}(\omega) \cdot S_{f_{ij}}^*(\omega)} = [\text{sinc}(\Theta_{w,i}) \cdot \text{sinc}(\Theta_{w,j})]^2
\]  

(36)

Here, \(S_{f_{ij}}(\omega)\) is \((i, j)\) component of sound source cross spectrum shown as follows;

\[
S_{f_{ij}}(\omega) = E <f_{ij}^*(\omega) \cdot f_{ij}(\omega)> = \tilde{f}_{w,i}^*(\omega) \cdot \tilde{f}_{w,j}(\omega) \cdot \text{sinc}(\Theta_{w,i}) \cdot \text{sinc}(\Theta_{w,j}) = M^* \tilde{w}_{ij}^*(\omega) \cdot \tilde{w}_{ij}(\omega) \cdot \text{sinc}(\Theta_{w,i}) \cdot \text{sinc}(\Theta_{w,j})
\]  

(37)

based on this equation, experimental verification of the proposed method for vortex shedding synchronization was executed. Detail of this process is mentioned in the reference [24]. Here, outline of this verification is mentioned.

A schematic view of the experimental apparatus is shown in Figure 14. The structure of this apparatus is similar to that of an actual power station heat exchanger. The configuration of in-line tube bank is shown in Figure 15. The tube bank consists of five or fifteen rows, with 50 tubes per row. The tube diameter, \(D\), is 9 mm. The tube pitch ratio of tube bank in the flow direction, \(L/D\), is ranged from 1.44 to 4.0, and the transverse direction, \(T/D\), is fixed to 2.0.

The sound pressure level (SPL) was measured using a microphone mounted outside the test apparatus as shown in Figure 14. The high peaks were observed in the spectrum of SPL when the acoustic resonance occurred at the tube bank. Fig. 16 shows all the peak frequencies of the SPL spectra plotted against the gap velocity. The size of the symbols shows the peak SPL levels.

Figure 17 shows the variations of peak SPLs at each tube pitch ratio, \(L/D\), plotted against the gap velocity. The acoustic resonance at 2nd mode occurred commonly for \(L/D = 2.8, 3.5, 4.0\) and the 4th mode was for \(L/D = 1.44, 1.62, 1.75, 1.87\). Therefore, we concentrated on the 2nd and 4th acoustic modes for the investigation of vortex shedding synchronization on acoustic resonance.

In order to detect lift forces on tubes, two tubes were equipped with pressure taps located on the opposed position of the tube surfaces. These tubes are located on the acoustic pressure nodes of the 2nd or 4th mode shape, as shown in Fig. 14 where the acoustic particle velocity became maximum values in these positions. These surface pressures are referred to as \(p_1L, p_1R, p_2L, p_2R\), respectively.

From these data, the coherence between the lift forces on two tubes is estimated. Figure 18 shows the comparison between the calculated and measured coherence at 2nd mode resonant frequency. The calculated coherence by equation (36), are in good agreement with measured coherence. It seems to support the validity of the proposed modeling method of vortex shedding synchronization phenomena.

We considered that the phase characteristics calculated from these formulations. Figure 20 show the phase relations for various parameters. Based on this diagram which suppose acceleration feedback shown in equation (9), \(\theta_{sp2,pSR}\) should be \(-0.5\pi\) at the 2nd resonant frequency. This value agreed well with the result of experimental one \(-0.41\pi\) shown in Fig. 19. It is considered that the acceleration feedback for the coupling condition between sound field and wake oscillator were effective. We are executing the experiments in various conditions of flow velocity, tube pitch and so on to verify this result.

7. SUMMARY

Formulas to estimate limit amplitude of resonance in tube bundles are derived. This formula incorporates interaction of vortex-acoustic oscillations. We demonstrated that the representative features of this resonance such as limit cycle of acoustic oscillation, lock-in of vortex shedding and phase delay of wake oscillator to acoustic excitation are realized. Modeling method of vortex shedding synchronization was verified experimentally. Based on these results the proposed modeling method seems applicable to simulate resonance attack in heat exchanger tube bundles.
8. REFERENCES


Fig.1 Overview of acoustic resonance

Fig.2 View showing vortex/sound interaction

Fig.3 Wake oscillator model

Fig.4 Block Diagram of Vortex-Sound Interaction

Parameters of wake oscillator

\[ w_0 : \text{amplitude} \]
\[ \phi_\omega : \text{phase lag to acoustic particle excitation} \]
\[ \pm \Theta_\omega : \text{scattering range of phase} \]

Fig.5 Overview of Modeling of Vortex-Resonance Interaction Model
Fig. 6  Probability density function of statistical component of phase of wake oscillator

Fig. 7  Power spectrum of acoustic particle displacement

Fig. 8  Influence of $\alpha$ on relation between $\theta_w$ and $R$

Fig. 9  Flow of Numerical Simulation

Table 1  Parameters for Numerical Simulation

<table>
<thead>
<tr>
<th>Order of Mode</th>
<th>2nd</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_0$</td>
<td>300Hz</td>
</tr>
<tr>
<td>Number of Tubes</td>
<td>7</td>
</tr>
<tr>
<td>$D$</td>
<td>10mm</td>
</tr>
<tr>
<td>Mesh Size (m)</td>
<td>$0.1 \times 0.1 \times 0.1$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.3%</td>
</tr>
<tr>
<td>$A$</td>
<td>12</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Fig. 14  Experimental apparatus

Fig. 15  Arrangement of tube bank

Fig. 16  Variation of peak frequencies against $U_g$

Fig. 17  Variation of peak SPL against $U_g$

Fig. 18  Comparison between calculated and measured coherence between two tap-equipped tube data

Fig. 20  Block diagram of sound field/vortices system