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Numerical study on the function of the register hole of the clarinet

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Abstract

The function of the register hole of the clarinet can be basically explained from the common property of two delay systems (J. Phys. Soc. Jpn, Vol.83, 124003 (2014)). Namely, if the strength of a short time delay is sufficiently small but non-negligible, the third harmonic is well sustained over a wide range of the short delay time. This fact indicates that the register hole with a small radius raises the pitch of the first register notes in a wide range more than an octave to the second register notes by a twelfth (19 semitones). However, the reflection function has many delay peaks even when only the register hole is opened. That is, the clarinet should be characterized as a multi-delay system. In this paper, we focus on the effect of a very short time delay, which characterizes the reflection from the discontinuity in the mouthpiece. We numerically found that the function of register hole survives even for such a three delay system including the reflection from the mouthpiece.

Keywords: Clarinet, Resister hole, Delayed Model

1 INTRODUCTION

It is well known that wind instruments are modelized by differential and difference equations with time delay [1, 2, 3, 4]. Actually, the time delay is involved in the model as a convolution of the reflection function and the past data of the acoustic pressure $p(t)$ in the mouthpiece and of the volume flow $u(t)$ injected into it [1]. The open end reflection makes a negative peak with a delay time t_o, which is given as $t_o = 2l_p/c_0$, where l_p and c_0 are pipe length and the speed of sound, respectively. If adding a bell to the open end, the peak becomes broad. The tone hole reflection also makes a negative peak with a delay time $t_t = 2l_t/c_0$, where l_t is the distance of the tone hole from the mouthpiece tip [1, 5]. Therefore, the clarinet with some tone holes opened is characterized as a multi-delay system.

The function of the register hole (key) of the clarinet is a long standing problem in the field of musical acoustics [4]. The register hole with a small radius of 1.5mm is used to play in the second register. Namely, it raises the pitch of most first-register notes by a twelfth (19 semitones), i.e., generating third harmonics, when opened: for the B-flat clarinet, each note from D3 (147.5 Hz) to D4♯ (312.5 Hz) in the first register changes to a corresponding note from A4 (442Hz) to A5♯ (936.6 Hz) in the second register. The register hole is placed nearly at a distance of 0.16m from the tip of the mouthpiece for the B Flat clarinet. Since the portion of the reed valve, i.e., sound generator, is separated from the pipe in dynamical models as shown in Figure 1, the effective length between the register hole and the entrance of the pipe is estimated as $l_r \approx 0.14$m by using an acoustic tube model [6, 7, 8].

In the previous study [8], to consider the function of the register hole, we introduced a simple model formed by a closed pipe with only a tone hole (register hole), which is driven by a sound generator attached to the closed end (see Figure 2). Let us introduce the notion of the effective length of the pipe l_o. That is, l_o is adjusted to generate the pitch of a given note in the first register as $l_o = c_0/4f$, where f is the frequency of the note. When $c_0 = 340$ m/s at a temperature of 288 K, l_o is estimated as 0.576 and 0.272 m for the notes D3 and D4♯, respectively. Namely, the register hole works in the range $(1.9 \le l_o/l_r \le 4.1)$. The pipe with the register hole, whose length l_o is changed depending on a given note, is analogous to a two-delay system.

In the previous study [8], we explained the function of the register hole from the common properties of two-delay systems [9, 10]. Let us consider a pipe with a small tone hole, which makes a short time delay with weak intensity(see Figure 2(a)). When the register hole (tone hole) is placed at 1/3 length of the pipe from the closed end (top picture of Figure 2(a)), the third-harmonic mode satisfies the boundary condition of this pipe: antinodes exists at the tone hole and the open end. Even in the cases that the tone hole is placed at the middle of the pipe (middle picture) and at the quarter (bottom picture), the third-harmonic mode can still be excited in the pipe, while the antinode shifts from the tone hole to the left and right, respectively. This is because if the tone hole is sufficiently small, it does not strictly require that one of the antinodes exists on it, but it still stimulates the excitation of the third-harmonic mode rather than the other harmonic modes. This fact is a common property of two-delay systems and explains the function of the register hole [8, 9, 10].

Let us consider the case of pipes with a large tone hole, i.e., the short time delay with high intensity (Figure 2(b)). If the tone hole is placed at 1/3 length of the pipe (top picture), the third-harmonic mode satisfies the boundary condition of this pipe. However, the large tone hole requires that oscillation modes satisfy the boundary condition caused by it: one of the antinodes is placed on it. In the cases shown by the middle and bottom pictures, there is no resonance mode that simultaneously satisfies both boundary conditions of the tone hole and the open end, because they make mismatched boundary conditions for harmonic modes. However, higher harmonic modes, which nearly satisfy both boundary conditions at the tone hole and at the open end, will be accepted reluctantly. This fact is explained by a common property of two-delay systems [8, 9, 10].

Thus, the above discussion indicates that the register hole should be sufficiently small but non-negligible. In the previous work [8], we focused on the theoretical analysis of the common properties of two-delay system by using a general two-delay system and discussed only one example of the clarinet model with a simplified reflection function. In this paper, we treat models including more realistic reflection functions and discuss the problem of how the change of the reflection function affects the function of register hole. Especially, we consider the effect of the mouthpiece reflection with a very short delay, which is caused by the mouthpiece discontinuity and plays an important role in reproducing the complicated mode-transitions observed in the experiment [6, 7, 11].

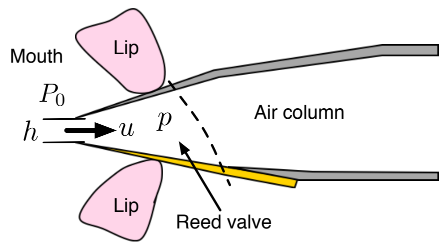

Figure 1. Cross section of the operating portion of the clarinet.

2 MODEL SYSTEMS

2.1 Delayed difference model of the clarinet

In this section, we introduce a delay difference model of the clarinet [2, 3, 8]. The operation of the clarinet is characterized with four dynamical values in Figure 1: the acoustic pressure in the mouthpiece p, the volume flow through the reed slit u, the height of the reed slit h and the blowing pressure in the mouth P_0.

By using the reflection function $r(t)$ [1, 8], the relationship between p and u is given by

$$p = Z_0 u + p_{inc}, \tag{1}$$

where p_{inc} is defined by

$$p_{inc}(t) = \int_0^\infty r(\tau)[p(t-\tau) + Z_0 u(t-\tau)]d\tau, \tag{2}$$

and Z_0 denotes the characteristic impedance defined by $Z_0 = \rho c_0 / S_c$, where ρ, c_0 and S_c are the air density,

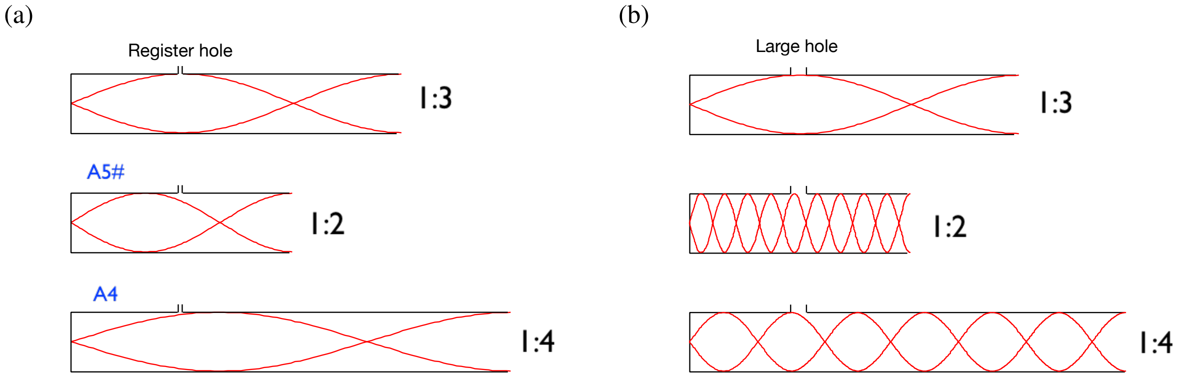

Figure 2. Function of the register hole. (a) Pipes with a small tone hole (register hole). (b) Pipes with a large tone hole.

the speed of sound and the area of the cross section at the entrance, respectively. The cross section S_c can be written as $S_c = \pi r_d^2$, where r_d is the effective radius of the pipe entrance.
Under quasi-static approximation [3, 8], the volume flow passing through the reed slit is obtained with Bernoulli's principle. Since the flow velocity in the mouth is negligibly small, Bernoulli's principle gives

$$P_0 = p + \frac{1}{2}\rho v^2, \tag{3}$$

where v is the flow velocity passing through the reed slit. The volume flow is given as $u = whv$, where w is the width of the reed slit.
The reed is driven by the pressure difference between the mouth and mouthpiece. Since the eigenfrequency of the reed is much larger than the acoustic frequency, the reed displacement $h - h_0$ is almost proportional to the pressure difference $p - P_0$,

$$k(h - h_0) = p - P_0 \equiv -\Delta p, \tag{4}$$

where h_0 is the height of the reed at rest and k is the effective stiffness of the reed. When the reed is closed, i.e., $h = 0$, the pressure is at $p = p_M$, which is obtained from

$$kh_0 = P_0 - p_M \equiv \Delta p_M. \tag{5}$$

From Eqs.(3), (4) and (5), the volume flow passing through the reed slit, $u = whv$, is given by [3, 8]

$$u = \begin{cases} u_0\left(1 - \frac{\Delta p}{\Delta p_M}\right)\sqrt{\frac{\Delta p}{\Delta p_M}} & (\Delta p < \Delta p_M) \\ 0 & (\Delta p \geq \Delta p_M), \end{cases} \tag{6}$$

where u_0 is defined by

$$u_0 = wh_0\sqrt{\frac{2kh_0}{\rho}}. \tag{7}$$

For the case of $\Delta p \geq \Delta p_M$, the reed is closed and no flow enters the mouthpiece, i.e., $u = 0$.
Combining eq.(6) with eq.(1), one can obtain the pressure $p(t)$ and the volume flow $u(t)$ at the present time t, if p_{inc} is calculated from the past values of p and u by eq.(2). To calculate the time evolution of p and u,

we consider the values of p and u at discrete times $t_n = n\Delta t$, namely $p_n = p(t_n)$ and $u_n = u(t_n)$, where Δt is a small time interval, which is taken as $\Delta t = 5\times10^{-6}$s in this paper. Then, p_{inc} at $t = t_n$ is given as

$$p_{inc,n} = \sum_{i=1}^{\infty} r_i(p_{n-i} + Z_0 u_{n-i})\Delta t, \tag{8}$$

and from eq.(6) and eq.(1) with $p_{inc,n}$, p_n and u_n are obtained; thus, $p_{inc,n+1}$ is obtained and the process is repeated over and over. This process is written as a delayed difference equation.

2.2 Reflection functions

In this paper, we use a simplified reflection function with three peaks [8],

$$r(t) = -\tilde{\alpha}_o f_r(t-t_o,\Delta t_o) - \tilde{\alpha}_r f_r(t-t_r,\Delta t_r) - \tilde{\alpha}_m f_r(t-t_m,\Delta t_m), \tag{9}$$

where the function $f_r(t-t',\Delta t)$ represents a reflection peak and is defined by

$$f_r(t-t',\Delta t) = \frac{1}{\Delta t}\exp\left(-\frac{1}{\Delta t}(t-t')\right)H(t-t'), \tag{10}$$

where $H(t)$ is the Heaviside function and Δt determines the width of the peak owing to dispersive reflection. When the direct current resistance of the pipe is ignored, the average pressure in the mouthpiece is equal to the pressure of the atmosphere; thus, the reflection function satisfies the following condition [1]:

$$\int_0^{\infty} r(t)dt = -1, \tag{11}$$

which is reduced into

$$\tilde{\alpha}_r + \tilde{\alpha}_o + \tilde{\alpha}_m = 1. \tag{12}$$

In eq.(9), the peaks at $t = t_o$, t_r and t_m indicate the reflections from the open end, register hole and mouthpiece discontinuity, whose heights are given as $\alpha_o = \tilde{\alpha}_o/\Delta t_o$, $\alpha_r = \tilde{\alpha}_r/\Delta t_r$ and $\alpha_m = \tilde{\alpha}_m/\Delta t_m$, respectively.

In this paper, we calculate three models with different reflection functions. The settings of Δt_o and α_m/α_o for the three models are shown in Table1. But, t_r and Δt_r of the register hole reflection as well as t_m and Δt_m of the mouthpiece reflection are common parameters for the three models as shown in Table2.

The Models 1 and 2 ignoring the mouthpiece reflection are characterized as two-delay systems, though the Model 3 including the mouthpiece reflection is regarded as a three-delay system. The height α_m and width Δt_m of mouthpiece reflection are determined by the reflection function of the mouthpiece model in our previous studies [6, 7]. The peak width of the open end reflection Δt_o is wider for the Models 2 and 3 than the Model 1. The peak width Δt_o of the Model 1 is almost the same as that of the cylindrical pipe without a bell [6, 7]. For the reflection function of the clarinet with a bell, the Models 2 and 3 with a wide peak width Δt_o seem to be more realistic. To consider the function of the register hole, α_r and t_o are set up as follows: α_r is fixed in the range $(0.001 \le \alpha_r/\alpha_o \le 1)$ while t_o is fixed in the range $(t_r \le t_o \le 5.5t_r)$ depending on the effective length of the pipe determined by a given note. Note that the ratio t_o/t_r is in the range $1.9 \le t_o/t_r \le 4.1$ for the real clarinets. Figure 3 shows the reflection functions for the Models 1, 2 and 3 at $\alpha_r/\alpha_o = 0.1$ and $t_o/t_r = 4$.

Table 1. Parameters of the models

Model	Δt_o[s]	α_m/α_o
Model 1	0.4×10^{-4}	0
Model 2	0.8×10^{-4}	0
Model 3	0.8×10^{-4}	1

Table 2. Common parameters of the reflection functions

t_r[s]	Δ_r[s]	t_m[s]	Δt_m[s]
8.25×10^{-4}	0.4×10^{-4}	1.25×10^{-4}	0.8×10^{-5}

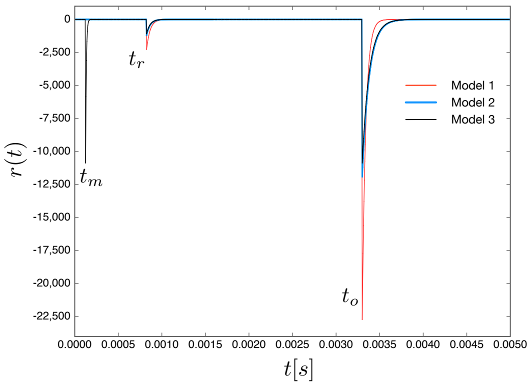

Figure 3. Reflection functions of the clarinet models at $\alpha_r/\alpha_o = 0.1$ and $t_o/t_r = 4$.

3 NUMERICAL CALCULATION

3.1 Setup of numerical calculations

We assume that the first excited mode in the range of the control parameter is the most dominant mode among possibly excited modes for the system, first, third, fifth modes and so on. Thus, we take the blowing pressure P_0 as a control and find the first excited mode, when P_0 is increased from 0 to 10kPa at a rate of 10Pa/s. Other parameter values are shown in Table 3.

Table 3. Parameter values

parameter	value
r_d	7.5×10^{-3}m
ρ	$1.2\text{kg}/\text{m}^3$
c_0	340m/s
w	1.4×10^{-2}m
h_0	6×10^{-4}m
k	12486993.75Pa/m

3.2 Numerical results

Before checking numerical results, let us remember the common properties of two-delay systems. Namely, in a neighborhood of $t_o/t_r = (2m+1)/(2n+1) \geq 1$, i.e., relevant condition, $(2m+1)$th mode is well sustained. However at $t_o/t_r = (2m)/(2n+1)$ or $(2m+1)/(2n)$, i.e., irrelevant condition, $2m$ or $(2m+1)$th mode is prohibited owing to the boundary condition and odd higher modes, which nearly satisfy the boundary conditions, are observed in its neighborhood. When the peak widths are sufficiently large, higher modes disappear and odd lower modes dominate: for example, the first, third and fifth modes appear in wide neighborhoods of $t_o/t_r = 1$, 3 and 5, respectively.

Figure 4 show pressure waves $p(t)$ at $\alpha_r/\alpha_o = 0$ and 0.1 for the Model 2 with $t_o/t_r = 4$ and $P_0 = 3000$Pa. At $\alpha_r/\alpha_o = 0$, the system is regarded as the single-delay system and a wave of the first mode is observed, while, at $\alpha_r/\alpha_o = 0.1$, a wave of the third mode is excited even though the condition $t_o/t_r = 4$ is irrelevant. This means that adding a delay with small intensity at $t = t_r$ stimulates the third mode and it works like a register hole.

Figure 5 shows phase diagrams of the excited modes in the parameter space of t_o/t_r and α_r/α_o for the Models

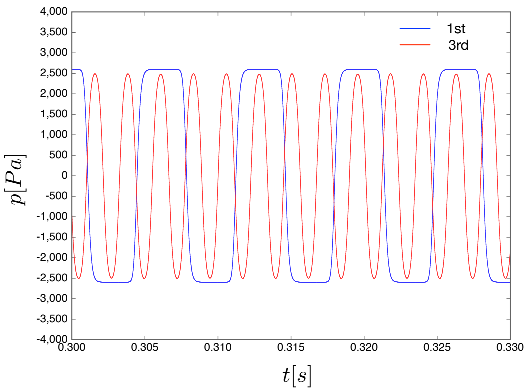

Figure 4. Pressure wave forms of the first harmonic mode at $\alpha_r/\alpha_o = 0$ and of the third harmonic mode at $\alpha_r/\alpha_o = 0.1$ for the Model 2 with $t_o/t_r = 4$ and $P_0 = 3000$Pa.

1, 2 and 3. For the Model 1, at $\alpha_r/\alpha_o = 1$, modes equal to and more than the fifth mode are observed in a neighborhood of the irrelevant condition $t_o/t_r = 2$ and the modes more than the fifth mode are excited near the irrelevant condition $t_o/t_r = 4$, which is explained by the common properties of two-delay systems [8, 9, 10]. With decreasing α_r/α_o, the higher modes near the irrelevant conditions gradually disappear, and at $\alpha_r/\alpha_o = 0.1$, the first, third and fifth modes occupy the ranges $(1.0 \leq t_o/t_r \leq 2.1)$, $(2.1 \leq t_o/t_r \leq 4.2)$ and $(4.2 \leq t_o/t_r \leq 5.5)$, respectively. With decreasing α_r/α_o further, the ranges of the third and fifth modes go to right and at $\alpha_r/\alpha_o = 0.01$ the first mode occupies the whole range. The function of the register is almost achieved at $\alpha_r/\alpha_o = 0.1$, where third modes appear in the range $(2.1 \leq t_o/t_r \leq 4.2)$.

For the Model 2, the areas of the higher modes around the irrelevant conditions shrink. Actually, at $\alpha_r/\alpha_o = 1$, there exists the area of the fifth mode in the left side of $t_o/t_r = 2$, which disappears below $\alpha_r/\alpha_o = 0.7$, while, in the right side, the higher modes appear only at $\alpha_r/\alpha_o = 1$. No higher mode appears around the irrelevant condition $t_o/t_r = 4$. Thus, the peak of the open end reflection with a wider width reduces the higher modes around the irrelevant conditions. At $\alpha_r/\alpha_o = 0.1$, third modes appear in the range $(2.4 \leq t_o/t_r \leq 4.7)$, which is not convenient for archiving the function of the register hole, because the first mode overcomes the third mode in the range $(t_o/t_r < 2.4)$.

For the Model 3, the areas of the higher modes around the irrelevant conditions are enhanced by the mouthpiece reflection. Actually, at $\alpha_r/\alpha_o = 1$, modes equal to and more than the fifth mode are observed in a neighborhood of the irrelevant condition $t_o/t_r = 2$, though no higher mode appears near the irrelevant condition $t_o/t_r = 4$. At $\alpha_r/\alpha_o = 0.1$, third modes appear in the range $(2.2 \leq t_o/t_r \leq 4.3)$. Thus, the mouthpiece reflection moves the area of the third mode to the left and the Model 3 is better than the Model 2 to archive the function of the register hole. As a result, the Model 3 at $\alpha_r/\alpha_o = 0.1$, which is the most realistic model among the three models, almost reproduces the function of the register hole. However, the register hole of the clarinet works in the range $(1.9 \leq t_o/t_r \leq 4.1)$, while for the Model 3, the first mode appears as a first excited oscillation in the range $(1.9 \leq t_o/t_r \leq 2.2)$. In general, the third mode is potentially excited even in this region and may be excited if one properly controls the attack, e.g., adjusting the embouchure. Anyhow, our result gives the answer to the questions why the diameter of the register hole is smaller than those of the other tone holes but not extremely small and why such a small register hole works in the wide range of the register.

4 CONCLUSIONS

In this paper, we investigated the function of the register hole with the models with two and three delays, and found that the basic function of the register hole is explained by the common properties of two-delay systems.

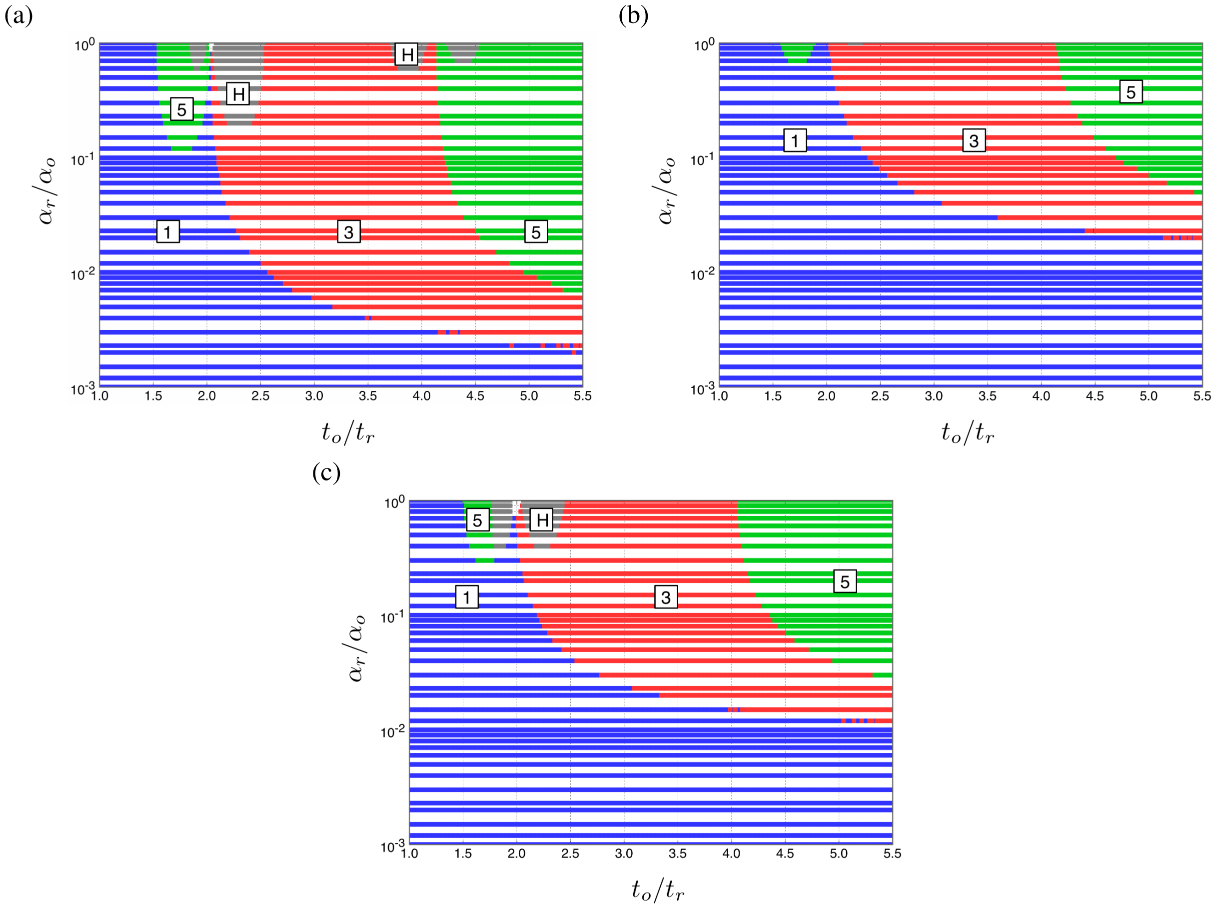

Figure 5. Phase diagrams of the first excited models in the parameter space of t_o/t_r and α_r/α_o. In the regions labeled '1', '3' and '5', first, third and fifth modes are observed, respectively, though modes equal to and more than seventh mode are excited in the regions labeled 'H'. (a) Model 1. (b) Model 2. (c) Model 3.

For the two-delay models, if the strength of the reflection from the register hole is sufficiently small but non-negligible, i.e., $\alpha_r/\alpha_o \approx 0.1$, the third mode is well sustained in the range $(2 \lesssim t_o/t_r \lesssim 4)$. This fact explains the setting of the position and radius of the register hole: the register hole is placed nearly at a quarter of the pipe length from the mouthpiece tip and its radius is about 1.5mm, which is much smaller than those of the other tone holes $(2.5-6.0\text{mm})$. The register hole with a large radius should induce higher modes around the irrelevant conditions $t_o/t_r = 2$ and 4.

The reflection caused by the bell is mimicked by the peak with a wide width, which reduces higher modes near $t_o/t_r = 2$ and 4, but shifts the range of the third mode at $\alpha_r/\alpha_o \approx 0.1$ to a positive direction, which is inconvenient for the register hole. The effect of the mouthpiece reflection with a very short delay is interesting. It induces higher modes around $t_o/t_2 = 2$ when α_r/α_o is large enough, but it shifts the range of the third mode at $\alpha_r/\alpha_o \approx 0.1$ to a negative direction and enhances the function of the register hole.

For the real clarinet, the register hole works in the range $(1.9 \leq t_o/t_r \leq 4.1)$, while for the Model 3 including the mouthpiece reflection, third modes appear in the range $(2.2 \leq t_o/t_r \leq 4.3)$. In general, the third mode is potentially excited even when the first mode dominates and it is expected that the third mode can be excited if properly controlling the attack, e.g., adjusting the embouchure [8]. For the real clarinet, the reflection function is much complicated owing to many opened and closed tone holes and should be characterized as a multiple-delay model [1, 5]. Nevertheless, the two-delay model and three-delay model including the mouthpiece reflection seem to well capture the underlying mechanism of the register hole. We postpone the study of the function of the register hole in terms of multiple-delay model with a real bore for future works.

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