The energy absorption properties of Helmholtz resonators enhanced by acoustic black holes

Xiaoqiang Zhou†,‡ and F. Semperlotti†

†School of Mechanical Engineering, Ray W. Herrick Laboratories, Purdue University, West Lafayette, IN 47907, USA
‡ The Institute of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

Helmholtz resonators typically exhibit narrow band absorption properties and optimal performance only in the neighborhood of the main resonance frequency of the cavity. In the past, several approaches have been proposed in order to improve the performance of HR systems, however their operating bandwidth remains one of the main limiting factors. This study addresses a novel design that combines a traditional HR with the concept of an Acoustic Black Hole (ABH). The HR cavity is partitioned by a flexible membrane having an embedded acoustic black hole. A fluid-structure interaction model was developed in order to simulate the coupled response of the system and numerical simulations showed that the novel design not only broadens the operating frequency range but can substantially increase the energy absorption.

Keywords: Helmholtz resonators; acoustic black hole; Wentzel-Kramer-Brillouin theory; broadband energy absorption.

1. INTRODUCTION

Helmholtz resonators (HR) are applied extensively in acoustic engineering as devices for passive control of noise and noise-induced vibrations. During the past decades there has been a considerable amount of research dedicated to the development and design of HR technology. Most of the research concentrated on tuning the resonance frequency, extending the resonance bandwidth as well as improving the performance in terms of transmission losses. In order to broaden the frequency bandwidth, HR in array configuration were investigated [1–8]. A series of theoretical works on HR array was performed by Sugimoto [1–3] who explored the dispersion properties of sound waves in a tube with HR arrays. In order to obtain a broader attenuation range, Wang and Mak [9] designed a duct-HR system where the HRs were periodically distributed on the duct. This research showed good potential for broadband noise control. Sugimoto [1] investigated also the nonlinear acoustic wave propagation in a duct with an array of HR, demonstrating the efficacy of this design in reducing the far field wave transmission. HR arrays were shown to provide an efficient approach to broaden the resonance frequency bandwidth and to increase the acoustic transmission loss. In practical engineering applications there are important limitations such as geometric constraints, system complexity and cost that should be taken into account. In order to overcome some of these issues, Bedout et al. [10] developed a HR that could be tuned by using a novel feedback and optimization system. Despite improved performance the system could only attenuate energy in selected bands therefore not providing a full broadband approach. In summary, despite many interesting research and approaches to improve the performance of HR systems exhibit narrow operational bandwidth while HR arrays have practical limitations.

In this study, we investigated a novel design consisting in a traditional single chamber HR having a tapered metallic membrane located in its resonant cavity. The taper was designed according to an axial-symmetric power-law profile, which is also known as an Acoustic Black Hole (ABH). The goal of this design is to obtain, by using completely passive means, an HR exhibiting broader operational bandwidth and enhanced transmission loss performance.

The Acoustic Black Hole is a structural feature that relies on a concept initially studied by Mironov [11] in tapered wedges. He observed that in a power-law tapered wedge elastic flexural waves are progressively

1email: fsemperl@purdue.edu
slowed down while approaching the tip of the wedge. In the ideal case, that is when the residual thickness vanishes at the tip of the wedge, the wave is not reflected back and the wedge appears as an ideal absorber.

Starting from this fundamental observation, Krylov [12–18] cast this idea in a three-dimensional element (i.e. the Acoustic Black Hole) that could be embedded in thin-walled structural elements and started a line of research that was followed by several other researchers [12, 14, 14, 15, 18–24].

In the following, we present the design of the HR coupled to an ABH tapered membrane (HR-ABH) and located in the resonance chamber. A fluid-structure interaction model is first develop and then used to perform numerical simulations in order to assess the performance of the new design. Numerical results will show that the system is able (i) to expand the operational band of the HR and (ii) to increase the transmission loss performance while still maintaining a very compact design.

2. MATHEMATICAL MODEL OF THE HR-ABH

In order to model the coupled HR-ABH, the system was divided into two parts: (1) a traditional HR and (2) a plate with an embedded ABH subject to fluid-structure interaction. The traditional HR stores energy while inducing small dissipation due to viscous losses in the fluid. More important, the cavity generates an oscillatory flow in the HR cavity that produces a resonance effect able to absorb energy from selected frequencies in the main duct. This process is not unlike a classical mechanical vibration absorber. When an ABH membrane is introduced into the HR cavity, the harmonic pressure produced by the resonating fluid drives the flexible ABH membrane inducing deformation of the flexible tapered structure. The vibration of the ABH membrane transforms the acoustic energy into mechanical energy and vice versa, therefore producing a continuous exchange of energy between the fluid and the flexible structure. The mechanical energy in the plate is associated with both propagating and standing stress waves that are affected by the presence of the ABH. In particular, the ABH induces both a wavenumber sweep effect [23], which is a key element to achieve broadband performance, and energy focusing that can be exploited for efficient energy absorption and attenuation.

Some of the details of the numerical model are summarized hereafter.

Our numerical model is based on the Wentzel-Kramer-Brillouin (WKB) approach. Two functions are separately defined to describe the fluid and the structure. For the fluid domain, we define a potential velocity as

$$\Phi(x, y, z, t) = \phi(x, y) \phi(z) e^{S(x, y)} e^{i\omega t}$$

(1)

For the solid structure, assuming the thickness of the plate much smaller than its in-plane dimensions, the transverse displacement field is defined as

$$w(x, y, t) = w(x, y) e^{S(x, y)} e^{i\omega t}$$

(2)

where, \(w(x, y)\) is the amplitude of the transverse displacement, \(S(x, y)\) is the spatially varying wavenumber [25, 26].
The governing equation describing the dynamic transverse response of an isotropic plate with an embedded ABH is

\[
\frac{\partial^2}{\partial x^2} \left( D_{h(x,y)} \frac{\partial^2 w}{\partial x^2} \right) + 2 \frac{\partial^2}{\partial x \partial y} \left( D_{h(x,y)} \frac{\partial^2 w}{\partial x \partial y} \right) + \frac{\partial^2}{\partial y^2} \left( D_{h(x,y)} \frac{\partial^2 w}{\partial y^2} \right) + \rho_j h (x, y) \frac{\partial^2 w}{\partial t^2} + P_f = 0 \tag{3}
\]

where \( D_{h(x,y)} = \frac{E h^2(x,y)}{12(1-\nu^2)} \) is the bending stiffness which is a function of plate thickness \( h(x,y) \), \( E \) and \( \nu \) denote the Young’s modulus and the Poisson’s ratio, \( P_f = P_f (x, y, z) \) is the uniformly distributed acoustic pressure applied on the ABH embedded plate.

The fluid in the duct and the cavity is assumed incompressible, inviscid, and irrotational, therefore the motion of the fluid is described by the Laplace equation

\[
\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{4}
\]

By substituting Eqn. (1) and enforcing boundary and continuity conditions, the acoustic potential in the fluid can be determined as

\[
\Phi (x, y, z, t) = \left[ e^{2k(a-y)} e^{-k(x+y-z) i} + e^{k(x+y+ct) i} \right] \left[ 2k^2 i \sin kz - (k + ki) \cos kz \right] e^{kz(1+i)+i\omega t} \tag{5}
\]

The calculation of the plate deformation is divided in two parts, that is the constant thickness part and the ABH variable thickness part. The details of the calculation are omitted, but the transverse deflection of the flexible plate in the constant thickness region is given by

\[
w(x, y) = -\frac{4ia_p b_p \rho_j \omega \phi(x, y) \phi(z)}{\pi^2 \left[ \frac{\pi}{a_p} \right]^4 + \left( \frac{\pi}{a_p} \right)^4 (M_0 - D_0) + \left( \frac{\pi}{b_p} \right)^4 (M_0 - D_0)} \sin \left( \frac{\pi x}{a_p} \right) \sin \left( \frac{\pi y}{b_p} \right) \tag{6}
\]

while the transverse deflection in the ABH region is given by

\[
w(r) = w_A(r) e^{S(r)} e^{i\omega t} \tag{7}
\]

where \( w_A(r) \) is the amplitude of the deformation and is given by

\[
w_A(r) = r^{-3m} \left[ A J^d_j(u) + C J^d_i(u) \right] - r^{3m} \frac{12i (1 - v^2) u^4 \rho_j \omega \phi(x, y) \phi(z)}{(ui)^2 E \varepsilon^3 S'} J^d_{ji}(u) \tag{8}
\]

where \( \rho_p \) is the density of the plate, \( J^d_j, J^d_y \) and \( J^d_{ji}, J^d_{yi} \) are the first and second kind Bessel’s functions of order \( d = \frac{2m}{m_0 - m} \), and \( S' = \frac{32 \left( 91 \omega^2 \rho_0 \frac{c_0^2}{\alpha^2} - 1210 E h_0 c_0 r \right)}{3 \pi E h_0^3} \).

Finally, the total transverse deformation of the ABH plate is

\[
w(x, y, t) = \begin{cases} w_{A_1} e^{S(x,y)} e^{i\omega t} & \text{on } \Omega_{ABH} \\ w_{A_2} e^{S(x,y)} e^{i\omega t} & \text{on } \Omega_{non-ABH} \end{cases} \tag{9}
\]

where \( w_{A_1} = w_A(r) \) and \( w_{A_2} = w(x, y) \) are the displacement amplitude of the ABH and constant thickness sections, respectively. The boundary conditions between the two sections are schematically indicated in Fig.2.

3. THE ENERGY BALANCE IN THE COUPLED SYSTEM

According to [27], the acoustic energy absorption by a Helmholtz resonator is given by

\[
E_{abs} = \frac{S_N}{2} \rho_0 c_0 (R_L + \Gamma_L) \frac{4|P_0|^2}{\rho_0 c_0^2 R_a} \tag{10}
\]
Figure 2: Schematic of the ABH plate indicating relevant geometric parameters and the interface between the ABH and non-ABH areas.

where, \( R_\alpha = (R_0 + R_L + \Gamma L)^2 + k_0^2 \left( L + 2 \delta - \frac{S_N}{\sqrt{k_0 \gamma}} \right)^2 \), \( R_0 = \frac{k_0^2 S_n}{2 \pi} \), \( \delta = \frac{3 \sqrt{S_N}}{4 \pi} \), \( R_L = \frac{k_0^2 S_N}{2 \pi} + \frac{S_N}{k_0 \gamma} \text{Re} \left( \frac{R_L}{k_0} \right) \), \( \Gamma = \sqrt{\frac{2 \pi k_0 L^2}{S_N}} \) is the dissipation factor at the walls of the tube, \( R_L \gamma = \frac{1}{\sqrt{2}} (\gamma - 1) \frac{S_n}{\sqrt{\gamma}} \sqrt{\frac{c_l}{\omega}} \), \( \gamma = \frac{C_P}{C_V} \), is the ratio of the specific heat capacities of the fluid, where \( C_P, C_V \) are the heat capacities per unit mass at constant pressure and volume, respectively. Assuming that the medium in the cavity is air, then \( \gamma = 1.401 \).

The infinitesimal energy exchanged between the plate and the fluid \( \delta E_{fp} \) is given by [27]

\[
\delta E_{fp} = \delta \left[ \frac{\text{Re} (P_f v^*)}{\omega} \right] \tag{11}
\]

where \( P_f \) is the distributed pressure generated by the fluid on the plate. The total energy exchanged \( E_{fp} \) is

\[
E_{fp} (\omega) = \frac{1}{\omega} \int_A \left[ \text{Re} (P_f v^*) \right] ds \tag{12}
\]

The integral is extended to the entire area \( A \) of the plate which includes both the constant thickness and the ABH area. The velocity in Eqn.(13) can be calculated based on the definition of the transverse displacement provided in Eqn.(10).

Considering the energy balance of the closed system (see Fig. 1 dashed box), and remembering that the energy stored in the different sections of the system is frequency dependent, the output energy at point B can be determined writing the following energy balance

\[
E_B = E_A - \left( E_{abs} + E_{fp} \right) \tag{13}
\]
4. NUMERICAL ANALYSIS

4.1 Energy stored in the Helmoltz Resonator

The model described above was used to perform numerical simulations in order to evaluate the energy absorbed by the new HR cavity. The energy absorbed can be directly related to the transmission losses which is a good indicator of the overall HR performance. Before analyzing the performance of the coupled system, we investigated the behavior of the individual components.

First, we analyzed the effect of the neck area $S_n$ on the energy stored in the HR cavity as a function of frequency. In this case study, the cavity does not include the tapered plate therefore the energy stored will be affected only by the dynamic behavior of the cavity. The energy was calculated according to Eqn. (10) and the results are presented in Fig. 3 in terms of a contour plot that shows the amount of stored energy as a function of the neck’s cross-sectional area $S_n$ and of the resonance frequency. Results are shown for a cavity described by the parameters $a_p = b_p = 0.2m, and m = 2.30$. As expected, the resonance frequency of the HR increases as $S_n$ increases while the energy stored in the HR shows an optimal value around $S_n = 0.004m^2$ and $f = 110$ Hz. This result is consistent with the behavior expected of a classical HR design and highlights the well-known narrowband energy absorption spectrum of a single cavity HR.

![Figure 3: Normalized energy stored in the HR as a function of the neck cross-sectional area and of the frequency.](image)

4.2 Energy Stored in the ABH Plate

The energy stored in the thin plate inside the HR cavity can also be evaluated. As a reminder, the plate is driven by the distributed pressure produced by the oscillating fluid in the cavity. The energy stored in the plate can be divided in the energy stored in the constant thickness area (labeled non-ABH) and in the tapered area (labeled ABH). The energy calculation was performed according to Eqn. (12). Results are shown in Figure 4 in terms of the normalized energy stored versus frequency. The energy spectrum indicates that the energy is stored very efficiently in the ABH area across the entire frequency range of interest, contrarily to what happens to the energy stored in the constant thickness section. This result already indicates that the introduction of the ABH plate is able to extract energy from the fluid in a more efficient way over a broad frequency range.
Figure 4: Normalized energy absorbed in the plate. The contributions of the flat and tapered parts of the plate are presented separately in order to clearly show the effect of the tapered region.

In general, the performance of the tapered plate is dependent on the parameters of the ABH and particularly on the taper coefficient \( m \). The effect of the taper coefficient was explored by performing a parametric study where the normalized energy stored in the plate was calculated as a function of the taper coefficient and of the driving frequency. The results shown in Fig. 5 indicate that when entering the ABH range (that is \( m \geq 2 \)) the ABH plate is able to extract a considerable amount of energy from the fluid. As expected, the ABH exhibit a cutoff frequency that depends on the size of both the host plate and the taper. Also of interest is the behavior outside the ABH regime \( (m < 2) \) that still shows considerable improvement in energy absorption. As an example, see the results at \( f = 150 \) Hz and \( m = 1.3 \).

Figure 5: Spectrum of the normalized energy absorbed by the ABH plate as a function of the taper coefficient \( m \). The white line marks the performance for the taper coefficient \( m=2.81 \) also shown in Fig. 4.

### 4.3 Energy Absorbed by the Coupled Structure

Finally, the overall performance of the coupled HR-ABH structure was calculated. Figure 6 shows the energy absorbed by the coupled system as a function of the taper parameter \( m \) and of the driving frequency.
The energy was calculated according to the term labeled ABH in eqn.13. As evident from a direct comparison, Figures 4 and 5 share many common features. This is not surprising considering that, as previously highlighted, the majority of the energy absorbed by the system is determined by the flexible tapered plate. The main difference is that, in the coupled results (Fig.5), an additional narrow absorption band appears at approximately 110 Hz. This band corresponds to the energy absorbed by the HR at the resonance frequency of the cavity and it is consistent with the expected behavior of a traditional HR design.

Figure 6: Normalized energy absorbed by the coupled HR-ABH system as a function of the taper coefficient $m$ and the driving frequency.

To further illustrate the performance of the novel design the spectrum of the absorbed energy is presented for a selected taper coefficient $m = 2.81$. Figure 7 shows the comparison of the absorbed energy versus frequency for the classical HR design (that is without ABH plate) and for the HR-ABH design. The performance of the two systems are equivalent up to about 125 Hz and are mostly driven by the performance of the resonant cavity which exhibits a resonance frequency around 110 Hz. This is also due to the fact the ABH design has a cut-off frequency approximately located in the same frequency range. As soon as the excitation frequency exceeds the cut-off of the ABH the performance are drastically enhanced and the HR-ABH design constantly outperforms the traditional design.

Figure 7: Normalized energy absorbed by the coupled structure for a taper coefficient $m = 2.81$. 

INTER-NOISE 2016
5. CONCLUSIONS

We have presented a novel design of a single cavity Helmholtz Resonator able to achieve broadband performance. The design relies on the use of a flexible plate-like element inserted in the cavity and whose thickness is tapered according to a power-law profile known as the Acoustic Black Hole. A fluid-structure interaction model was developed in order to study the performance of the system and the partition of the absorbed energy in the different elements of the resonator. In particular, the energy exchange between the fluid and the structure allows converting acoustic into elastic energy which generates stress waves propagating into the ABH plate. As the waves propagate, the ABH produces a wavenumber sweep effect that enables effective energy absorption over a broad frequency range. Numerical results shows that the interaction between the acoustic pressure field produced by the resonating fluid and the wave propagation properties of the tapered structure allows achieving a broadband absorption even with a single cavity design.

REFERENCES


